

Aufgabe 1:

$$(a) \quad \vec{B}(\vec{r}) \quad ; \quad \vec{r} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \quad \text{für} \quad \begin{array}{l} x < a(t) \\ y < a(t) \end{array}$$

Induktion eines unendlich langen Leiters: (Ebene  $z=0$ )

$$\vec{B}_{1,2}(\vec{r}) = \frac{\mu_0 \cdot I}{2\pi d(\vec{r})} \cdot \vec{e}_z \quad \text{mit } d(\vec{r}) \hat{=} \text{Abstand zum Leiter}$$

$$\vec{B}_1(\vec{r}) = \frac{\mu_0 \cdot I}{2\pi} \cdot \vec{e}_z \cdot \frac{1}{a(t) - x}$$

$$\vec{B}_2(\vec{r}) = \frac{\mu_0 \cdot I}{2\pi} \cdot \vec{e}_z \cdot \frac{1}{a(t) - y}$$

somit

$$\vec{B}(\vec{r}) = \vec{B}_1(\vec{r}) + \vec{B}_2(\vec{r}) = \frac{\mu_0 \cdot I}{2\pi} \cdot \vec{e}_z \cdot \left( \frac{1}{a(t) - x} + \frac{1}{a(t) - y} \right)$$

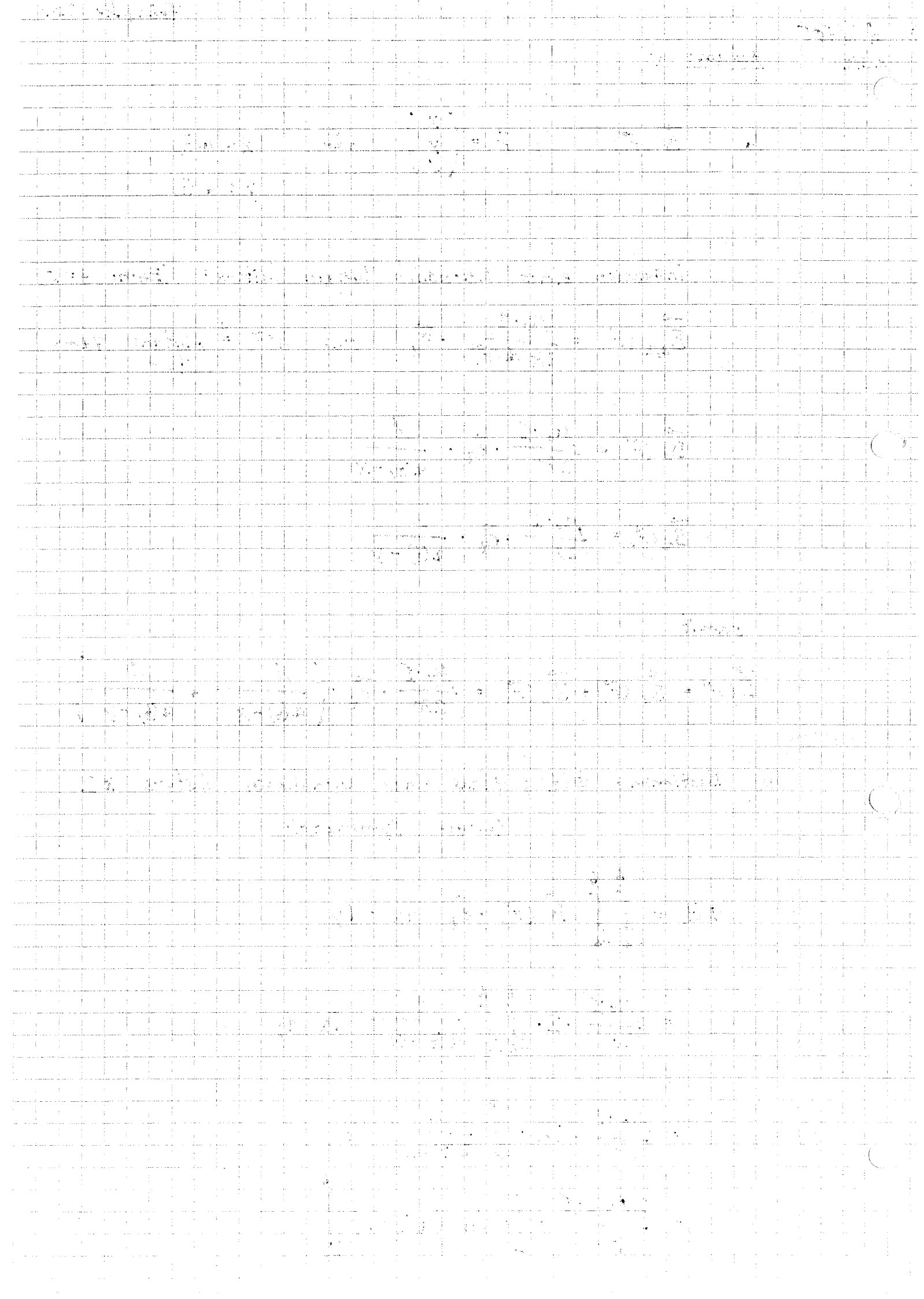
(b) Bestimme  $\Phi(t)$ ; Fluss eines unendlichen Leiters  $\times 2$   
(wegen Symmetrie)

$$\Phi(t) = \int_{-b}^b \int_{-b}^b \vec{B}(\vec{r}) \cdot \vec{e}_z \, dx \cdot dy$$

$$= \frac{\mu_0 \cdot I}{2\pi} \cdot 2 \cdot \int_{-b}^b \int_{-b}^b \frac{1}{a(t) - x} \, dx \, dy$$

$$= \frac{\mu_0 \cdot I}{\pi} \cdot 2b \cdot \int_{-b}^b \frac{1}{a(t) - x} \, dx$$

$$= \frac{2\mu_0 b \cdot I}{\pi} \cdot \left[ -\ln(a(t) - x) \right]_{-b}^b$$



A1/F05  
5.2/2

$$= \frac{2\mu_0 b \cdot I}{\pi} \cdot \left( -\ln(a(t) - b) + \ln(a(t) + b) \right)$$

$$= \frac{2\mu_0 b \cdot I}{\pi} \cdot \ln \left( \frac{a(t) + b}{a(t) - b} \right)$$

(c)  $u(t) = - \frac{d\phi(t)}{dt}$  mit  $\frac{da(t)}{dt} = v$

$$\Rightarrow u(t) = - \frac{2\mu_0 b \cdot I}{\pi} \cdot \frac{d}{dt} \left( \ln(a(t) + b) - \ln(a(t) - b) \right)$$

$$= -2\mu_0 b \cdot \frac{I}{\pi} \left( v \cdot \frac{1}{a(t) + b} - v \cdot \frac{1}{a(t) - b} \right)$$

$$= 2\mu_0 I v \frac{b}{\pi} \cdot \left( \frac{1}{a(t) - b} - \frac{1}{a(t) + b} \right)$$

(d)  $u(t) \equiv 0$  wenn  $\frac{d\phi}{dt} = 0 \Rightarrow \phi = \text{const}$

aus (b)  $\phi = 2\mu_0 \frac{b}{\pi} \cdot I(t) \cdot \ln \left( \frac{b + a(t)}{-b + a(t)} \right) = \text{const} \stackrel{!}{=} K$

$$\Rightarrow I(t) = \frac{K \cdot \pi}{2\mu_0 b} \cdot \frac{1}{\ln \left( \frac{b + a(t)}{-b + a(t)} \right)}$$

$$I(t) = I_0 \cdot \ln \left( \frac{a(t) + b}{a(t) - b} \right)^{-1}$$

mit  $I_0$  beliebiger Strom

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

## Aufgabe 2:

$$a) \quad \underline{\vec{E}}(f, z) = E(f) \cdot \exp(-i k_0 z) \cdot \vec{e}_\rho = E_f \cdot \vec{e}_\rho$$

Helmholtzgleichung:  $(\Delta + k_0^2) \cdot \vec{E}(\rho, z) = \vec{0} \quad (*)$

Da  $\vec{E}$  nur Fkt. von  $\rho$  und  $\tau$  und  $\vec{E} \parallel \vec{p}_p$  ist:

$$\Delta \vec{E}(r, z) = \left\{ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r \cdot E_r) \right) + \frac{\partial^2 E_z}{\partial z^2} \right\} \cdot \vec{e}_r + 0$$

$\Delta$  in Zylinderkoordinaten nach Formelsammlung

$$= \left\{ \frac{\partial}{\partial y} \left( \frac{1}{y} \frac{\partial}{\partial y} (y \cdot E_y) \right) - k_0^2 \cdot E_y \right\} \cdot \vec{e}_y$$

$$(*) : \vec{\phi} = (\Delta + K_0^2) \vec{E}(y, z) \Rightarrow \phi = \underbrace{\frac{\partial}{\partial y} \left( \frac{1}{y} \frac{\partial}{\partial y} (y \cdot E_y) \right)}_{(**)}$$

$$\boxed{\vec{\nabla} \vec{E}(\varphi, \tau) = \emptyset} \Rightarrow \longrightarrow = \emptyset$$

Formelsumme (\*\*\*)

(sonst wie)

(sonst wäre  $E_g \sim \rho$  auch  
Lsg. für (\*\*))

$$(**) \text{ \& } (***) \Rightarrow \phi = \frac{\partial}{\partial \varphi} (\varphi \cdot E_\varphi) \Rightarrow E_\varphi \sim \frac{1}{\varphi} \quad \text{also } E(\varphi) \sim \frac{1}{\varphi}$$

$P_1, P_2$  Punkte auf der Oberfläche vom Innenleiter, bzw. Innenseite des Außenleiters

$$U_0 = \int_{P_1}^{P_2} \vec{E}(P, \vec{r}) \cdot d\vec{r} = \int_a^b E(\rho) d\rho = \int_a^b \frac{A}{\rho} d\rho = A \cdot \ln \frac{b}{a}$$

$$\Rightarrow A = \frac{U_0}{\ln \frac{b}{a}} \Rightarrow E(\varphi) = \frac{U_0}{\ln \left| \frac{b}{a} \right|} \cdot \frac{1}{\varphi}$$

1. The first part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 2. The second part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 3. The third part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 4. The fourth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 5. The fifth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 6. The sixth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 7. The seventh part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 8. The eighth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 9. The ninth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...  
 10. The tenth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...

$$(b) \quad \vec{\nabla} \times \vec{E} = -j\omega\mu_0 \vec{H} \quad (= -\mu_0 \frac{\partial}{\partial t} \vec{H})$$

$$\Rightarrow \vec{\nabla} \times \vec{E}(\varphi, z) = -j\omega\mu_0 \vec{H}(\varphi, z)$$

$$\begin{aligned} \vec{\nabla} \times \vec{E}(\varphi, z) &= 0 + \vec{e}_\varphi \cdot \left\{ \frac{\partial}{\partial z} (E(\varphi) \cdot \exp(-jk_0 z)) - 0 \right\} + 0 \\ &= \vec{e}_\varphi \cdot \{-jk_0 E(\varphi) \exp(-jk_0 z)\} = -j\omega\mu_0 \vec{H}(\varphi, z) \\ \vec{H}(\varphi, z) &= \frac{k_0}{\omega\mu_0} E(\varphi) \exp(-jk_0 z) \cdot \vec{e}_\varphi = \sqrt{\frac{\epsilon_0}{\mu_0}} \cdot \frac{U_0}{\ln\left(\frac{b}{a}\right)} \cdot \frac{1}{\varphi} \cdot \exp(-jk_0 z) \cdot \vec{e}_\varphi \end{aligned}$$

$$\begin{aligned} (c) \quad \underline{U}_+(z) &= \int_{P_1'}^{P_2'} \vec{E}(\varphi, z) d\vec{r} = \int_{P_1'}^{P_2'} \frac{U_0}{\ln\left(\frac{b}{a}\right)} \exp(-jk_0 z) \cdot \frac{1}{\varphi} d\varphi \\ &= \frac{U_0}{\ln\left(\frac{b}{a}\right)} \exp(-jk_0 z) \underbrace{\int_a^b \frac{1}{\varphi} d\varphi}_{= \ln \frac{b}{a}} \end{aligned}$$

$$\underline{U}_+(z) = U_0 \cdot \exp(-jk_0 z)$$

$$\underline{I}_+(z) = \oint_{C_0} \vec{H} d\vec{r} = \int_0^{2\pi} \vec{H}(\varphi=a, z) \cdot a \cdot d\varphi$$

$\hookrightarrow C_0$  z.B. Kurve entlang der Oberfläche des Innenleiters  
( $\Rightarrow \varphi=a$ ,  $\vec{H} d\vec{r} = 0 + (\vec{H} \cdot \vec{e}_\varphi) a \cdot d\varphi$ )

$$= \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{2\pi}{\ln \frac{b}{a}} U_0 \cdot \exp(-jk_0 z)$$

$$\underline{Z_L} = \frac{\underline{U}_+(z)}{\underline{I}_+(z)} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\ln\left(\frac{b}{a}\right)}{2\pi}$$

1. The first part of the document is a list of names and addresses.

2. The second part of the document is a list of names and addresses.

3. The third part of the document is a list of names and addresses.

4. The fourth part of the document is a list of names and addresses.

5. The fifth part of the document is a list of names and addresses.

6. The sixth part of the document is a list of names and addresses.

7. The seventh part of the document is a list of names and addresses.

8. The eighth part of the document is a list of names and addresses.

9. The ninth part of the document is a list of names and addresses.

10. The tenth part of the document is a list of names and addresses.

11. The eleventh part of the document is a list of names and addresses.

12. The twelfth part of the document is a list of names and addresses.

13. The thirteenth part of the document is a list of names and addresses.



(d)  $z = 0$  :  $\vec{E}, \vec{H}$  stetig

$z = h$  :  $\vec{E} \equiv 0$ ,  $\vec{E}_z \times (0 - \vec{H}_z) = \vec{\gamma}_z$

$z = 0$  :  $\underline{u}(z), i(z)$  stetig

$z = h$  :  $\underline{u}(h) \equiv 0$

(e)  $z \leq 0$  :  $\underline{u}(z) = u_0 \exp(-j k_0 z) + u_{0-} \exp(j k_0 z)$

$\underline{i}(z) = \frac{u_0}{Z_L} \exp(-j k_0 z) - \frac{u_{0-}}{Z_L} \exp(j k_0 z)$

$0 < z < h$  :  $\underline{u}_2(z) = u_{2+} \exp(-j k z) + u_{2-} \exp(j k z)$

$\underline{i}_2(z) = \frac{u_{2+}}{Z_{L2}} \exp(-j k z) - \frac{u_{2-}}{Z_{L2}} \exp(j k z)$

Grenzbed. von (d) :  $\underline{u}_2(h) \equiv 0$  (\*)

$\underline{u}(0) = \underline{u}_2(0)$  (\*\*)

$\underline{i}(0) = \underline{i}_2(0)$  (\*\*\*)

(\*)  $\Rightarrow u_{2-} = -u_{2+} \exp(-2 j k h)$

$\Rightarrow \underline{u}_2(z) = u_{2+} \exp(-j k h) \cdot (\exp(-j k [z-h]) - \exp(j k [z-h]))$

(\*\*)  ~~$u_0 + u_{0-}$~~   $u_0 + u_{0-} = u_{2+} \underbrace{(1 - \exp(-2 j k h))}_{=: \alpha}$

(\*\*\*)  $\Rightarrow \frac{u_0}{Z_L} - \frac{u_{0-}}{Z_L} = \frac{u_{2+}}{Z_{L2}} (1 + \exp(-2 j k h)) \Rightarrow u_{0+} - u_{0-} =$   
 $\underbrace{u_{2+} \frac{Z_L}{Z_{L2}} (1 + \exp(-2 j k h))}_{=: \beta}$

Handwritten text on a grid background, appearing to be a list or index of items, possibly related to a collection or inventory. The text is written in a cursive script and is organized into several columns and rows. The entries are mostly illegible due to the quality of the scan and the handwriting.

$$(**) \text{ d}(***) \Rightarrow \frac{u_0 + u_{0-}}{\alpha} = \frac{u_0 - u_{0-}}{\beta} \Rightarrow u_{0-} = u_0 \frac{\alpha - \beta}{\alpha + \beta}$$

$$\sim \Rightarrow 2u_0 = u_{2+} (\alpha + \beta) \Rightarrow u_{2+} = u_0 \cdot \frac{2}{\alpha + \beta}$$

$$u_{2-} = -u_0 \cdot \frac{2 \exp(-2jkh)}{\alpha + \beta}$$

Reflections coeff.  $\Gamma = \frac{u_{0-}}{u_0} = \frac{\alpha - \beta}{\alpha + \beta} = \frac{Z_{L2} j \sin(Kh) - Z_L \cos(Kh)}{Z_{L2} j \sin(Kh) + Z_L \cos(Kh)}$

$$\left\{ \alpha = \exp(-jkh) \cdot \{2j \sin(Kh)\}; \beta = \exp(-jkh) \{2 \cos(Kh)\} \cdot \frac{Z_L}{Z_{L2}} \right\}$$

$$\underline{u}(z) = u_0 \cdot \exp(-jK_0 z) + \Gamma \cdot u_0 \cdot \exp(jK_0 z)$$

$$\underline{i}(z) = \frac{u_0}{Z_L} \exp(-jK_0 z) - \frac{\Gamma u_0}{Z_L} \exp(jK_0 z)$$

$$\underline{u}_2(z) = \frac{2u_0}{[1 - \exp(-2jKh)] + \left[ \frac{Z_L}{Z_{L2}} (1 + \exp(-2jKh)) \right]} \cdot \left\{ \exp(-jKz) - \exp(jKz - 2jKh) \right\}$$

$$\underline{i}_2(z) = \frac{2u_0}{Z_{L2} [1 - \exp(-2jKh)] + Z_L [1 + \exp(-2jKh)]} \cdot \left\{ \exp(-jKz) + \exp(jKz - 2jKh) \right\}$$

1. The first part of the report is a general introduction to the project.

2. The second part is a detailed description of the methodology used.

3. The third part is a discussion of the results.

4. The fourth part is a conclusion.

5. The fifth part is a list of references.

6. The sixth part is a list of appendices.

7. The seventh part is a list of figures.

8. The eighth part is a list of tables.

9. The ninth part is a list of abbreviations.

10. The tenth part is a list of symbols.

11. The eleventh part is a list of acronyms.

12. The twelfth part is a list of footnotes.

# Lösung A3

a)  $\Delta \underline{F}(\rho, z) + k^2 \underline{F}(\rho, z) = 0$  ;  $\Delta \underline{G}(\rho, z) + k^2 \underline{G}(\rho, z) = 0$  ;  $k^2 = \omega^2 \epsilon \mu$  (1)

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \underline{f}(\rho) e^{-j\beta^2 z} \right) + \frac{\partial^2}{\partial z^2} (\underline{f}(\rho) e^{-j\beta^2 z}) + k^2 \underline{f}(\rho) e^{-j\beta^2 z} = 0 \quad (2)$$

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{d \underline{f}(\rho)}{d \rho} \right) + \rho^2 \underline{f}(\rho) = 0 \quad ; \quad \rho^2 = k^2 - \beta^2 \quad (3)$$

$$\frac{1}{\rho} \frac{d}{d \rho} \left( \rho \frac{d \underline{g}(\rho)}{d \rho} \right) + \rho^2 \underline{g}(\rho) = 0 \quad ; \quad \text{analog} \quad (4)$$

(3), (4): Besselsche DGL nullter Ordnung ( $\rho \neq 0$  nach Voraussetzung)

Lösungen:  $\underline{f}(\rho) = A J_0(\rho \rho) + B N_0(\rho \rho) \quad (5)$

$$\underline{g}(\rho) = C J_0(\rho \rho) + D N_0(\rho \rho) \quad (6)$$

b) Die Felder und damit  $\underline{f}, \underline{g}$  dürfen für  $\rho \rightarrow 0$  nicht singular werden

$$\Rightarrow \boxed{B=0, D=0} \quad (7)$$

Grenzbedingung auf  $G_1$  (elektr. Wand;  $\varphi=0, \varphi=\pi$ ):

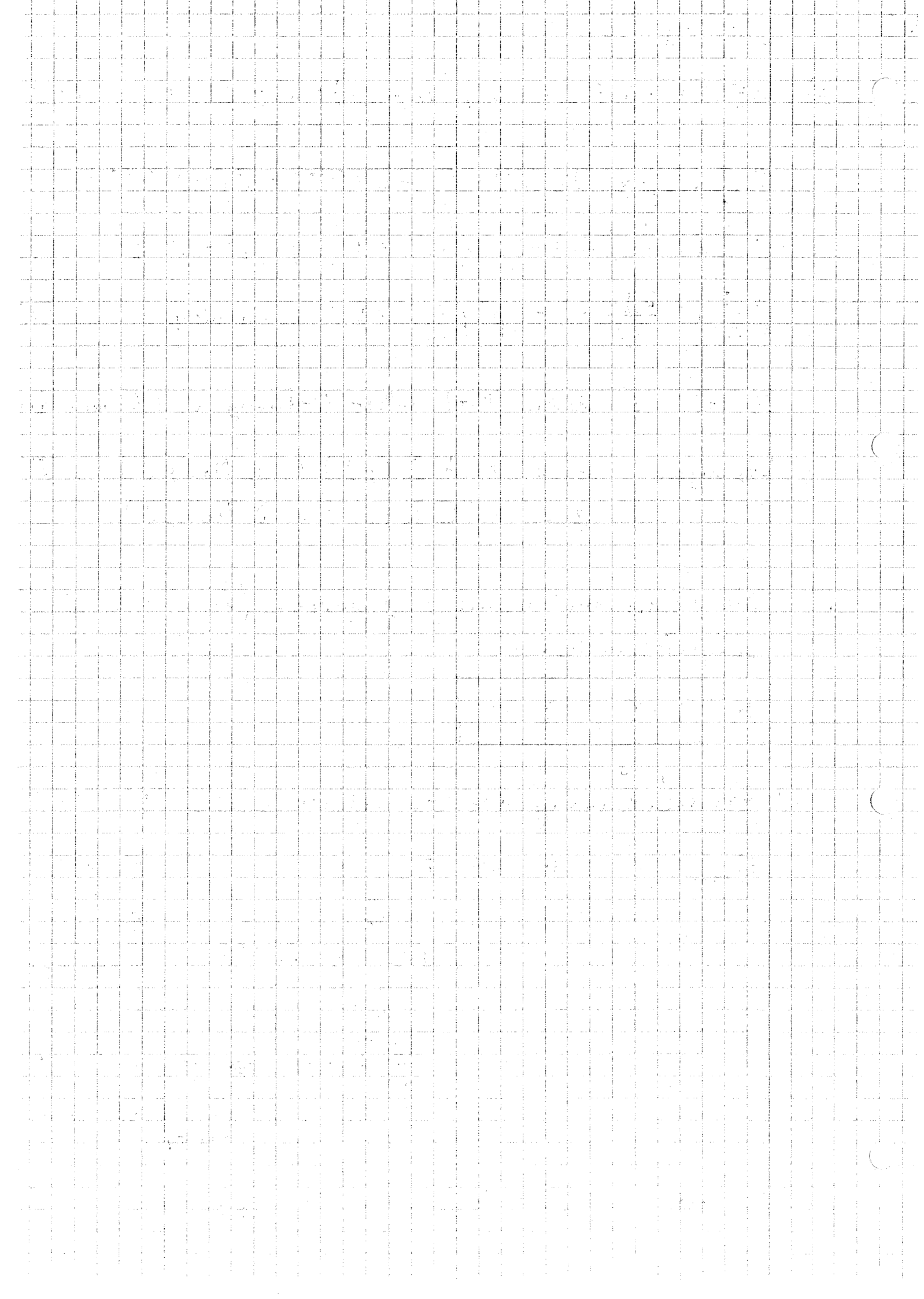
$$\underline{E}_{\text{tang}} = 0 \quad \Rightarrow \quad \alpha) \quad \underline{E}_z = 0 \quad \beta) \quad \underline{E}_\rho = 0$$

$$\Downarrow \\ \underline{g}(\rho) = 0 \quad (C=0)$$

$$\Downarrow \\ \frac{d \underline{g}(\rho)}{d \rho} = 0 \quad \Rightarrow \quad \underline{E}_\rho = 0 \quad (8)$$

überall, also auch auf  $G_1$

Somit:  $\underline{f}(\rho) = A J_0(\rho, \rho) ; \quad \underline{g}(\rho) = 0 \quad (9)$



c) Grenzbedingung auf  $G_2$  (magn. Wand;  $\beta = \alpha$ ):

$$\underline{H}_{\text{tang}} = 0 \Rightarrow \alpha) \underline{H}_z = 0 \quad \beta) \underline{H}_\rho = 0$$

$$\Downarrow \quad \Downarrow$$

$$(10) \quad \underline{f}(\beta = \alpha) = 0 \quad \Rightarrow \frac{d\underline{f}}{d\beta} = 0$$

$$\Downarrow$$

$$(11) \quad \underline{f}'_0(p\alpha) = 0 \quad \text{erfüllt wegen } (P)$$

Die Gleichung (11) legt die möglichen Werte von  $p$  fest; da die Nullstellen der Besselfunktion reell, abzählbar sind, gilt das auch für die  $p$

$$p = p_1, p_2, p_3, \dots, p_\ell, \dots \quad p_\ell \text{ reell} > 0 \quad (12)$$

$$\text{Aus } k^2 - \beta^2 = p^2 \text{ folgt } k^2 - p_\ell^2 = p_\ell^2$$

$$\text{also } p_\ell = \sqrt{k^2 - p_\ell^2}$$

$$= \sqrt{\omega^2 \epsilon \mu_0 - p_\ell^2} \quad ; \quad \ell = 1, 2, 3, \dots \quad (13)$$

d) Es gibt nur Wellenausbreitung in  $+z$ -Richtung für  $\text{Re}\{p_\ell\} > 0$  (14)

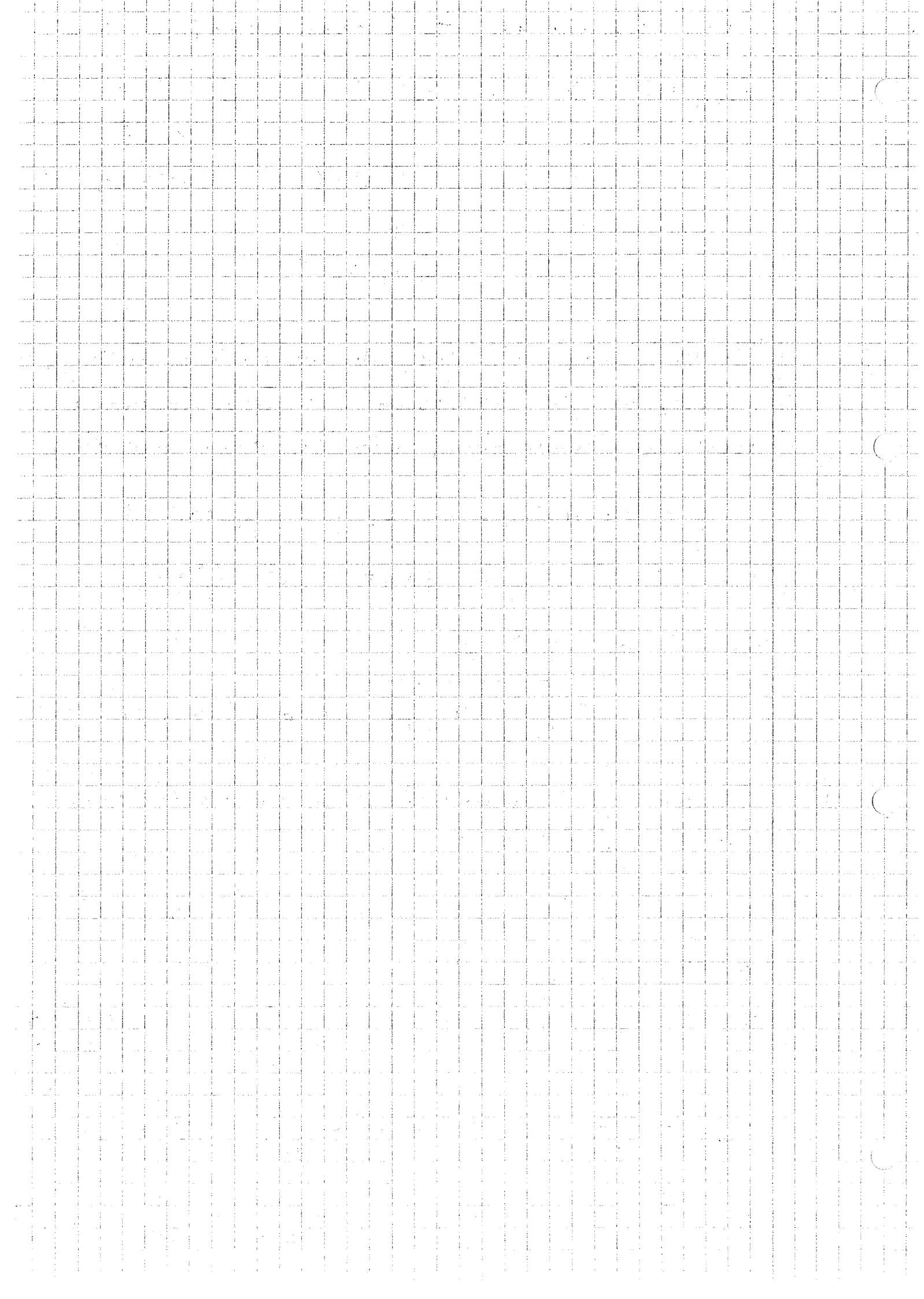
$$\text{aus (13): } \text{Re}\{p_\ell\} > 0, \text{ falls } \omega^2 \epsilon \mu_0 > p_\ell^2$$

~~Wellenausbreitung~~

$$\Rightarrow \text{Grenzfrequenz für Wellentyp "}\ell\text{" : } \omega_\ell = \frac{p_\ell}{\sqrt{\epsilon \mu_0}} \quad (15)$$

$$v_{\text{Gr}, \ell} = \frac{\partial \omega}{\partial p_\ell} \quad ; \quad (13) \Rightarrow \frac{1}{\sqrt{\epsilon \mu_0}} \sqrt{p_\ell^2 + p_\ell^2} = \omega \quad (16)$$

$$v_{\text{Gr}, \ell} \stackrel{(16)}{=} \frac{1}{\sqrt{\epsilon \mu_0}} \frac{p_\ell}{\sqrt{p_\ell^2 + p_\ell^2}} \stackrel{(16)}{=} \frac{p_\ell}{\epsilon \mu_0 \omega} = \frac{1}{\epsilon \mu_0} \sqrt{\epsilon \mu_0 - \left(\frac{p_\ell}{\omega}\right)^2}$$





$$v_{Gr,e} = \frac{1}{\sqrt{\epsilon \mu_0}} \sqrt{1 - \left(\frac{\omega_e}{\omega}\right)^2} \quad (17)$$

$$\begin{aligned} e) \quad \vec{E}_e &= + \frac{df}{dp} e^{-j\beta_e z} \vec{e}_\varphi \\ &= \vec{e}_\varphi A_e p_e \eta'_0(p_e \rho) e^{-j\beta_e z}; \quad A_e = A_e^* \text{ o.B.d.A. (18)} \end{aligned}$$

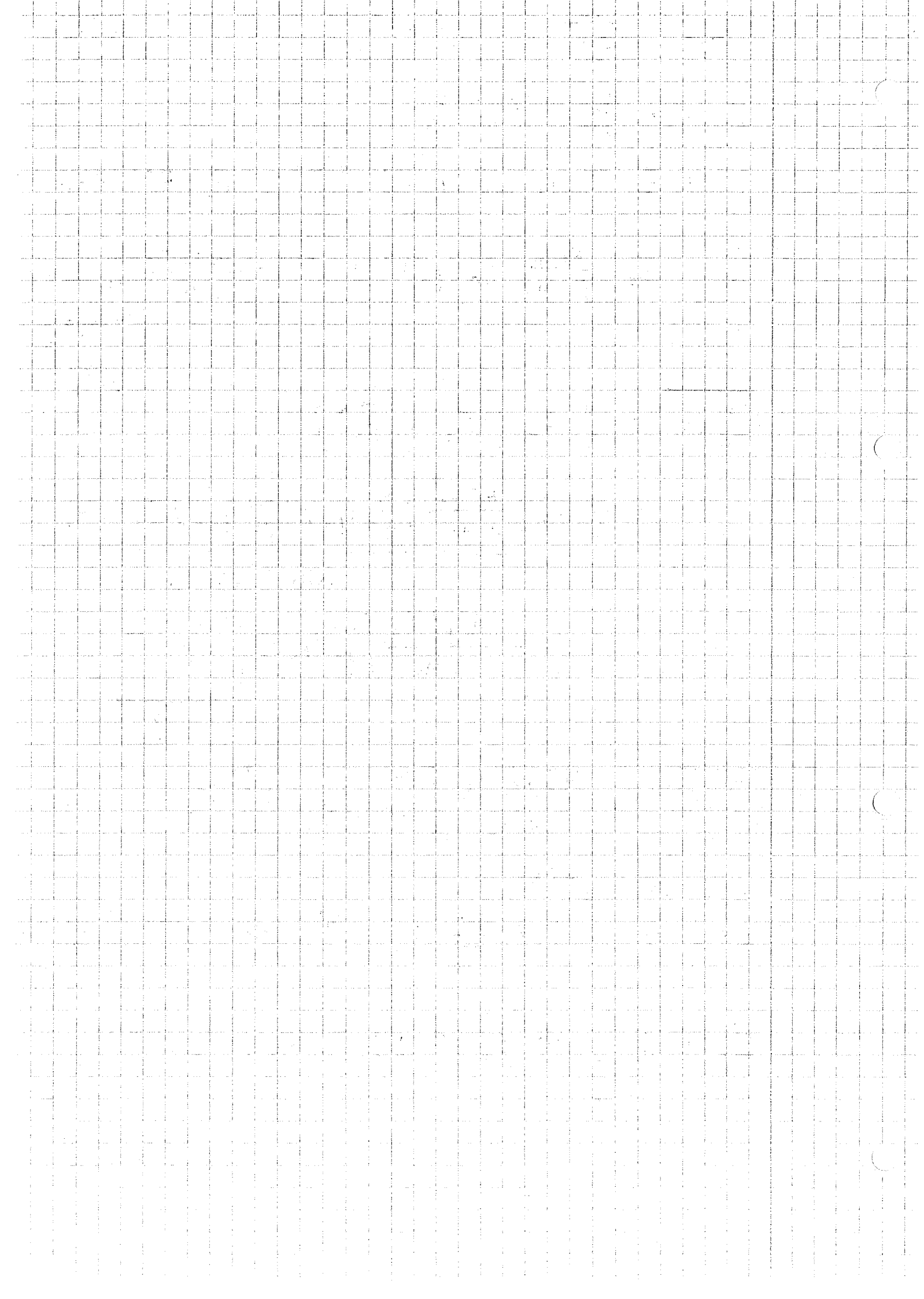
$$\vec{H}_e = \vec{e}_z \frac{p_e^2 A_e}{j\omega\mu_0} \eta_0(p_e \rho) e^{-j\beta_e z} - \frac{\beta_e p_e A_e}{\omega\mu_0} \eta'_0(p_e \rho) e^{-j\beta_e z} \vec{e}_\varphi \quad (19)$$

$$\begin{aligned} \vec{S}_e(t) &= \frac{1}{2} \operatorname{Re} \{ \vec{E}_e \times \vec{H}_e^* \} \\ &= \frac{1}{2} \operatorname{Re} \left\{ \underbrace{\frac{A_e^2 p_e^3}{\omega\mu_0} j \eta_0(p_e \rho) \eta'_0(p_e \rho) \vec{e}_\varphi \times \vec{e}_z}_{\text{rein imaginär}} - \frac{A_e^2 p_e^2 \beta_e}{\omega\mu_0} \left[ \eta'_0(p_e \rho) \right]^2 \underbrace{\vec{e}_\varphi \times \vec{e}_\varphi}_{-\vec{e}_z} \right\} \end{aligned}$$

$$\vec{S}_e(t) = \frac{A_e^2 p_e^2 \beta_e}{2\omega\mu_0} \left[ \eta'_0(p_e \rho) \right]^2 \vec{e}_z \quad (20)$$

$$\begin{aligned} z=0 &\Rightarrow e^{-j\beta_e z} = 1 \\ t = \frac{T}{8} &\Rightarrow \omega t = \frac{2\pi}{T} \frac{T}{8} = \frac{\pi}{4} \Rightarrow e^{j\omega t} = \frac{1}{\sqrt{2}} (1+j) \quad (21) \end{aligned}$$

$$\begin{aligned} \left. \vec{E}_e(t) \right|_{\substack{z=0 \\ t=\frac{T}{8}}} &= \vec{E}_e(0, \frac{T}{8}) = \operatorname{Re} \{ \vec{E}_e e^{j\omega t} \}_{\substack{z=0 \\ t=\frac{T}{8}}} \\ &= \vec{e}_\varphi A_e p_e \eta'_0(p_e \rho) \frac{1}{\sqrt{2}} \quad (22) \end{aligned}$$



$$\begin{aligned}
 \vec{H}_e(t) \Big|_{\substack{z=0 \\ t=\frac{T}{8}}} &= \vec{H}_e(0, \frac{T}{8}) = \operatorname{Re} \left\{ \underline{H}_e e^{j\omega t} \right\}_{\substack{z=0 \\ t=\frac{T}{8}}} \\
 &= \frac{\rho_e^2 A_e}{\omega \mu_0 \sqrt{2}} \int_0 (p_e \rho) \vec{e}_z \\
 &\quad - \frac{\rho_e \rho_e A_e}{\omega \mu_0 \sqrt{2}} \int_0 (p_e \rho) \vec{e}_z
 \end{aligned}$$

$$\begin{aligned}
 \vec{S}_e \Big|_{\substack{z=0 \\ t=\frac{T}{8}}} &= \vec{E}_e(0, \frac{T}{8}) \times \vec{H}_e(0, \frac{T}{8}) \\
 &= \frac{\rho_e^3 A_e^2}{2\omega \mu_0} \int_0' (p_e \rho) \int_0 (p_e \rho) \\
 &\quad + \frac{A_e^2 \rho_e^2 \rho_e}{2\omega \mu_0} \left[ \int_0' (p_e \rho) \right]^2 \vec{e}_z \quad (23)
 \end{aligned}$$

