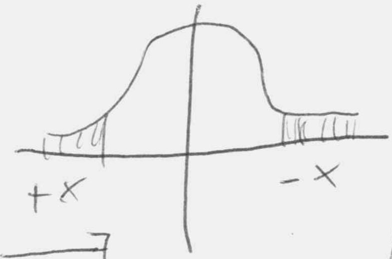
1 Stat. (Disk.) 7 15.06.09



$P(X \leq x) = \phi(x)$

$P(X \leq -x) = \phi(-x)$

Beispiel: 1)

$$P(X \leq 2{,}5) = P\Bigg(\underbrace{\frac{X-1}{2}}_{W(0,1)} \leq \frac{2{,}5-1}{2}\Bigg)$$

$$= \phi\left(\frac{2{,}5-1}{2}\right)$$

aus Tabelle ablesbar

$$= \phi(0{,}75) = 0{,}773373$$

2) $P(X \leq a) = 0{,}85$

$$\Leftrightarrow P\Bigg(\underbrace{\frac{X-1}{2}}_{\sim W(0,1)} \leq \frac{a-1}{2}\Bigg) = 0{,}85 \qquad \Leftrightarrow \phi\left(\frac{a-1}{2}\right) = 0{,}85$$

$$\Leftrightarrow \frac{a-1}{2} = \phi^{-1}(0{,}85)$$

$$\Leftrightarrow a = 2\underbrace{\phi^{-1}(0{,}85)}_{\text{Tabelle} = 1{,}04} + 1 \qquad \Rightarrow a = 3{,}08$$

$\boxed{P19}$

$x \leq 0$ oder $Y \leq 5$ | $0 < x < 12$ und $5 < y < 8$

$0 < x < 12$ und $Y \geq 8$ | $X \geq 12$ und $5 < y < 8$

$x \geq 12$ und $y \geq 8$

P 19

a) $x \leq 0$ oder $Y \leq 5$

$$G_{(X,Y)}(x,y) \equiv G(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y}\underbrace{g(s,t)}_{=0}\,dt\,ds$$

$$= 0$$

P 19

(2)

a) F.

- $0 < x < 12$ und $5 < y < 8$

$$G(x,y) = \int_0^x \int_5^y \underbrace{g(s,t)}_{1/36} \, dt\, ds = \int_0^x 1/36\,(y-5)\, ds$$

$$= 1/36\, x\,(y-5)$$

$0 < x < 12$ und $y \geq 8$

$$G(x,y) = \int_0^x \int_5^8 g(s,t)\, dt\, ds = \dots = \frac{1}{12} x$$

$x \geq 12$ und $5 < y < 8$

$$G(x,y) = \int_0^{12} \int_5^y g(s,t)\, dt\, ds = \dots = \frac{1}{3}(y-5)$$

$x \geq 12$ und $y \geq 8$

$$G(x,y) = \int_0^{12} \int_5^8 g(s,t)\, dt\, ds = \dots = 1$$

$\Rightarrow$ Also

$$G(x,y) = P(X \leq x, Y \leq y) = \begin{cases} 0 & , x \leq 0 \text{ oder } y \leq 5 \\ 1/36\, x\,(y-5) & , 0 < x < 12 \text{ und } 5 < y < 8 \\ 1/12\, x & , 0 < x < 12 \text{ und } y \geq 8 \\ \frac{1}{3}(y-5) & , x \geq 12 \text{ und } 5 < y < 8 \\ 1 & , x \geq 12 \text{ und } y \geq 8 \end{cases}$$

b). $P(X \leq 5, Y \leq 7) \overset{\text{Def.}}{=} G(5,7) = \frac{1}{36}(7-5)\cdot 5 = \frac{5}{18}$ (3)

$\cdot\ P(1 < X \leq 5, 5 < Y \leq 6{,}5) = \int_1^5 \int_5^{6,5} \frac{1}{36}\, dy\, dx = \frac{1}{6}$

$\cdot\ P(X \leq 5, 6 < Y \leq 7) = \int_0^5 \int_6^7 \frac{1}{36}\, dy\, dx = \frac{5}{36}$

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c). $g_X(x,y) = \int_{-\infty}^{\infty} g(x,y)\, dy = \int_5^8 \frac{1}{36}\, dy = \frac{1}{12}$

$\Rightarrow g_X(x,y) = \begin{cases} \frac{1}{12}, & x \in (0,12) \\ 0, & x \notin (0,12) \end{cases}$

$\cdot\ g_Y(x,y) = \int_{-\infty}^{\infty} g(x,y)\, dx = \int_0^{12} \frac{1}{36}\, dx = \frac{1}{3}$

$\Rightarrow g_Y(x,y) = \begin{cases} \frac{1}{3}, & y \in (5,8) \\ 0, & y \notin (5,8) \end{cases}$

P20

$X, Y \sim U[12,13]$, d.h.

$f_X(x) = \begin{cases} 1, & x \in [12,13] \\ 0, & \text{sonst.} \end{cases}$

$f_Y(y) = \begin{cases} 1, & y \in [12,13] \\ 0, & \text{sonst} \end{cases}$

$f_{X,Y}(x,y) \underset{X,Y \text{ st. unabh.}}{=} f_X(x) \cdot f_Y(y) = \begin{cases} 1, & x,y \in [12,13] \\ 0, & \text{sonst.} \end{cases}$

P20

(4)

a) $P(12 < X \le 12{,}5 \,,\, 12 < Y \le 12{,}5) \overset{!}{\underset{}{=}}$ (X, Y st. unabh.) $P(X \le 12{,}5) \cdot P(Y \le 12{,}5)$

$$= \int_{12}^{12{,}5} 1\,dx \cdot \int_{12}^{12{,}5} 1\,dy = \frac{1}{4}$$

b) $P(X \le 12{,}25 \,,\, Y \ge 12{,}75) \underset{!}{=} P(X \le 12{,}25) \cdot P(Y \ge 12{,}75)$

X, Y st. un.

$$= \int_{12}^{12{,}25} 1\,dx \cdot \int_{12{,}75}^{13} 1\,dy = \frac{1}{16}$$

c) $P(X < Y) = 1 - P(X \ge Y)$

$$= 1 - P(X > Y)$$

X, Y ~~st. un.~~ besitzen gleiche symm. Vert.

$$= 1 - P(X < Y)$$

$$\Leftrightarrow \quad 2P(X < Y) = 1$$

$$\Leftrightarrow \quad P(X < Y) = \frac{1}{2}$$