RÜ 2

Dienstag, 19. April 2011
15:56

H08

A1

1.1) $\alpha_r = \alpha_p$

Snellius: $\sin\alpha_p \underbrace{\sqrt{\varepsilon_{r1}}}_{2} = \sin\beta \underbrace{\sqrt{\varepsilon_{r2}}}_{3}$

$$\beta = \arcsin\left(\frac{\sin\alpha_p}{\sqrt{\varepsilon_{r2}}}\sqrt{\varepsilon_{r1}}\right)$$

1.2) Reflexionsebene ist die Ebene von einfallenden und reflektierten Wellenzahlvektor aufgespannt ($\vec{k}_p ; \vec{k}_r$)

=) Reflexionsebene $y = const$

1.3) totale Transmission falls

α_p = Brewsterwinkel

(α_B tritt nur für $E_{\parallel}$ auf)

$$\alpha_B = \arctan\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \arctan(3)$$
$$= 71{,}56°$$

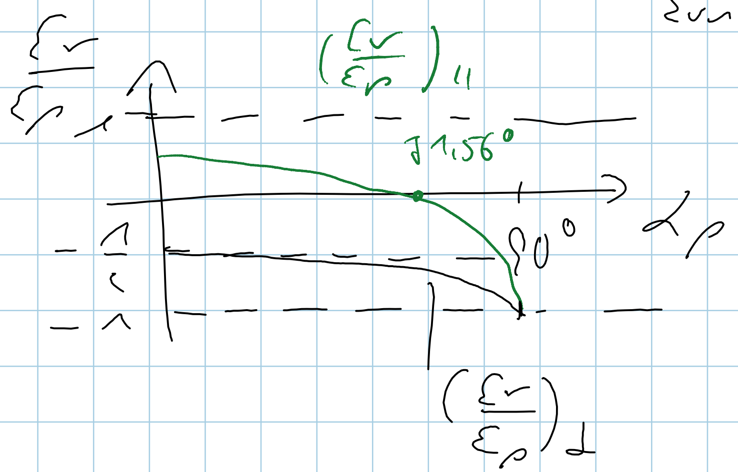
1.4)

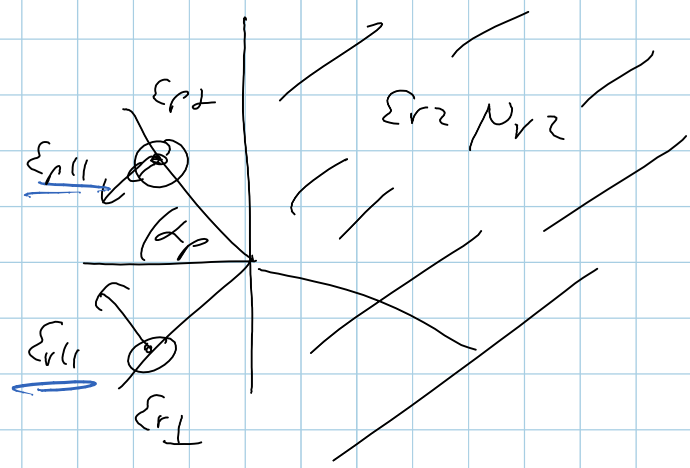
$$\left(\frac{E_r}{E_e}\right)_\perp = \frac{\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\cos\alpha_e - \cos\beta}{\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\cos\alpha_e + \cos\beta}$$

$$= \frac{\cos\alpha_e - \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}} - \sin^2\alpha_e}}{\cos\alpha_e + \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}} - \sin^2\alpha_e}}$$

$$\left(\frac{E_r}{E_e}\right)_\perp\Bigg|_{\alpha_e=0^\circ} = \frac{1-\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}}{1+\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}} = \frac{1-3}{1+3} = -\frac{1}{2}$$

$$\left(\frac{E_r}{E_e}\right)_\perp\Bigg|_{\alpha_e=90^\circ} = \frac{-\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}-1}}{\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}-1}} = -1$$





$$\left(\frac{E_r}{E_e}\right)_\parallel = -\frac{\sqrt{1-\frac{\varepsilon_{r1}}{\varepsilon_{r2}}\sin^2\alpha_e} - \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\cos\alpha_e}{\sqrt{1-\frac{\varepsilon_{r1}}{\varepsilon_{r2}}\sin^2\alpha_e} + \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\cos\alpha_e}$$

$$\left(\frac{E_r}{E_e}\right)\Big| \quad = \quad 1-\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} \quad 1-3 \quad 1$$

$$\left(\frac{\varepsilon_r}{\varepsilon_p}\right)_{\parallel}\Bigg|_{\alpha_p=0°} = -\frac{1-\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}}{1+\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}} = -\frac{1-3}{1+3} = \frac{1}{2}$$

$$\left(\frac{\varepsilon_r}{\varepsilon_p}\right)_{\parallel}\Bigg|_{\alpha_p=90°} = -\frac{\sqrt{1-\frac{1}{9}}}{\sqrt{1-\frac{1}{9}}} = -1$$

$$\left(\frac{\varepsilon_r}{\varepsilon_p}\right)_{\parallel}\Bigg|_{\alpha_p=71{,}56°} = 0$$

<u>1.2)</u>

<u>1.2.1)</u> $R >> 2\cdot H \Rightarrow \alpha_p \approx 180°$

$$\left(\frac{\varepsilon_r}{\varepsilon_p}\right)_{\parallel,\alpha_p=90°} \approx -1 = e^{j180°}$$

<u>1.2.2</u> $E(r) = A\cdot\frac{e^{-jkr}}{r}$

$$E_{ges} = A\cdot\frac{e^{-jkR}}{R} + A\,\frac{e^{-jkR_2}}{R_2}\,e^{j\varphi_E}$$

$$R_2 = \sqrt{(2H)^2 + R^2}$$

mit $R >> 2H$ gilt: ~~Nenner~~ Nenner $R_2 \approx R$

Zähler: $R_2 = R\sqrt{1+\left(\frac{2H}{R}\right)^2}$

$\approx R\left(1+\frac{1}{2}\left(2\frac{H}{R}\right)^2\right)$

$$E_{ges} = A \cdot \frac{e^{-jkR}}{R} + A \cdot \frac{e^{-jkR\left(1+\frac{1}{2}\left(2\frac{H}{R}\right)^2\right)}}{R} e^{j\varphi_E}$$

$$E_{ges} = A \frac{e^{-jkR}}{R} \left(1 + e^{-jkR\frac{1}{2}\left(2\frac{H}{R}\right)^2} \underbrace{e^{-j\varphi_E}}_{-1}\right)$$

$$E_{ges} = A \frac{e^{-jkR}}{R} \left(1 - \underbrace{e^{-jkR\frac{1}{2}\left(2\frac{H}{R}\right)^2}}_{1-jkR\cdot 2\left(\frac{H}{R}\right)^2}\right)$$

$$E_{ges} = A \frac{e^{-jkR}}{R} \left(+jkR\, 2 \left(\frac{H}{R}\right)^2\right)$$

$$= A \frac{e^{-jkR}}{R^2} k 2 j H^2$$

2.3)

$$a = 10 \log \left|\frac{P_{ges}}{P_1}\right|$$

$$\left|\frac{P_{ges}}{P_1}\right| = \frac{|E_{ges}|^2}{|E_1|^2} = \frac{\left|A \frac{\overset{=1}{\cancel{e^{-jkR}}}}{R^2} 2\cancel{j}kH^2\right|^2}{\left|A \frac{e^{-jkR}}{R}\right|^2}$$

$$= \frac{A^2 \frac{1}{R^4} 4k^2 H^4}{A^2 \frac{1}{R^2}} = \frac{4k^2H^4}{R^2}$$

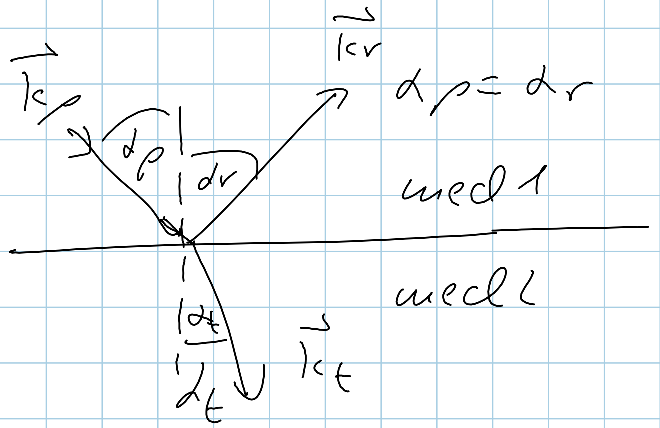
mit $k = \frac{2\pi}{\lambda} = 15{,}71$

$$\Rightarrow \left|\frac{\rho_{ges}}{\rho_n}\right| = 9{,}87 \cdot 10^{-4}$$

$$\Rightarrow \alpha = -30\,dB$$

FO9 A.1

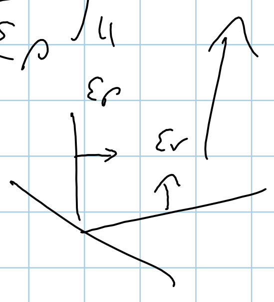
1.1



$$\frac{\sin \alpha_t}{\sin \alpha_p} = \sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}}$$

2.1 x-z-Ebene (von $\vec{k}_p$ und $\vec{k}_r$ aufgespannt)

2.2)
$$\left(\frac{\varepsilon_r}{\varepsilon_p}\right)_{\parallel} = -\frac{\sqrt{\frac{\mu_2}{\mu_1} - \frac{\varepsilon_1}{\varepsilon_2}\sin^2\alpha_p} - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\cos\alpha_p}{\sqrt{\frac{\mu_2}{\mu_1} - \frac{\varepsilon_1}{\varepsilon_2}\sin^2\alpha_p} + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\cos\alpha_p}$$



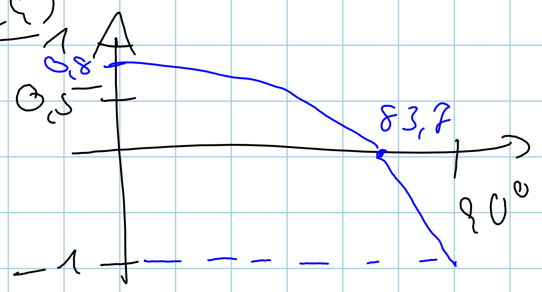
a) $$\left|\frac{\varepsilon_r}{\varepsilon_p}\right|_{\parallel, \alpha_p = 0°} = -\frac{1-9}{1+9} = \frac{8}{10}$$

b) $\left(\frac{E_r}{E_p}\right)_{\parallel}$ $\alpha_p = 90° = -1$

2.3 Brewsterwinkel α_B

$$\alpha_B = \arctan\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \arctan(9) = 83{,}66°$$

2.4)



2.5) $\underline{\vec{E}}_t = \underline{\vec{E}}_{t0}\, e^{-jk_2 z}$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2} \approx \omega\sqrt{\frac{\mu_2\cdot\varkappa}{j\omega}}$$

$$= (1-j)\sqrt{\frac{\omega\mu_2\varkappa}{2}}$$

$$\underline{\vec{E}}_t = \underline{\vec{E}}_{t0}\, e^{-j(1-j)\sqrt{\frac{\omega\mu_2\varkappa}{2}}\cdot z}$$

$$|\underline{\vec{E}}_t| = |\underline{E}_t(z=0)|\,\left|e^{-j(1-j)\sqrt{\frac{\omega\mu_2\varkappa}{2}}\cdot z}\right|$$

$$E_t = E_t(z=0)\, e^{-\sqrt{\frac{\omega\mu_2\varkappa}{2}}\cdot z} \overset{!}{=} 10\mu\frac{V}{m}$$

$$E_t(z=0) = \frac{E_t}{e^{-\sqrt{\frac{\omega\mu_2\varkappa}{2}}z}} = \frac{10\mu\frac{V}{m}}{1{,}38\cdot10^{-4}} = 72{,}3\, mV/m$$

1.2 b) $\left(\frac{E_t}{\dots}\right) = 2\cdot\sqrt{\frac{\mu_2}{\mu_1}}\cos\alpha_p$

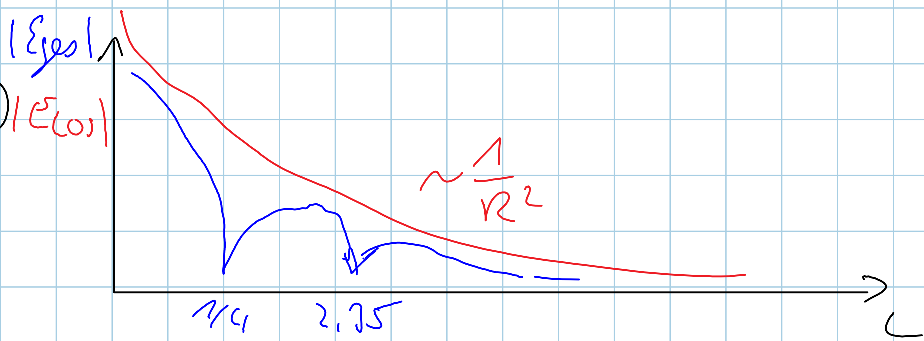
1.2.6) $\left(\frac{E_t}{E_p}\right)_{\parallel} = \frac{2\cdot\sqrt{\frac{\mu_2}{\mu_1}}\cos\alpha_p}{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\cos\alpha_p + \sqrt{\frac{\mu_2}{\mu_1} - \frac{\varepsilon_1}{\varepsilon_2}\sin^2\alpha_p}}$

$\left(\frac{E_t}{E_p}\right)_{\parallel\, \alpha_p = 0°} = \frac{2}{9}$

$\Rightarrow E_p = \frac{9}{2} E_t = \frac{9}{2}\cdot 32{,}3\,\frac{mV}{m} = 145{,}3\,\frac{mV}{m}$

1.3)

1.3.1)



a) im Winkelbereich $0 \leq \alpha \leq 83{,}7°$

Grenzschicht keine Phasenänderung auf dem Bodenpfad

$\Rightarrow$ Ein minimum ergibt sich daher wenn

Wegunterschied beider Strecken

$\frac{\Delta L}{\lambda} = \frac{\ell}{2}(2n-1) \quad n \in \mathbb{N}$ beträgt

$(\Delta L = (2n-1)\frac{\lambda}{2} \qquad \frac{1}{2}, \frac{3}{5}, \frac{5}{2}$

$\Delta L = \frac{L}{\sin\alpha_p} - L = \frac{\lambda}{2}(2n-1) \quad n \in \mathbb{N}$

$$\sqrt{L^2+(2h)^2} - L = \frac{\lambda}{2}(2n-1) \qquad n \in \mathbb{N}$$

$$\Rightarrow \cancel{L^2} + (2h)^2 = \frac{\lambda^2}{4}(2n-1) + \cancel{L^2}$$
$$+ \lambda L (2n-1)$$

$$\Rightarrow L = \frac{(2h)^2 - \frac{\lambda^2}{4}(2n-1)^2}{\lambda(2n-1)}$$

b) $n=1 \qquad \frac{L}{\lambda} = 2{,}75$

$n=2 \qquad \frac{L}{\lambda} = 0{,}25$

$n=3 \qquad \frac{L}{\lambda} < 0 \quad ↯$

<u>1.3.2</u> $\quad \frac{L_1}{\lambda} = 0{,}25$

$$\alpha_p = \arctan\left(\frac{\frac{1}{2}L_1}{h}\right) = \arctan\left(\frac{\frac{1}{2}\frac{L_1}{\lambda}}{\frac{h}{\lambda}}\right) = 8{,}21^\circ$$

Reflexionsfaktor am Boden

$$\left(\frac{E_r}{E_r}\right)_{\parallel} = 0{,}7982 \approx 0{,}8$$

$$E_{ges} = A_0\left(\frac{e^{-jkL_1}}{\quad} + r\,\frac{e^{-jk\sqrt{L_1^2+(2h)^2}}}{\quad}\right)$$

$$\underline{E}_{ges} = \underbrace{A_0 \left(\frac{e^{-jkL_1}}{L_1}\right.}_{\underline{E}_{LOS}} + \underbrace{r \left.\frac{e^{-jk\sqrt{L_1^2+(2h)^2}}}{\sqrt{L_1^2+(2h)^2}}\right)}_{E_r}$$

$$\left| \frac{E_{ges}}{E_{LOS}} \right| = \left| 1 + r \underbrace{\frac{e^{-jk\sqrt{L_1^2+(2h)^2}}}{e^{-jkL_1}}}_{=-1 \text{ „Gegenphase"}} \cdot \frac{L_1}{\sqrt{L_1^2+(2h)^2}} \right|$$

$$= \left| 1 - r \frac{L_1}{\sqrt{L_1^2+(2h)^2}} \right| = \left| 1 - \frac{r}{\sqrt{1+\left(\frac{2h}{L_1}\right)^2}} \right|$$

$$\text{mit } \frac{2h}{\lambda} \cdot \frac{\lambda}{L_1} = \frac{2\sqrt{3}}{2} \frac{4}{1} = 4\sqrt{3}$$

$$= 0{,}886$$

$$a \mathrel{\hat{=}} -1{,}05 \text{ dB}$$