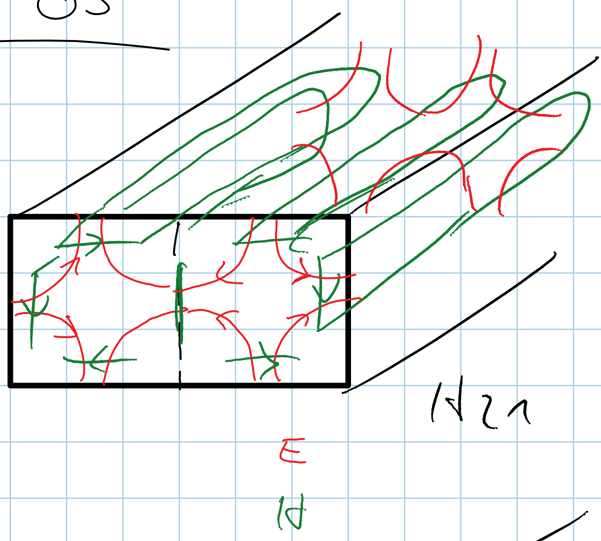
# RÜ 7

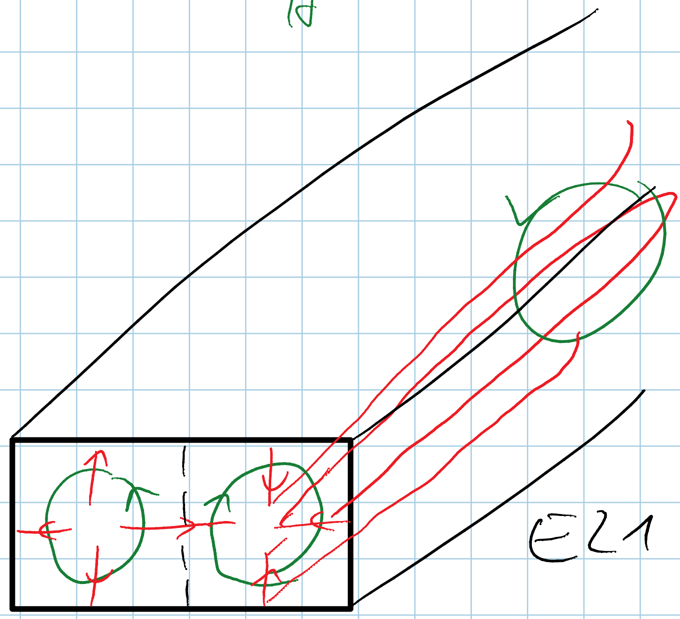
Dienstag, 24. Mai 2011
16:02

## Herbst 05

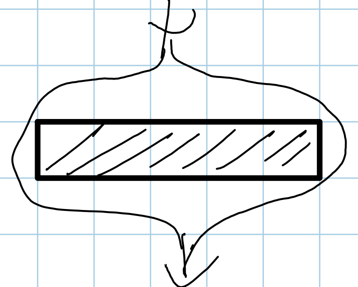
### 1.5

1.1)

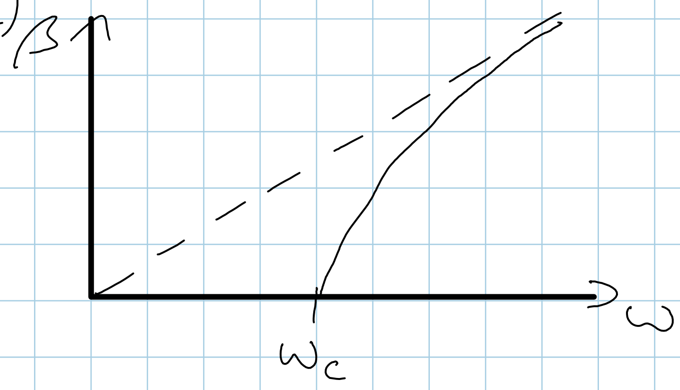



1.2) Oberflächenströme müssen senkrecht zum Schlitz verlaufen



**1.3)**




$$\beta = \sqrt{\left(\frac{\omega}{c_0}\right)^2 \cdot \varepsilon_r \mu_r - p_\vartheta^2}$$

mit $p_\vartheta = \pi \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{\frac{1}{2}}$

**2.1)** $f_{c,H10} = 5\,\mathrm{GHz}$ $\qquad \frac{a}{b} = 2$

$$p_\vartheta = \pi \frac{1}{a}$$

$$f_{c,H10} = \frac{p_{H10}\, c_0}{2\pi \sqrt{\varepsilon_r \mu_r}} = \frac{\pi\, c_0}{2\pi a} \overset{!}{=} 5\,\mathrm{GHz}$$

$$\Rightarrow a = 0{,}03\,\mathrm{m} = 3\,\mathrm{cm}$$

$$\Rightarrow b = 1{,}5\,\mathrm{cm}$$

**2.2)** $k^2 = p^2 + \beta^2$

$$\Rightarrow \left(\frac{2\pi f_0}{c_0}\right)^2 = \left(\frac{2\pi}{2a}\right)^2 + \left(\frac{2\pi}{\lambda_H}\right)^2$$

mit $a = \frac{\lambda_H}{2}$ folgt

$$\left(\frac{2\pi f_0}{c_0}\right)^2 = \left(\frac{2\pi}{2a}\right)^2 + \left(\frac{2\pi}{2a}\right)^2$$

$$f_0 = c_0 \sqrt{\left(\frac{1}{2a}\right)^2 + \left(\frac{1}{2a}\right)^2} = \frac{c_0}{2a}\sqrt{2}$$

$$= f_{c,H_{10}} \cdot \sqrt{2} = 3{,}031\ \mathrm{GHz}$$

3.1) $$P = \frac{1}{2}\int_A (E \times H^*)\, d\vec{A}$$

$$= \frac{1}{2}\int_0^b \int_0^a \frac{|E_y|^2}{Z_{F H_{10}}}\, dx\, dy$$

$$|E| = |E_y|$$

$$|H| = \frac{|E_y|}{Z_{F H_{10}}}$$

$$|E_y| = |E_{max}|\, |\sin(\pi \tfrac{x}{a})|$$

$$\Rightarrow P = \frac{1}{2}\int_0^b \int_0^a \frac{|E_{max}|^2 \sin^2(\pi \tfrac{x}{a})}{Z_{F H_{10}}}\, dx\, dy$$

$$= \frac{|E_{max}|^2\, a\, b}{4 \cdot Z_{F,H_{10}}}$$

mit $$Z_{F H_{10}} = \frac{Z_0}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = Z_0\sqrt{2}$$

$$= \frac{|E_{max}|^2\, a\, b}{4 \cdot Z_0 \sqrt{2}}$$

mit

$$|E_{max}| = \frac{2\pi f_0 \mu_0}{\pi/a} \cdot |H_{z\,max}| = 533{,}14 \, \frac{V}{m}$$

$$\Rightarrow P \approx 60\,mW$$

$$|E_{max}| = \left| -\frac{j\omega\mu}{\pi/a} F \right| = \frac{2\pi f_0 \mu_0}{\pi/a} |H_{z,max}|$$

3.2) $|H_x| \overset{!}{=} |H_z|$

$$\underbrace{\frac{\frac{2\pi}{\lambda_H}}{\pi/a}}_{=1} \quad a = \frac{\lambda_H}{2}$$

$$\frac{\frac{2\pi}{\lambda_H}}{\pi/a} |H_{z,max}| \cdot |\sin(\tfrac{\pi}{a} x)| = |H_{z\,max}| \cdot |\cos(\tfrac{\pi}{a} x)|$$

$$\Rightarrow |\sin(\tfrac{\pi}{a} x)| \overset{!}{=} |\cos(\tfrac{\pi}{a} x)|$$

$$\Rightarrow x_1 = \frac{1}{4} a \qquad x_2 = \frac{3}{4} a$$

3.3) $f_0' > f_0$

$\Rightarrow$ $|H_z|$ bleibt konstant

$\Rightarrow$ $|H_x| \sim \beta \sim f$ d.h. $|H_x|$ steigt an jedem Ort

a) Somit die Ebenen, in den $|H_z| = |H_x|$ gilt, verschieben sich in Richtung der Wände

# Herbst 2020

## A. 2

2.1) $a = 47{,}549\,mm$

$$\frac{a}{b} = 2{,}147$$

2.1.1) $K \to \infty$

a) $\varepsilon_r = 1 \to f_{cHmn} = \frac{1}{2\sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$$f_{cH10} = \frac{c_0}{2a} \Rightarrow 3{,}1546\ GHz$$

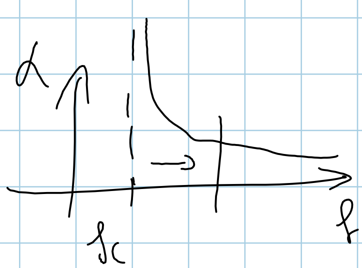
$$f_{cH20} = 2 \cdot \underset{(\varepsilon_r=1)}{f_{cH10}} = 6{,}3093\ GHz$$

b) $\varepsilon_r = 4 \Rightarrow f_{cH10} = \frac{f_{cH10}}{\sqrt{\varepsilon_{r_2}}} = 1{,}5773\ GHz$

$$f_{cH20} = 3{,}1547\ GHz$$

## A. 2.1.2

$f_{cH10} < f < f_{cH20}$




$\longrightarrow f \to f_{cH10}$ steigt die Dämpfung $\alpha_{H10}$ stark an

$f \to f_{cH20} \to$ bereits ist die

$H_{20}$-Mode ausbreitungsfähig, da die Abmessungen a & b des Hohlleiters vergrößern sich wenn die $H_{20}$ Mode in die Wände eindringen kann.

2.2.1) $f = 3\ GHz$

$$|H_3|_{max} = 1{,}9644\ A/m$$

$$\alpha_{M,n} = P_{M,n} \sqrt{1 - \left(\frac{f}{f_{c\,mn}}\right)^2}$$

$$\alpha_{H10} = \frac{\pi}{a} \cdot \sqrt{1 - \left(\frac{3}{3{,}15}\right)^2} = 20{,}4334\ \frac{Np}{m}$$

2.2.2)

$$\underline{\vec{S}} = \underline{\vec{E}} \times \underline{\vec{H}}^*$$

$\underline{H_{10}}$

$$\underline{E}_x = 0$$

$$\underline{E}_y = -j \frac{\omega \mu_0}{(\pi/a)} \cdot F \cdot \sin\left(\frac{\pi}{a} x\right) e^{-\alpha z}$$

$$\underline{H}_y = 0$$

$$\underline{H}_x = -\frac{\alpha}{(\pi/a)^2} F \cdot \frac{\pi}{a} \sin\left(\frac{\pi}{a} x\right) e^{-\alpha z}$$

$$\underline{S}_z = \underline{E} \cdot \underline{H}^* \, (\vec{e}_y \times \vec{e}_x)$$

$$= \frac{j\omega\mu_0\alpha}{(\pi/a)^2} \cdot |H_z|^2_{max} \sin^2\left(\frac{\pi}{a}x\right) e^{-\alpha 2 z} \vec{e}_z$$

2.2.3) Blindleistung $\Rightarrow Q(z=0) = \frac{1}{2} \mathrm{Im}\left\{ \iint_A \underline{S} \, d\vec{A} \right\}$

$$Q(z=0) = \frac{1}{2} \frac{\omega\mu_0\alpha}{(\pi/a)^2} |H_{zmax}|^2 \cdot \int_0^a \int_0^b \sin^2\left(\frac{\pi}{a}x\right) dx\, dy$$

$$\Rightarrow \frac{1}{2} \frac{\omega\mu_0\alpha}{(\pi/a)^2} |H_z|^2_{max} \cdot b \int_0^a \sin^2\left(\frac{\pi}{a}x\right) dx$$

$$\Rightarrow \int \sin^2(kx)\, dx = \frac{x}{2} - \frac{1}{4k}\sin(2kx)$$

$$Q(z=0) = \frac{ab}{4} \frac{\omega\mu_0\alpha}{(\pi/a)^2} \cdot |H_{zmax}|^2 = 0{,}1127\,W$$

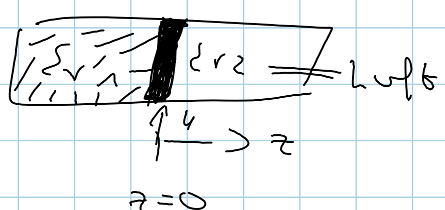
2.2.4)

$$Q(z=\delta) = \cancel{Q(z=0)}\, e^{-2}$$

$$= \cancel{Q(z=0)}\, e^{-2\alpha\delta}$$

$$e^{-2} = e^{-2\alpha\delta} \Rightarrow \delta = \frac{1}{\alpha} = 0{,}049\,m$$

2.3) $\varepsilon_{yp}, \varepsilon_{yr}, \varepsilon_{yt}$




2.3.1)

Grenzbedingung $E_{tan} \rightarrow$ stetig

$$\Rightarrow E_{yp} + E_{yr} = E_{yt}$$

$$\Rightarrow 1 + r = t$$

2.3.2)

$$\alpha = 20{,}4334 \ Np/m$$

2.3.3)

$$Z_{F1} = \frac{\omega\mu}{\beta} = Z_0 \frac{1}{\sqrt{\varepsilon_r}} \cdot \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$= 221{,}593 \ \Omega$$

$$Z_{F2} = \frac{j\omega\mu_0}{\alpha} = j\, Z_0 \frac{1}{\sqrt{\left(\frac{f_{c,H10}}{f}\right)^2 - 1}} = j\, 11592 \ \Omega$$

2.3.4)

$$r = \frac{Z_{F2} - Z_{F1}}{Z_{F2} + Z_{F1}} = 0{,}9295 + j\, 0{,}3688$$

$$|r| = 1$$