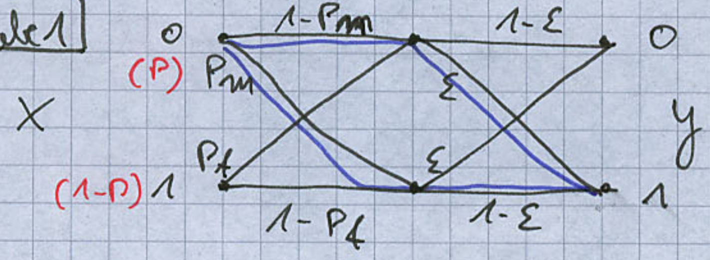
Aufgabe 1)

Input



Transmit	Receive
0	1
1	0

a)
$$\tilde{P}_m = P_m(1-\varepsilon) + (1-P_m)\varepsilon$$
$$= P_m + \varepsilon - 2\varepsilon P_m$$

$$\tilde{P}_f = P_f + \varepsilon - 2\varepsilon P_f$$

b) Transmit (P) , (1-P)

$$P_e = P\tilde{P}_m + (1-P)\tilde{P}_f$$

c) $P[x=0|e]$

$$= \frac{P[e|x=0]\,P(x=0)}{P[e|x=0]\,P(x=0) + P[e|x=1]\,P(x=1)}$$

$$= \frac{P\tilde{P}_m}{P\tilde{P}_m + (1-P)\tilde{P}_f}$$

Aufgabe 2

a) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

Immer die Formel aufschreiben, da bei TI auch für die richtige Idee Punkte gibt. nur Ergebnis nie geht!!!

$$\int_1^2 \frac{\mu}{x}\,dx$$

$$= [\mu \ln x]_1^2$$

$$= \mu \ln 2 = 1$$

$$\mu = \frac{1}{\ln 2}$$

b) $E(x) = \int_{-\infty}^{+\infty} x f(x)\, dx$

$$= \int_{1}^{2} \frac{1}{x \ln 2} x \, dx$$

$$= \frac{x}{\ln 2} \Big|_{1}^{2}$$

$$= \frac{1}{\ln 2}$$

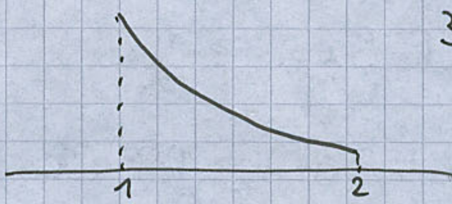
$$V(x) = \int_{-\infty}^{+\infty} (x - \mu)^2 f(x)\, dx$$

$$= \frac{3}{\ln 2} - \frac{1}{(\ln 2)^2}$$

c) $F_X(x) = \int_{-\infty}^{x} f(x)\, dx$

3 Bereiche

$$= \int_{1}^{x} f(x)\, dx \qquad 1 \leq x < 2$$



$$= \int_{1}^{x} \frac{1}{\ln 2}\, dx = \frac{\ln x}{\ln 2}$$

$$F_X(x) = \begin{cases} 0, & x < 1 \\ \frac{\ln x}{\ln 2}, & 1 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

d) From the Transformation Theorem

$$T: x \longmapsto \ln x$$

$$T^{-1}: y \longmapsto e^{y}$$

$$\cancel{\frac{dF(x)}{dx}} \quad \frac{dT^{-1}(y)}{d_y} = e^{y}$$

$$f(y) = \frac{d T^{-1}(y)}{dy} f_X(T^{-1}(y)) \underset{[1,2]}{\mathbb{1}}(e^y)$$

$$= e^y \frac{1}{\ln 2\, e^y} \mathbb{1}_{[1,2]}(e^y)$$

$$= \frac{1}{\ln 2} \underset{(0,\ln 2)}{\mathbb{1}}(y)$$

$$E(y) = \int_{-\infty}^{\infty} y(f(x))\, dy$$

$$= \int_0^{\ln 2} \frac{y}{\ln 2}\, dy$$

$$= \frac{\ln 2}{2}$$

<u>Aufgabe 3</u>

a) $$D(P\|q) = \sum_{i=1}^{3} P_i \log \frac{P_i}{q_i}$$

$$= \frac{1}{2}\log\frac{3}{2} + \frac{1}{3}\log\frac{3}{3} + \frac{1}{6}\log\frac{6}{6}$$

$$= \frac{1}{6}\log\frac{3}{2}$$

b) $$D(P\|q) = \frac{1}{2}\log\frac{3}{2} + \frac{1}{3}\log\frac{1}{3t} + \frac{1}{6}\log\frac{1}{6(\frac{2}{3}-t)}$$

$$\frac{d\, D(P\|q)}{dt} = -\frac{1}{3t} + \frac{1}{6(2/3-t)} = 0$$

$$9t - 4 = 0$$

$$t = +\frac{4}{9}$$

check:

$$\frac{d(9t-4)}{dt} = 9 > 0$$

$$\frac{d^2 D(P\|q)}{dt^2} > 0$$

c) $D(r \| p) \geq 0$

Equality holds when $r = p$

$D(r \| p)$ is minimized for $r = (\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$

" is maximized for $r = (0, 0, 1)$

Aufgabe 4

a) Given $f(x, y, z) = f_1(y \mid x, z) \, f_2(x) \, f_3(z)$

$$P(x, y, z) \stackrel{?}{=} P(x) P(z)$$

$$P(x, y, z) = \underline{P(y \mid x, z)} \, \boxed{p(x, z)}$$

$$= \underbrace{\underline{f_1(y \mid x, z)}}_{\text{Same}} \, \underbrace{\boxed{f_2(x) \, f_3(z)}}_{\text{Same}}$$

b)

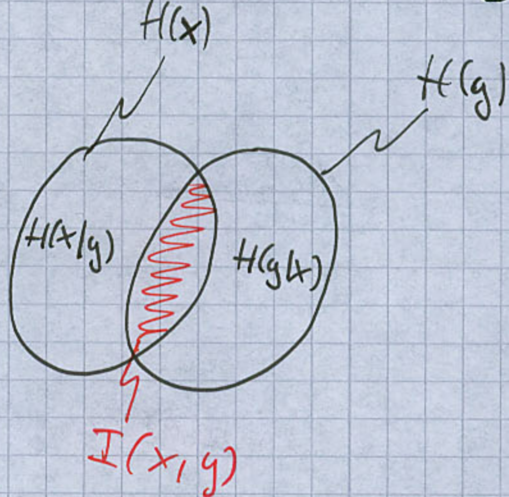
$I(x, y \mid z)$

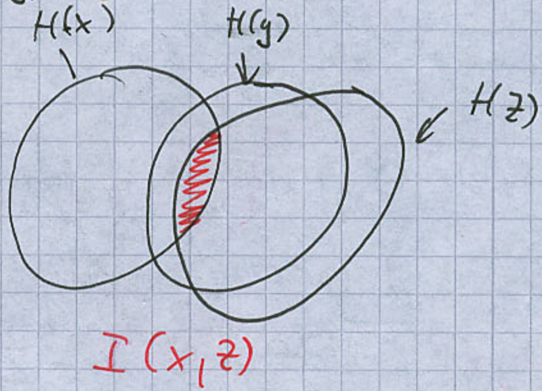
$= H(x \mid z) - H(x \mid y, z)$

$= H(x) - H(x \mid y, z)$

$\geq H(x) - H(x \mid y)$ $\qquad H(x \mid y) \geq H(x \mid y, z)$

$= I(x, y)$





$I(x, z) \leq I(x, y)$

$I(x, z) \leq I(y, z)$

Aufgabe 5

$y = \{0,1,2,3\} \quad x = \{0,1,2,3\}$

a) $H(y) \leq \log_2 |y| = \log_2 4 = 2$

auf 2 achten

b) $H(y|x) = -\sum_{x \times y} P(x,y) \log_2 P(y|x)$

$= -\sum_{x \times y} P(y|x) P(x) \log_2 P(y|x)$

$= -P(x=0) \log_2 \frac{1}{2} - P(x=1) \log_2 \frac{1}{2}$
$- P(x=2) \log_2 \frac{1}{2} - P(x=3) \log_2 \frac{1}{2}$

$= 1$

c) $H(y) = 2$ holds if y is gleichverteilt

y ist gleichverteilt falls:

① $P(x=0) = P(x=2) = \frac{1}{2}$

② $P(x=1) = P(x=3) = \frac{1}{2}$

③ $P(x=0) = P(x=1) = P(x=2) = P(x=3) = \frac{1}{4}$

d) $C = \max_{P_x} I(x,y) = \max_{P_x} (H(y) - H(y|x))$

$= \max_{P_x} H(y) - 1$

$= 2 - 1 = 1$

Aufgabe 6

$y = x + N_1$

$C = \frac{1}{2} \ln(1 + \frac{4}{1}) = \frac{\ln 5}{2}$

$\frac{1}{2} \log_2 (1 + SNR)$

$\mu \in [0,1]$

$y = x + \underbrace{\mu N_1 + (1-\mu N_2)}_{=N}$

$\left[= (x + N_1)(\mu) + (x + N_2)(1-\mu) \right]$

$Var(N) = Var(\mu N_1 + (1-\mu) N_2)$

$= Var(\mu N_1 + Var((1-\mu) N_2)$

$= \mu^2 Var(N_1) + (1-\mu)^2 Var(N_2)$

$= 3\mu^2 - 4\mu + 2$

$C = \frac{1}{2} \ln(1 + \frac{4}{3\mu^2 - 4\mu + 2}) = \frac{\ln 7}{2}$ erst nach Rechnung hingeschrieben

$\left[\frac{dc}{d\mu} \right.$ nur über dem Kram in ln $\left. \right]$ wird Max von $3\mu^2 - 4\mu + 2$ min wird

$\frac{d\, 3\mu^2 - 4\mu + 2}{d\mu} = 6\mu - 4 = 0$

$\mu = \frac{2}{3}$

$\frac{d\, 6\mu - 4}{d\mu} = 6 > 0$ => Minimum

Aufgabe 7

a) $\sqrt{2}\, e^{j\frac{\pi}{4}} = 1+i$

$\sqrt{2}\, e^{j\frac{5\pi}{4}} = -1-i$

$$H^* H = \begin{pmatrix} 1-i & 1 & 0 & -1+i \\ 0 & 0 & \sqrt{8} & 0 \\ 1-i & -1+i & 0 & 1 \end{pmatrix} \begin{pmatrix} 1+i & 0 & 1+i \\ 1 & 0 & -1-i \\ 0 & \sqrt{8} & 0 \\ -1-i & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

b) $r_1 = 5 \quad r_2 = 8 \quad r_3 = 5$

$$= \left(r - \frac{40}{5}\right)^+ + \left(r - \frac{40}{8}\right)^+$$ [Channel $\frac{5}{40}$ $\frac{\text{gain}}{\text{noise}}$]

$$+ \left(r - \frac{40}{5}\right)^+ = 2$$

$$r - 5 = 2$$

$$r = 7$$

$$(7-8)^+ + (7-5)^+ + (7-8)^+ = 0 + 2 + 0 = 2$$

$$C = \sum_{i=1}^{3} \left[\ln\left(\frac{r \cdot r_i}{6}\right)\right]^+ = \ln \frac{7 \cdot 8}{40} = \ln \frac{7}{5}$$

[alle r's müssen μ's sein]

μ_1: $5x = 5x$

$5y = 8y \Rightarrow y = 0 \qquad u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$5z = 5z$

μ_2: $8x = 5x$

$8y = 8y \qquad z = 0 \qquad u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$8z = 5z$

μ_3 . . . . $u_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$$Q = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right) \begin{pmatrix} 0 & & \\ & 2 & \\ & & 0 \end{pmatrix} \left(\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \right)$$

$$= \begin{matrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{matrix}$$

komplexes Signal kein $\frac{1}{2}$

c) $\cancel{\frac{1}{2}} \log(1 + SNR)$

$$\log\left(1 + \frac{2}{\sigma^2}\right) = \log \frac{7}{5}$$

$$1 + \frac{2}{\sigma^2} = \frac{7}{5}$$

$$5 + \frac{10}{\sigma^2} = 7$$

σ = sigma

$$\sigma^2 = 5$$

Aufgabe 8

min $10x_1 + 2{,}5x_2$

s.t. $6x_1 + 3x_2 \leq 30$

$6x_1 + 2x_2 \geq 24$

$x_1 \geq 3$

$x_2 \geq 3$

$x_1, x_2 \in \{1, 2, 3, \ldots\}$ gibt nur ganze Chips

$\in \mathbb{N}$

max $-10x_1 - 2{,}5x_2$

s.t. $6x_1 + 3x_2 \leq 30$

$-6x_1 - 2x_2 \leq -24$

$-x_1 \leq -3$

$-x_2 \leq -3$

$x_1, x_2 \in \mathbb{N}$