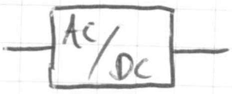
Power Electronics FTA

Rep. / Revision Course

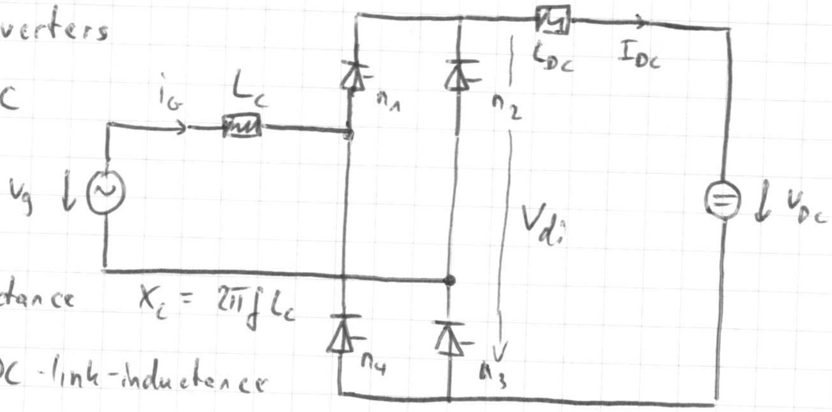
05.02.

Marco Stieneker

Line Commutated Converters



B2C



V_g grid voltage

L_C commutation reactance $X_C = 2\pi f L_C$

L_{DC} smoothing or DC-link-inductence

$X_{DC} = 2\pi f L_{DC}$

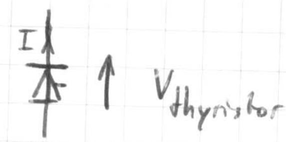
V_{DC} = output voltage

general assumptions

- V_g purely sinusoidal, no internal resistance
- V_{DC} is constant, in steady-state $\overline{V}_{di} = V_{DC}$ $(\overline{V}_{L_{DC}} = 0)$
- n_1 & n_3 are fired at $\omega t = \alpha$
- n_2 & n_4 " " " $\omega t = \alpha + \pi$
- all other parasitic effects are neglected
- perodic operation

Thyristor

- can be fired, if $V_{thyristor} > 0$

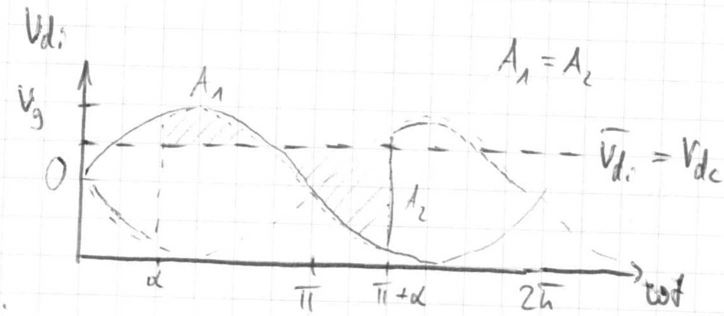


- turns off automatically at zero crossing of the current
- $I_{Thyristor} > 0$ (always!)

① Idealized Calculations

a) $L_C = 0$

b) $L_{DC} \to \infty \Rightarrow I_{DC} = const.$



$$\overline{V}_{di} = \frac{1}{2\pi} \int_0^{2\pi} V_g(\omega t)\, d(\omega t)$$

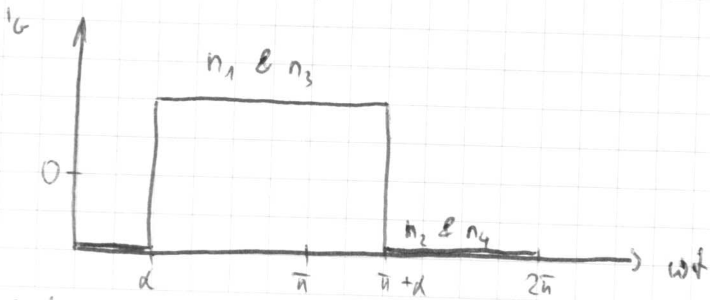
$$= \frac{1}{2\pi} \int_0^{2\pi} \sqrt{2}\, V_g \sin(\omega t)\, d(\omega t)$$

Symmetric

$$\stackrel{\downarrow}{=} \frac{1}{\pi} \int_\alpha^{\alpha+\pi} \sqrt{2}\, V_g \sin(\omega t)\, d(\omega t)$$

$$= \left[\frac{\sqrt{2}}{\pi} V_g \left(-\cos(\omega t)\right)\right]_\alpha^{\alpha+\pi}$$

$$= \frac{\sqrt{2}}{\pi} V_g \left[\cos\alpha - \underbrace{\cos(\alpha+\pi)}_{-\cos\alpha}\right] = \frac{2\sqrt{2}}{\pi} \cdot V_g \cdot \cos\alpha$$



$0° < \alpha < 90° \Rightarrow \bar{V}_{di} > 0$, $I_{DC} > 0 \Rightarrow$ Rectifier

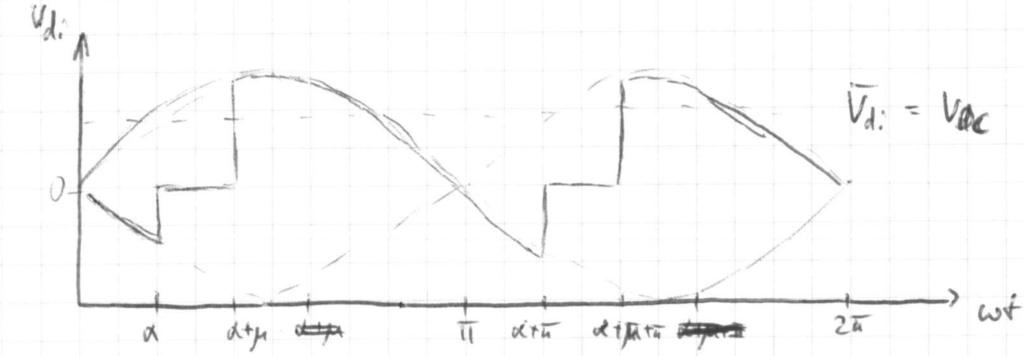
$90° < \alpha < 180° \Rightarrow \bar{V}_{di} < 0$, $I_{DC} > 0 \Rightarrow$ Inverter

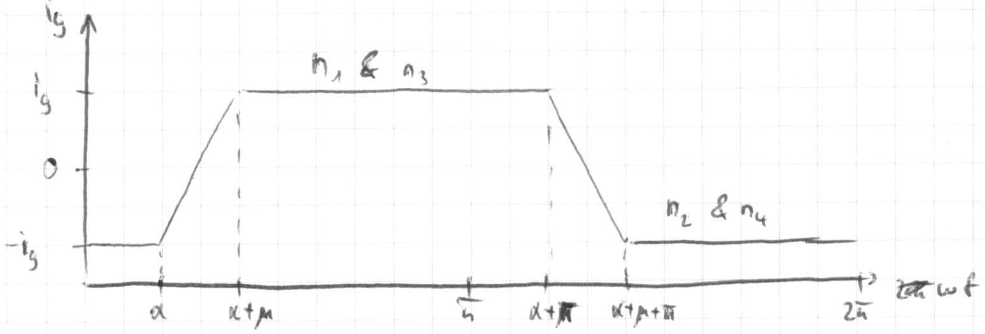
② Conventional Calculations

a) $L_c \neq 0 \Rightarrow$ no instantaneous commutation $\Leftrightarrow$ all thyristors conduct during commutation

$\Rightarrow V_{Lc} = V_g \quad \Rightarrow$ Input of $B_2C = 0V$

b) $L_{DC} \to \infty \Rightarrow I_{DC} = \text{const.}$





Calculation of the Commutation

$$V_{L_c} = L_c \cdot \frac{di_g(t)}{dt} \overset{V_{Lc}=V_g}{\Rightarrow} i_g(t) = \int_{t=\frac{\alpha}{\omega}}^{t=\frac{\alpha+\mu}{\omega}} \frac{V_g(t)}{L_c}\, dt + \underbrace{i_g\left(t = \frac{\alpha}{\omega}\right)}_{-I_{DC}}$$

Integration / Substitution

$y := \omega t \quad y' = \frac{dy}{dt} = \omega \Leftrightarrow dt = \frac{d\omega t}{\omega}$ wenn $y = \omega t \Rightarrow dt = \frac{d\omega t}{\omega}$

$\Rightarrow$ new boundaries multiplied by ω

$$i_g(y) = \int_{\alpha}^{\alpha+\mu} \frac{V_g(y)}{L_c} \frac{dy}{\omega} - I_{DC} \Rightarrow i_g(\omega t) = \int_{\alpha}^{\alpha+\mu} \frac{V_g(\omega t)}{\omega L_c}\, d\omega t - I_{DC}$$

with $i_g(\alpha+\mu) = I_{DC}$

$$\Rightarrow 2 I_{DC} = \int_{\alpha}^{\alpha+\mu} \frac{V_g(\omega t)}{\omega L_c}\, d(\omega t)$$

$\to$ calculation of commutation angle μ

$\to$ " " output voltage

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③ Detailed Calculation

a) $L_c \neq 0$ $L_{DC} \not\to \infty$ ⇒ Exercise II

<u>B6C</u> general assumptions:

- three phase system is balanced
- V_{DC} is constant $\bar{V}_{di\alpha} = V_{DC}$
- $n_1 \dots n_6$ are ideal thyristors

– firing angles: n_1 α, n_2 $\alpha + 60°$, n_3 $\alpha + 120°$, n_4 $\alpha + 180°$, n_5 $\alpha + 240°$, n_6 $\alpha + 300°$

⇒ always two thyristors are on-state $1,2 \to 2,3 \to 3,4 \to 4,5 \to 5,6 \to 6,1 \to 1,2 \dots$

① Idealized Calculations

a) $L_c = 0$

b) $L_{DC} \to \infty$ ⇒ $I_{DC} = \text{const.}$

Period $= 60° \mathrel{\hat{=}} \frac{\pi}{3}$ $V_{di\alpha}(t) = V_+ - V_-$ $\hat{=}$ line to line Voltage

$V_{LL} = \{ V_{12}, V_{23}, V_{31}; -V_{12}, -V_{23}, -V_{31} \}$

$$\bar{V}_{di\alpha} = \frac{1}{T/6} \int_{\alpha/\omega}^{(\alpha+60°)/\omega} V_{12}(t)\,dt = \underbrace{\frac{3\sqrt{2}}{\pi} V_{12}}_{\text{line to line rms}} \cdot \cos\alpha$$

$\alpha < 90°$ ⇒ Rectifier $\alpha > 90°$ Inverter

② Conventional Calculations

a) $L_c \neq 0$ b) $L_{DC} \to \infty$ → compare B2C

$$\bar{V}_{di\alpha} = \frac{3\sqrt{2}}{\pi} V_{LL} \left(\cos\alpha - \frac{X_{Lc} I_{DC}}{\sqrt{2}\, V_{LL}} \right)$$

calculation of the current

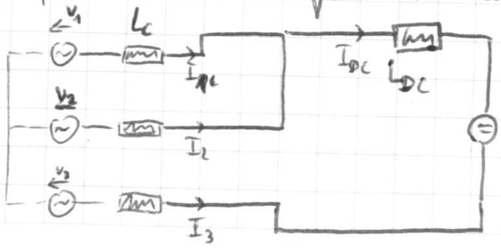
- commutation from n_1 to n_3 $(\omega t = \alpha + 120°)$

n_1 & n_2 are conducting

⇒ $V_+ = V_1$ ⇒ $V_- = V_3$ ⇒ $V_{di} = V_{13}$

$\omega t = \alpha + 120°$ → n_3 is fired

→ new equivalent circuit for $\alpha + 120° \le \omega t \le \alpha + \mu + 120°$



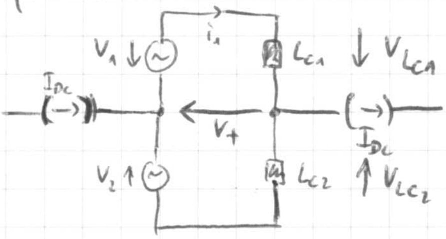
before commutation: $\omega t = \alpha$

$I_1 = I_{DC}$, $I_2 = 0$, $I_3 = -I_{DC}$

after commutation: $\cancel{I_1 >}$ $\omega t = \alpha + \mu$

$I_1 = 0 \quad , \quad I_2 = I_{DC} \quad , \quad I_3 = - I_{DC}$ $\quad \rightarrow$ no voltage drop across L_{c3}, $\frac{dI}{dt} = 0$

new equivalent circuit



$i_1 + i_2 = I_{DC} = \text{const}$

$n_1 \rightarrow n_3 \qquad V_{10}(t) - V_{20}(t) = L_c \frac{di_1(t)}{dt} - L_c \frac{di_2(t)}{dt}$

$$V_{12}(t) = 2 \cdot L_c \frac{di_1(t)}{dt}$$

$$i_2(t) = I_{DC} - i_1(t)$$

$$\Rightarrow V_{12}(\omega t) = 2 X_{L_c} \frac{di_1(\omega t)}{\omega t}$$

$$\Rightarrow i_1(\omega t) = \frac{1}{2X_c} \int_{\alpha}^{\omega t} \sqrt{2}\, V_{grid} \sin(\omega t)\, d(\omega t) = \sqrt{2}\, V_{grid}/2X_c \cdot (\cos\alpha - \cos\omega t)$$

i_1 for a half period

$$i_1(\omega t) = \begin{cases} \frac{\sqrt{2} \cdot V_{grid}}{2 X_c} (\cos\alpha - \cos\omega t) & \alpha < \omega t < \alpha + \mu \\ I_{DC} & \alpha + \mu < \omega t < \alpha + \frac{2\pi}{3} \\ I_{DC} - \frac{\sqrt{2}\, V_{grid}}{2 X_c} \left[\cos \cancel{\alpha}(\alpha \cancel{+\frac{2\pi}{3}}) - \cos\left(\omega t - \alpha + \frac{2\pi}{3}\right)\right] & \alpha + \frac{2\pi}{3} < \omega t < \alpha + \frac{2\pi}{3} + \\ 0 & \text{otherwise} \end{cases}$$

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Exercise B

B.1) Smoothing inductance to reduce the current ripple

B.2) $I_{DC} = 2kA$ $V_{DC} = 260 kV$ $R_{DC} = 0$ $\bar{V}_{L_{DC}} = 0$ $V_{DC} = \bar{V}_{di\alpha} = \frac{3\cdot\sqrt{2}}{\pi}\sqrt{3}\cdot V_{10} \left(\cos\alpha - \frac{\omega L_\sigma \cdot I_{DC}}{\sqrt{3}\cdot\sqrt{2}\cdot V_{10}}\right)$

$V_{dc} = \ldots = \frac{3\sqrt{2}}{\pi}\sqrt{3}\cdot 220kV\left(\cos\alpha - \frac{2\pi\, 50Hz\cdot 1{,}5mH\cdot 2kA}{\sqrt{3}\cdot\sqrt{2}\; 220kV}\right)$

$\hookrightarrow \cos\alpha = 0{,}507 \Rightarrow \alpha = 59{,}536°$

$\cos(\alpha) - \cos(\alpha+\mu) = \frac{2 X_c \cdot I_{DC}}{\sqrt{2}\sqrt{3} V_{10}}$

$\Rightarrow \mu = \cos^{-1}\left(\cos\alpha - \frac{2 X_c I_{DC}}{\sqrt{2}\sqrt{3} V_{10}}\right) - \alpha = 0{,}0232°$

B.3) see diagram

B.4) $\alpha = \cancel{60°}\ 59{,}536°$ $R_{DC} = 0{,}01\,\Omega$

From DC $P_{AC} = V_{DC,r}\cdot I_{DC,r} + I_{DC,r}^2\cdot R_{DC}$

$= 260kV\cdot 2kA + (2kA)^2\cdot 0{,}01\,\Omega = 520.04\,MW$

$P_{AC} = 3\cdot V_{10}\cdot {}^1I_1\cdot\cos\alpha$ $Q = 3\cdot V_{10}\cdot {}^1I_1 \sin\alpha$

${}^1I_1 = \frac{V_{DC,r}\cdot I_{DC,r} + I_{DC,r}^2\cdot R_{DC}}{3 V_{10}\cdot\cos\alpha} = 1.554\,kA$ $= 884.123\,MVar$

B.5) $S = 3\cdot V_{10}\cdot\sqrt{\frac{2}{3}}\cdot I_{DC,r} = 1077\,MVA$

$S^2 = P^2 + Q^2 + D^2$

$D = \sqrt{S^2 - P^2 - Q^2} = 320.386\,MVA$

B.6) see solution

B.7) γ, the hold-off interval

B.8) $R_{DC} \neq 0$ $V_{DC,min} = 0$ $V_{DC,max} = \frac{3\sqrt{2}}{\pi}\sqrt{3}\, V_{10}\left(1 - \frac{\omega L_\sigma\cdot I_{DC}}{\sqrt{3}\sqrt{2}\cdot V_{10}}\right) - R_{DC}\cdot I_{DC}$

$= 513.72\,kV$

B.9) losses should be minimized $(I_{DC}^2\cdot R_{DC})$

$\Rightarrow V_{DC,max}$

$P = V_{DC}\cdot I_{DC} = 500\,MW \Rightarrow I_{DC} = 973.33\,A$

$\Rightarrow P_V = I_{DC}^2\cdot R_{DC} = (973.33A)^2\cdot 0{,}01\,\Omega = 9.473\,kW$

B.10)

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Corrections Day 1

Thyristor Ex. 8

$S_{ac} = \sqrt{3} \cdot U_{LL} \cdot U_{rms}$ ↳ not $\hat{I}$ ↳ $\sqrt{2/3}\, I_{DC}$

Current as in Fig 5.18 (LN) rectangular waveform

$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2(\omega t)\, d\omega t} = \ldots = \sqrt{\frac{2}{3}}\, I_{DC}$

Day 2 DC-DC Converter

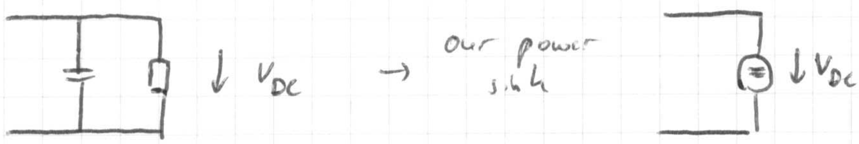
A) Buck Converter B) Boost Converter

Steady State analysis is based on a periodic operation with a period $T_s = \frac{1}{f_s}$

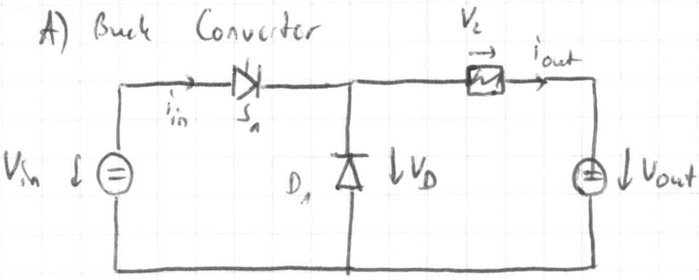
All systems quantities have to be same at $t=0$ and $t=T_s$

$\Rightarrow 0 = \int_0^{T_s} V_L(t)\,dt$ $\Rightarrow 0 = \int_0^{T_s} i_C(t)\,dt$

i.e. real loads



A) Buck Converter



- always: $V_{in} > V_{out}$
- idealized (no losses): $\bar{P}_{in} = \bar{P}_{out}$
- two different operation modes

1) Continous Conduction Mode (CCM)

$0 \le t < dT_s$ d duty cycle S_1 on D off

$\Rightarrow V_L = V_{in} - V_{out} > 0$

$\Rightarrow$ current increases $V_L = L \cdot \frac{di_{out}}{dt}$

$dT_s \le t < T_s$ S_1 off D_1 on

$\Rightarrow V_L = -V_{out} < 0$

$\Rightarrow$ current decreases with $V_L = L \cdot \frac{di_{out}}{dt} < 0$

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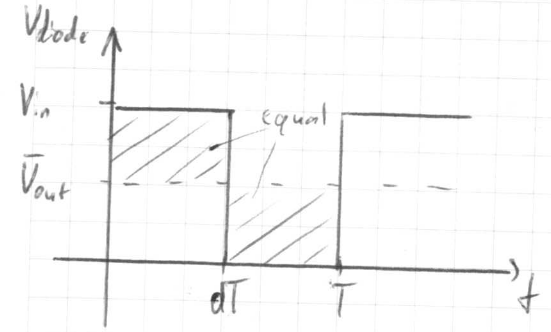
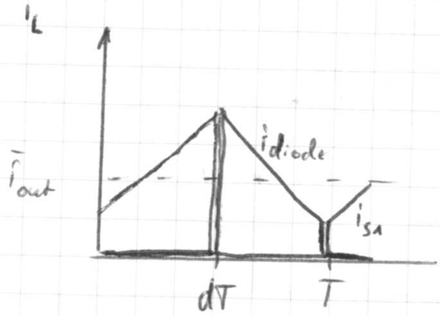




Since $V_L \overset{!}{=} 0 = dT_s(V_{in} - V_{out}) + (1-d)T_s(-V_{out}) = dT_s V_{in} - T_s V_{out}$

$\Rightarrow V_{out} = d V_{in}$ $\qquad \bar P_{in} = \bar P_{out}$

$\Rightarrow \bar i_n = d \bar i_{out}$

2) Discontinuous Conduction Mode (DCM)

→ $i_{out} = 0$ during period

$0 \le t < dT_s$ $\qquad S_1$ on $\quad$ D off

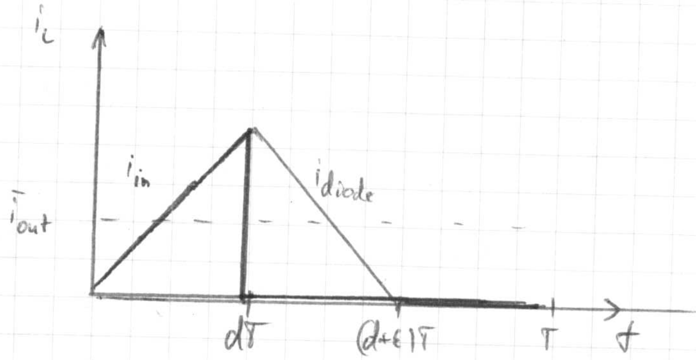
$\qquad V_L = V_{in} - V_{out} \Rightarrow \frac{di_{out}}{dt} > 0$

$dT_s \le t < (t+\varepsilon)T_s$ $\qquad S_1$ off $\quad$ D on

$\qquad V_L = -V_{out} \Rightarrow \frac{di_{out}}{dt} < 0$

$(t+\varepsilon)T_s \le t < T_s$ $\qquad S_1$ off $\quad$ D off

$\qquad V_L = 0V \qquad V_{Diode} = V_{out} \qquad V_{S1} = V_{in} - V_{out} \qquad i_L = 0A$!



Calculation of DCM

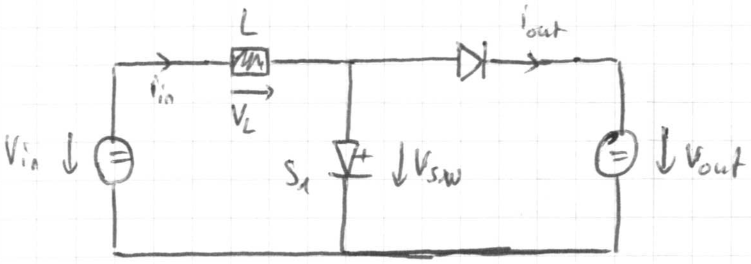
$(V_{in} - V_{out})\, dT_s = V_{out} \cdot \varepsilon T_s \Rightarrow \frac{V_{out}}{V_{in}} = \frac{d}{d+\varepsilon}$ for DCM

$\bar i_{out} = \bar i_L = \frac{\hat i}{2} \underbrace{(d+\varepsilon)}_{<1}$

with $\hat i = V_{out} \frac{\varepsilon T_s}{L} \Rightarrow \bar i_{out} = \frac{V_{out} \cdot \varepsilon \cdot T_s}{2L}(d+\varepsilon) = \frac{V_{in}}{2L} \cdot T \cdot d \cdot \varepsilon$

$\Rightarrow \varepsilon$ is load dependent

Boost Converter



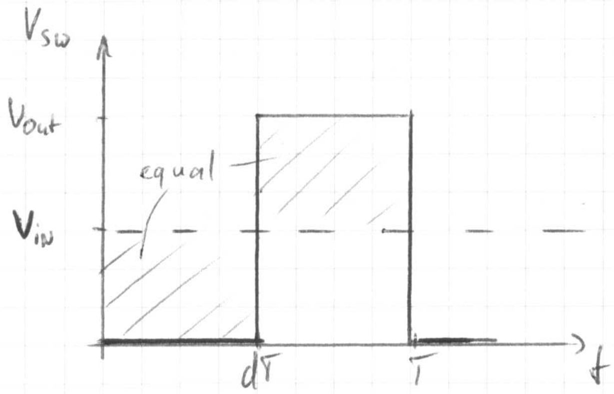
- always: $V_{in} < V_{out}$
- no losses

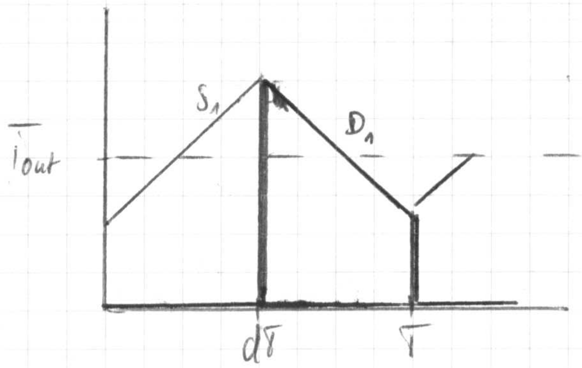
1) <u>CCM</u> $\quad 0 \le t < dT_s \quad$ S_1 on $\quad D$ off

$V_L = V_{in} > 0 \implies \frac{di_{in}}{dt} > 0$

$dT_s \le t < T_s \quad$ S_1 off $\quad D$ on

$\implies V_L = V_{in} - V_{out} < 0 \implies \frac{di_{in}}{dt} < 0$





Since $\bar{V}_L = 0$ in steady state

$d V_{in} + (1-d)(V_{in} - V_{out}) = 0$

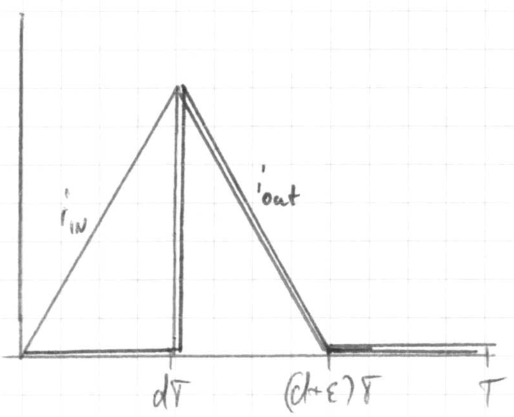
$\Leftrightarrow \quad V_{out} = \frac{1}{1-d} V_{in}$

$\bar{P}_{in} = \bar{P}_{out} \implies \bar{i}_{out} = (1-d)\,\bar{i}_{in}$

2) DCM $\quad 0 < t < dT_s \quad$ S_1 on $\quad D$ off $\quad V_L = V_{in} > 0 \quad \frac{di_L}{dt} > 0$

$dT_s \le t < (d+\varepsilon)T_s \quad$ S_1 off $\quad D$ on $\quad V_L = V_{in} - V_{out} < 0 \quad \frac{di_L}{dt} < 0$

$(d+\varepsilon)T_s \le t < T_s \quad$ S_1 off $\quad D$ off $\quad V_L = 0 \quad i_L = 0 \quad V_{SW} = V_{IN} \quad V_{Diode} = V_{out} - V_{in} < 0$



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$V_{in}\, d\, T_s = (V_{out} - V_{in})\, \varepsilon\, T_s$

$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{d+\varepsilon}{\varepsilon}$

$\bar{i}_{out} = \frac{\hat{i}_{out}}{2}\, \varepsilon$

with $\hat{i}_{out} = \frac{V_{in} \cdot d \cdot T_s}{L}$ $\Rightarrow$ $\bar{i}_{out} = \frac{V_{in} \cdot d \cdot \varepsilon \cdot T}{2 \cdot L}$

Ideal Inductor Model

- Magnetic flux density B
- " field strength H $B = \mu \cdot H$
- permebility $\mu = \mu_r \cdot \mu_0$
- magnetic flux ϕ $\phi = \iint_{A_c} B \cdot dA$

$u_L = -N \cdot \frac{d\phi}{dt} = L \cdot \frac{di}{dt}$

- $W = \frac{1}{2} \cdot L \cdot I^2$
- Number of turns N
- magnetic path length l_m
- cross sectional area of the core A_c
- length of airgap l_{airgap}

$L = \frac{B \cdot A_c \cdot N}{i}$ ~~with~~ (*)

with $\mu_{air} \approx 1$, $\mu_{r,core} \gg 1$

$\Rightarrow L = \underbrace{\frac{A_c \cdot \mu_0}{l_{airgap}}}_{A_L} \cdot N^2$

A_L = characteristic core parameter

- $i = \frac{B \cdot l_{airgap}}{N \cdot \mu_0}$

Design approach

(*) implies that B_{max} is reached for I_{max}

Always $B < B_{sat}$

$k_{cu} = \frac{N \cdot A_{cu}}{A_w}$

According Ex. 5.7

$L = 8\,mH$ $\quad I_{max} = 700\,mA$ $\quad B_{max} = 300\,mT$ $\quad A_c = 146\,mm^2$ $\quad A_w = 75\,mm^2$

$k_{cu} = 0.866$ $\quad D = 0.75\,mm$

	l_{airgap} (mm)	A_L (nH)
1	0	5050
2	0.30	630
3	0.51	400
4	0.68	315

$$N_{max} = \frac{A_w \cdot k_{cu}}{A_{cu}} = \frac{75\,mm^2 \cdot 0{,}866}{\left(\frac{0.75\,mm}{2}\right)^2 \cdot \pi} = 147.01$$

$$L \approx A_L \cdot N^2$$

$$N = \sqrt{\frac{L}{A_L}} \Rightarrow \text{(1) } N = 40 \quad \text{(2) } N = 113 \quad \text{(3) } N = 141 \quad \text{(4) } N = 159$$

Number (4) is not possible because of N_{max}

<u>Ex. 5.8</u>

$$I_{max} \approx \frac{B_{max} \cdot l_{airgap}}{N \cdot \mu_0} \quad \text{with} \quad A_L = \frac{A_c \cdot \mu_0}{l_{airgap}}$$

$$\approx \frac{B_{max} \cdot l_{airgap} \cdot A_c}{N \cdot A_L \cdot l_{airgap}} = \frac{B_{max} \cdot A_e}{N \cdot A_L}$$

(1) $I_{max} = \frac{300\,mT \cdot 146\,mm^2}{40 \cdot 5050\,nH} = 217\,mA$

(2) $I_{max} = \frac{300\,mT \cdot 146\,mm^2}{113 \cdot 630\,nH} = 615\,mA$

(3) $I_{max} = \frac{300\,mT \cdot 146\,mm^2}{141 \cdot 400\,nH} \approx 777\,mA \Rightarrow$ (3)

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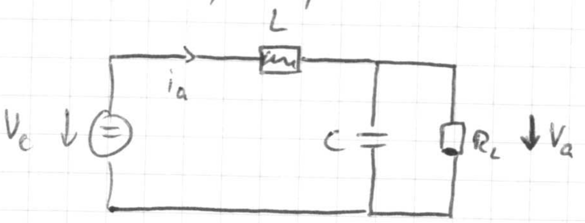
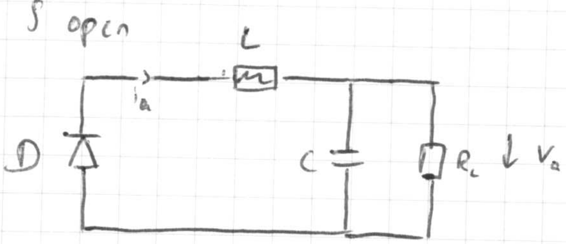
Exercise F) F.1) S closed



S open



~~F.2) steady-state~~ $\cancel{\bar{I}_a = \frac{V_a}{R_L}}$

F.2) S closed $\frac{di_a}{dt} = \frac{V_e - V_a}{L}$ S open $\frac{di_a}{dt} = -\frac{V_a}{L}$

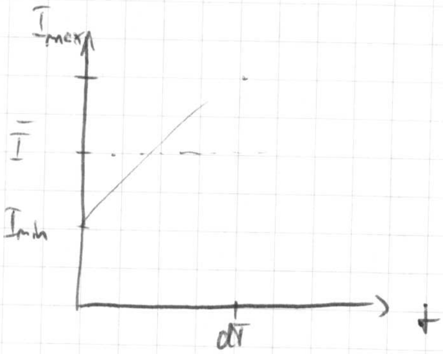
F.3) steady state $\bar{I}_a = \frac{V_a}{R_L}$

$I_{a,min} > 0$ (CCM)

$I_{a,min} = \frac{V_a}{R_L} - \left(\frac{1}{2} \frac{di_a}{dt} \cdot dT\right)$

with $d = \frac{V_a}{V_e}$

$\Rightarrow I_{a,min} = \frac{V_a}{R_L} - \frac{1}{2}\left(\frac{V_e - V_a}{L} \frac{V_a}{V_e} \cdot T\right)$



F.4)(I) $V_e \to L$; $V_e \to R$

(II) $V_e \to L$; $V_e \to R$; $V_e \to C$

(III) $L \to C$; $L \to R$

(IV) $L \to R$; $C \to R$

F.5) $P = W_{tot} \cdot \frac{1}{T}$

F.6) $P_{out} = const$ $\eta = 50\%$

$P_{tot} = P_{loss} + \frac{V_a^2}{R_L}$ $P_{loss} = \frac{V_a^2}{R_L}$

$W_{tot} \cdot \frac{1}{T} = P_{loss} \approx \frac{V_a^2}{R_L}$ $\Leftrightarrow$ $f_{50\%} = \frac{V_a^2}{R_L} \cdot \frac{1}{W_{tot}}$

F.7) MOSFET conduction losses

Inductance copper losses

Hysteresis "

Half-Bridge Inverter

S_1 on $\quad S_2$ off $\quad \Rightarrow \quad V_{pwm}(t) = \frac{V_{DC}}{2}$

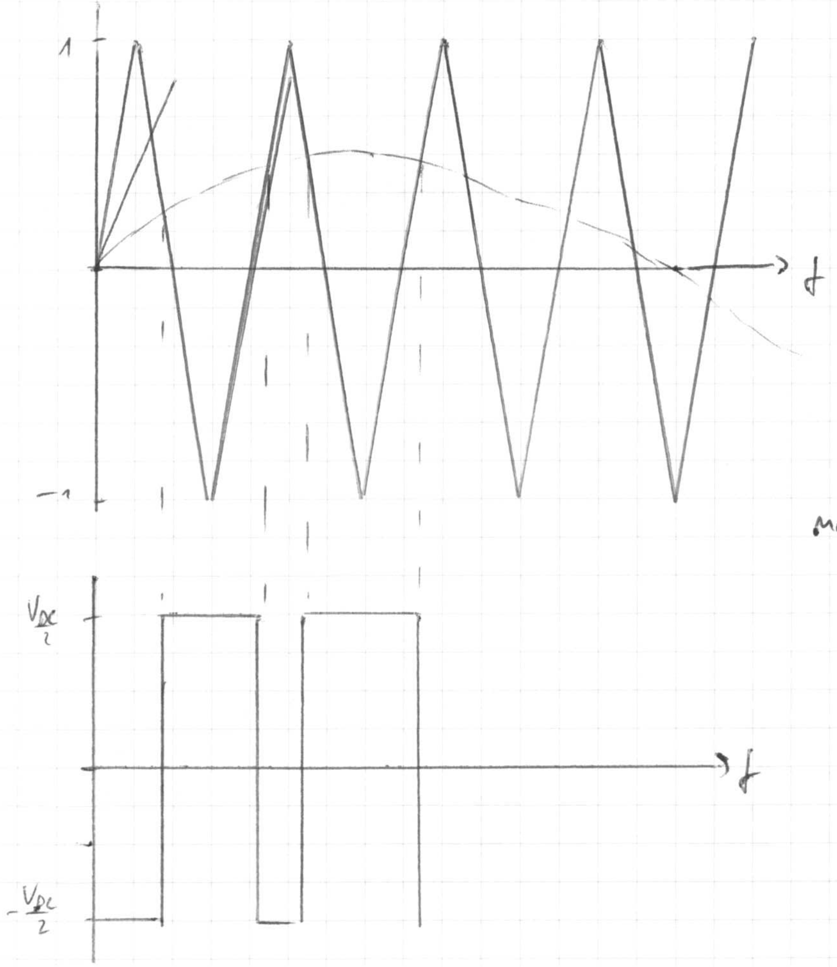
S_1 off $\quad S_2$ on $\quad \Rightarrow \quad V_{pwm}(t) = -\frac{V_{DC}}{2}$

regardless of $i_{out}(t)$

Generation of $v_{pwm}(t)$ by switching S_1 & S_2 according to modulation scheme.

Target: $\bar{v}_{pwm}(t)$ identical with $v^*_{load}(t)$ (AC or DC)

$m<1$



$f_{sw} \gg f$

$V_{load} > V_{carrier}$

$\Rightarrow V_{pwm}(t) = \frac{V_{DC}}{2}$

$V_{load} < V_{carrier}$

$\Rightarrow V_{pwm}(t) = -\frac{V_{DC}}{2}$

if $f_{sw} \gg f \Rightarrow \bar{v}_{pwm} = v^*_{load}$

modulation index $m = \frac{\hat{v}^*_{load}}{\frac{V_{DC}}{2}}$

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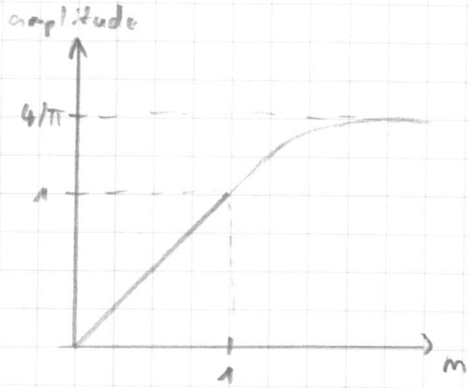
- Overmodulation: $m > 1$ (see Lecture Notes) } schemes in LE
- Overmodulation: square wave operation }
- $m < 1$ $m > 1$

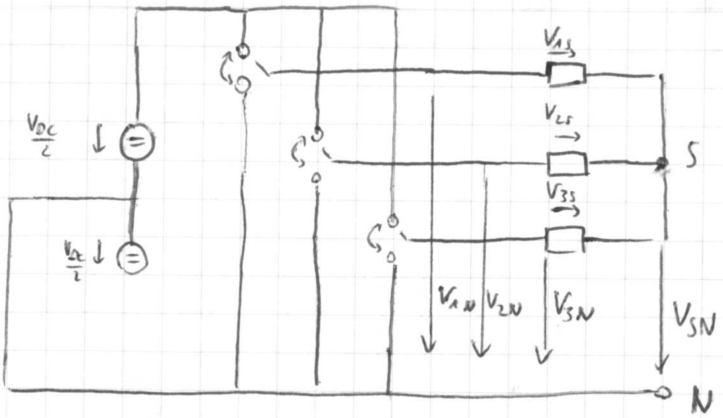
easy to filter difficult to filter



$$\hat{v}_{PWM}(t) = \frac{2}{\pi}\int_0^{\pi} \frac{V_{DC}}{2} \sin\gamma \, d\gamma$$
$$= \frac{4}{\pi} \cdot \frac{V_{DC}}{2}$$

$$\frac{\frac{4}{\pi}\cdot\frac{V_{DC}}{2}}{\frac{V_{DC}}{2}} = \frac{4}{\pi} \approx 1.3$$

Space Vector Modulation



Following voltages have to be generated

$V_{1S}^{*} = \hat{v} \cdot \cos(\omega t)$ → symmetric system

$V_{2S}^{*} = \hat{v} \cdot \cos(\omega t - \frac{2\pi}{3})$ → transformation onto two axis system

$V_{3S}^{*} = \hat{v} \cdot \cos(\omega t - \frac{4\pi}{3})$

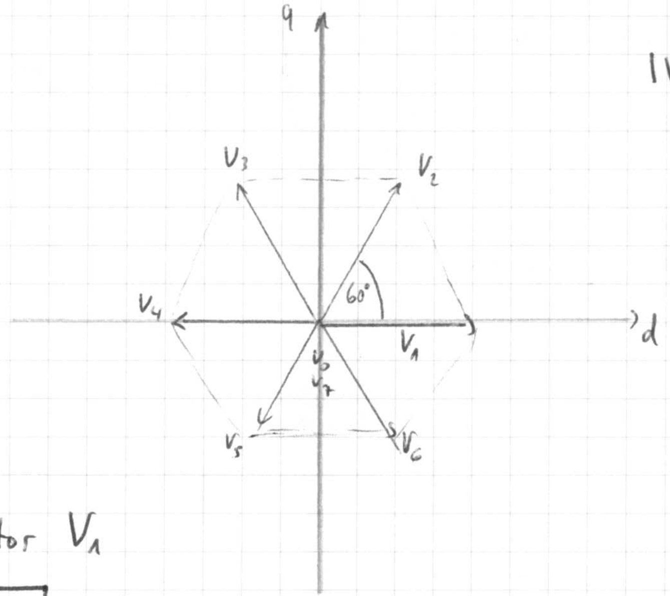
Def: $\underline{v}(t) = \frac{2}{3}\left(V_{1S}(t) + \underline{a}\, V_{2S}(t) + \underline{a}^2 V_{3S}\right)$

$\underline{a} = e^{j\frac{2\pi}{3}}$ $e^{j\varphi} = \cos(\varphi) + j\sin(\varphi)$

$V_d(t) + j\, V_q(t)$

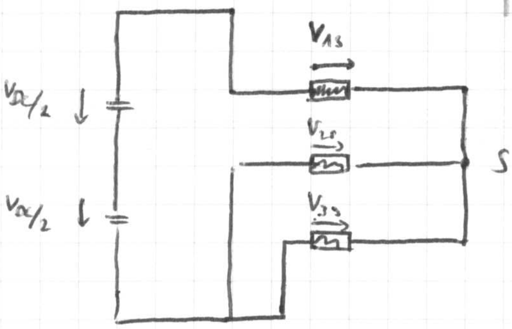
$$\Rightarrow \begin{pmatrix} V_d \\ V_q \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} V_{1S} \\ V_{2S} \\ V_{3S} \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix}\begin{pmatrix} V_{1S} \\ V_{2S} \\ V_{3S} \end{pmatrix}$$

$V_{1N}, V_{2N}, V_{3N} = \begin{cases} V_{DC}/2 \\ -V_{DC}/2 \end{cases}$ $\Rightarrow 2^3 = 8$ states ⟨ 6 active, 2 zero

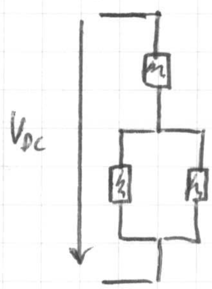


$$|V| = \frac{2}{3} V_{DC}$$

State Vector V_1



$\frac{V_{1S}}{V_{DC}}$	$\frac{V_{2S}}{V_{DC}}$	$\frac{V_{3S}}{V_{DC}}$
$2/3$	$-1/3$	$-1/3$



$$\begin{pmatrix} V_d \\ V_q \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\frac{\sqrt{3}}{2} \end{pmatrix} \cdot \begin{pmatrix} 2/3\, V_{DC} \\ -1/3\, V_{DC} \\ -1/3\, V_{DC} \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 2/3\, V_{DC} \\ 1/3\, V_{DC} \\ -1/3\, V_{DC} \end{pmatrix} \Rightarrow V_d = \frac{2}{3} V_{DC}$$

$$V_q = 0$$

$\Rightarrow$ Radius of inner circle gives the maximum amplitude

$$|\hat{V}_{max}| = \sqrt{(2/3\, V_{DC})^2 - (1/3\, V_{DC})^2} = \frac{1}{\sqrt{3}} V_{DC} = 1.15\, V_{DC}$$

$\Rightarrow$ sine triangle modulation

$$|\hat{V}_{max}| = \frac{V_{DC}}{2} \quad \text{(without zero - sequence)}$$

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Exercise K K.1) I, II, III, IV

K.2) $m \leq 1 = \frac{\hat{V}_{10}}{\frac{V_{DC}}{2}}$

$\Rightarrow V_{DC,min} = \sqrt{2} \cdot 2 \cdot V_{10} = 650.53\,V$

$\Rightarrow V_{bat} = \frac{V_{DC}}{30} = 21.68\,V$

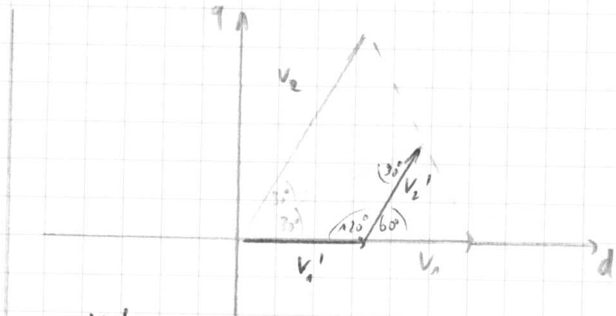
K.3) Harmonics content of the output

K.4) $V_{DC,min} = \sqrt{3} \cdot \sqrt{2} \cdot 230\,V = 563.38\,V$

$\Rightarrow V_{bat} = \frac{V_{DC,min}}{30} = 18,78\,V$

$m = 1 \Rightarrow$ vector v^* is on the inner circle

K.5) $\varphi^* = 30°$ $|V^*| = \sqrt{2} \cdot 230\,V$ $|V_1| = |V_2| = 720\,V \cdot \frac{2}{3}$ $f_s = \frac{1}{T_s} = 5\,kHz$



Projection of v_s^* on V_1 & V_2

$\Rightarrow$ law of sine

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\frac{V^+}{\sin 120°} = \frac{V_2'}{\sin 30°} \Rightarrow V_2' = V^+ \frac{\sin 30°}{\sin 120°} = 187.79\,V$

$\frac{V^+}{\sin 120°} = \frac{V_1'}{\sin 30°} \Rightarrow V_1' = V^+ \frac{\sin 30°}{\sin 120°} = 187.79\,V$

Duty cycle $T_1 = \frac{V_1'}{V_1} \cdot \frac{T_s}{2} = \frac{187.79\,V}{\frac{2}{3}\,720\,V} \cdot \frac{200\,\mu s}{2} = 39.1\,\mu s$

$T_2 = \frac{V_2'}{V_2} \cdot \frac{T_s}{2} = \ldots = 39.1\,\mu s$

$\frac{T_s}{2} = T_1 + T_2 + T_0 + T_7$

$\Rightarrow T_7 + T_0 = 21,8\,\mu s \Rightarrow T_0 = T_7 = 10.9\,\mu s$