a) $\vec{k}_I = \vec{e}_z \cdot \omega\sqrt{\varepsilon\mu_0}$

$\vec{k}_{II} = -\vec{e}_z \cdot \omega \cdot \sqrt{\varepsilon\mu_0}$

$k_I = \omega\sqrt{\varepsilon\mu_0} \quad k_{II} = +\sqrt{\omega\mu_0}$

(Vorzeichenwechsel muss konsistent mit späterer Rechnung sein)

b) $\vec{E} = \mathrm{Re}\{\vec{E}_0 \cdot e^{j\omega t} \cdot e^{-jk_I z} + \vec{E}_0 e^{j\varphi} e^{j\omega t} \cdot e^{jk_{II} z}\}$

$= \vec{E}_0 \cdot \mathrm{Re}\{e^{j\omega t} \cdot e^{-jk_I z} + e^{j\varphi} e^{j\omega t} \cdot e^{jk_{II} z}\}$

$= E_0 \vec{e}_y [\cos(\omega t - k_I z) + \cos(\omega t + k_{II} z + \varphi)]$

c) $\vec{E}(z = \frac{1}{3} l_z, t) = 0$ insbes. auch bei $t = 0$

$\Rightarrow \cos(-k_I z)|_{z=\frac{1}{3}l_z} + \cos(k_{II} z + \varphi)|_{z=\frac{1}{3}l_z} = 0$

$\Rightarrow$
$\cos(-k_I \cdot \frac{1}{3} l_z) + \cos(k_{II} \frac{1}{3} l_z + \varphi) = 0$

Die Phase beider cos-Fkt muss sich um $(2n+1)\pi \quad n = 0, \pm 1, \pm 2 \ldots$ unterscheiden

$\varphi = -\frac{2}{3} l_z k_I + (2n+1)\pi$, da $n = 0, \pm 1, \pm 2 \ldots$ beliebig wie n

$-k_I \frac{1}{3} l_z = k_I \frac{1}{3} l_z + \varphi - (2n+1)\pi$

$\Rightarrow \varphi = (2n+1)\pi - k_I (\frac{1}{3} l_z + \frac{1}{3} l_z)$

$\Rightarrow \varphi = -\frac{2}{3} l_z k_I + (2n+1)\pi$

Gesucht: $\vec{H}(z = \frac{1}{3} l_z, t)$ mit φ wie bestimmt

$\vec{H} = 2 \cdot \vec{H}_I = \frac{2}{Z}(\vec{e}_z \times \vec{E}_I)$

$= 2 \cdot \sqrt{\frac{\varepsilon}{\mu_0}} E_0 (\vec{e}_z \times \vec{e}_y) \cos(\omega t - k_I z)$

$= 2 \cdot \sqrt{\frac{\varepsilon}{\mu_0}} E_0 (-\vec{e}_x) \cos(\omega t - k_I \cdot \frac{1}{3} l_z)$

d) $\vec{E} = E_0 \cdot \vec{e}_y [\cos(\omega t - k_I z) + \cos(\omega t + k_{II} z + \varphi)$

$= E_0 \cdot \vec{e}_y [\cos(\omega t - k_I z) + \cos(\omega t + k_{II} z + (2n+1)\pi - \frac{2}{3} l_z k_I)]$

$= E_0 \vec{e}_y [\cos(\omega t - k_I z) + \cos(\pi + \omega t + k_{II} z - \frac{2}{3} l_z k_I)]$

$$\vec{E} = E_0 \vec{e}_y \left[\cos(\omega t - k_I z) - \cos(\omega t + k_{II}(z - \tfrac{2}{3} l_z)\right]$$

$$\vec{H} = E_0 \sqrt{\frac{\varepsilon}{\mu_0}} \left[(\vec{e}_z \times \vec{e}_y)\cos(\omega t - k_I z) - (-\vec{e}_z \times \vec{e}_y)\cos(\omega t + k_{II}(z - \tfrac{2}{3} l_z))\right]$$

$$= E_0 \sqrt{\frac{\varepsilon}{\mu_0}} \left[-\vec{e}_x \cos(\omega t - k_I z) - \vec{e}_x(\cos(\omega t + k_{II}(z - \tfrac{2}{3} l_z))\right]$$

$$= -E_0 \cdot \vec{e}_x \sqrt{\frac{\varepsilon}{\mu_0}} \left[\cos(\omega t - k_I z) + \cos(\omega t + k_{II}(z - \tfrac{2}{3} l_z))\right]$$

geht auch über $\operatorname{rot}\vec{E} = -\dot{\vec{B}} = -\mu_0 (j\omega)\vec{H}$

$$\vec{H} = -\frac{\operatorname{rot}\vec{E}}{j\omega}$$

e) $$U(z,t) = \int_{y=-\frac{a}{2}}^{\frac{a}{2}} \vec{E} \cdot \vec{e}_y \cdot dy$$

$$= a E_0 \left[\cos(\omega t - k_I z) - \cos(\omega t + k_{II}(z - \tfrac{2}{3} l_z))\right]$$

$I = ?$

$\operatorname{rot}\vec{H} = \vec{J} + \dot{\vec{D}}$ // $\oint \vec{H} d\vec{l} = \iint (\vec{J} + \dot{\vec{D}}) dA$

wegen TEM bzw $\vec{E} \parallel \vec{e}_y$ und $d\vec{A} \parallel \vec{e}_z$

$\Rightarrow \oint \vec{H} dl = I$

Nur Beiträge innerhalb des Dielektrikums

Umlaufsinn $x = \frac{b}{2} \dots (-\frac{b}{2})$

$$I = \int_{+\frac{b}{2}}^{-b/2} \vec{H}\, \vec{e}_x\, dx = +b E_0 \sqrt{\frac{\varepsilon}{\mu_0}} \left[\cos(\omega t - k_I z) + \cos(\omega t + k_{II}(z - \tfrac{2}{3} l_z))\right]$$

ufgabe 2

a) $\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t} = -j\omega\mu_0 \vec{H} \Rightarrow \vec{H} = j\frac{1}{\omega\mu_0}\nabla\times\vec{E}$

$\nabla\times(E_x\,\vec{e}_x) = \frac{\partial}{\partial z}E_x\cdot\vec{e}_y - \frac{\partial}{\partial y}E_x\,\vec{e}_z$

$\vec{H}_e = j\frac{1}{\omega\mu_0}E_0\,(-j\sqrt{1-\delta^2}\,k_1\sin(\delta k_1 y)\cdot e^{-j\sqrt{1-\delta^2}k_1 z}\cdot\vec{e}_y$

$-k_1\delta\cos(\delta k_1 y)\cdot e^{-j\sqrt{1-\delta^2}k_1 z}\cdot\vec{e}_z$

$= j\frac{1}{\omega\mu_0}E_0\,e^{-j\sqrt{1-\delta^2}k_1 z}\left(-j\sqrt{1-\delta^2}\,k_1\sin(\delta k_1 y)\vec{e}_y - k_1\delta\cos(\delta k_1 y)\cdot\vec{e}_z\right)$

$\vec{H}_r = j\frac{1}{\omega\mu_0}E_{r0}\,e^{+j\sqrt{1-\delta^2}k_1 z}\left(+j\sqrt{1-\delta^2}\,k_1\sin(\delta k_1 y)\vec{e}_y - k_1\delta\cos(\delta k_1 y)\vec{e}_z\right)$

$\vec{H}_d = j\frac{1}{\omega\mu_0}E_{d0}\,e^{-j\sqrt{1-\delta_d^2}k_2 z}\left(-j\sqrt{1-\delta_d^2}\,k_2\sin(\delta_d k_2 y)\vec{e}_y - k_2\delta_d\cos(\delta_d k_2 y)\vec{e}_z\right)$

b) $\overline{\vec{S}_2(t)} = \frac{1}{2}\text{Re}\{\vec{E}_d\times\vec{H}_d^*\} = \frac{1}{2}\text{Re}\{|E_{d0}|^2\frac{1}{\omega\mu_0}(\sin^2(\delta_d k_2 y)\sqrt{1-\delta_d^2}\,k_2$

E_{d0} kann komplex sein

$-jk_2\delta_d\cos(\delta_d k_2 y)\sin(\delta_d k_2 y)\}\underbrace{(\vec{e}_x\times\vec{e}_z)}_{-\vec{e}_y}$

$\Rightarrow \overline{\vec{S}_2(t)} = \frac{1}{2}|E_{d0}|^2\frac{1}{\omega\mu_0}(\sin^2(\delta_d k_2 y)\sqrt{1-\delta_d^2}\,k_2\,\vec{e}_z)$

c) $\sqrt{1-\delta_d^2} = 0 \Rightarrow \delta_d^2 = 1 \Rightarrow \delta_d = 1$

gemäß Aufgabenstellung: $\delta_d > 0$

d) E_{tan} ist stetig, H_{tan} ist stetig bei $z = 0$

$\Rightarrow$ alle exp-Terme 1

$E_{xe} + E_{xr} = E_{xd}$ ①

$E_{ye} + H_{yr} = H_{yd}$ ②

① $E_0\sin(\delta k_1 y) + E_{r0}\sin(\delta k_1 y) = E_{d0}\sin(k_2 y)$

② $E_0\cdot(-j\sqrt{1-\delta^2}\,k_1\sin(\delta k_1 y)) + j\sqrt{1-\delta^2}\,k_1\sin(\delta k_1 y)E_{r0} \overset{!}{=} 0$

weil $\sqrt{1-\delta_d^2} = 0$

$\Rightarrow E_0 = +E_{r0}$ in 1 einsetzen

$$E_{d0} = \frac{\sin(\delta k_1 y)}{\sin(k_2 y)} \cdot 2E_0 \qquad \left[\begin{array}{l}\text{muss für alle} \\ y \text{ gelten}\end{array}\right]$$

$\Rightarrow \sin(\delta k_1 y) = \sin(k_2 y)$

$\Rightarrow \delta k_1 = k_2$

$\Rightarrow \delta = \frac{k_2}{k_1} = \frac{1}{\sqrt{2}} \qquad \left[\frac{\sin(c \cdot y)}{\sin(c \cdot y)} = 1 \right]$

$\Rightarrow E_{d0} = 2 \cdot E_0$

e) $E_d(t) = 2 \cdot E_0 \sin(k_2 y) \, Re\{ e^{-j \cdot 0 \cdot z + j\omega t} \} \vec{e}_x = 2 \cdot E_0 \sin(k_2 y) \vec{e}_x \cos(\omega t)$

$\vec{H}_d(t) = Re\{ \vec{H}_d(t) \} = Re\{ j \frac{1}{\omega \mu_0} \cdot 2 \cdot E_0 \, e^{-j \cdot 0 \cdot z + j\omega t} \cdot (-k_2 \cos(\delta_d k_2 y) \cdot \vec{e}_z) \}$

$= + \frac{k_2}{\omega \mu_0} \cdot 2 \cdot E_0 \cdot \cos(k_2 y) \cdot \sin(\omega t) \, \vec{e}_z$

mit $k_2 = \omega \sqrt{\varepsilon_0 \mu_0} \Rightarrow \frac{k_2}{\omega \mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}} = \frac{1}{Z_F}$

$H_d(t) = \frac{1}{Z_F} \cdot 2 \cdot E_0 \cdot \cos(k_2 y) \sin(\omega t) \cdot \vec{e}_z$

$\vec{S}_2(t) = \vec{E}_d \times \vec{H}_d = \frac{4 E_0^2}{Z_F} \cdot \cos(k_2 y) \sin(k_2 y) \cos(\omega t) \sin(\omega t) \underbrace{(\vec{e}_x \times \vec{e}_z)}_{-\vec{e}_y}$

∘ mit $\cos x \cdot \sin x = \frac{1}{2} \sin(2x)$

$\vec{S}_2(t) = -\frac{E_0^2}{Z_F} \sin(2 k_2 y) \sin(2 \omega t) \, \vec{e}_y$

A3 F09

$\underline{E}_x = 0$

$\underline{E}_y = -\frac{\partial f}{\partial z} = j\beta \tilde{f} e^{-j\beta z}$

$\underline{E}_z = 0$

$\underline{H}_x = \frac{1}{j\omega\mu}\left(k^2 + \frac{\partial^2}{\partial x^2}\right)f = \frac{\beta^2}{j\omega\mu} \tilde{f} e^{-j\beta z}$

$\underline{H}_y = 0$

$\underline{H}_z = -\frac{\beta}{\omega\mu}\frac{\partial f}{\partial x} = -\frac{\beta}{\omega\mu}\frac{\partial \tilde{f}}{\partial x} e^{-j\beta z}$

$\tilde{f}'' + p_i^2 \tilde{f} = 0$

$p_i^2 = k_i^2 - \beta^2$

a) $\Delta \underline{f}_i + k_i^2 \underline{f}_i = 0$ mit $k_i^2 = \omega^2 \varepsilon_i \mu_0 = \varepsilon_{ri} k_0^2$

$\Delta_{tr} \tilde{\underline{f}}_i + p_i^2 \tilde{\underline{f}}_i = 0$ mit $p_i^2 = k_i^2 - \beta^2$

$$\boxed{\tilde{\underline{f}}_i'' + p_i^2 \tilde{\underline{f}}_i = 0}$$

$\tilde{\underline{f}}_1(x) = \underline{A}_1 \cos(p_1 x) + \cancel{\underline{B}_1 \sin(p_1 x)}$

$\tilde{\underline{f}}_2(x) = \underline{A}_2 \cdot \cos(p_2(2a-x)) + \cancel{\underline{B}_2 \sin(p_2(2a-x))}$

b) magnetische Wände bei $x=0$ und $x=2a$

$\vec{H}_{tan} = 0$

$\underline{H}_{1z}(x=0) = 0 \Rightarrow \underline{B}_1 = 0$

$\underline{H}_{2z}(x=2a) = 0 \Rightarrow \underline{B}_2 = 0$

$H_{1z}(x) = \frac{\beta}{\omega\mu_0} p_1 (\underline{A}_1 \sin(p_1 x) - \cancel{\underline{B}_1 \cos(p_1 x)}) e^{-j\beta z}$

$H_{2z}(x) = -\frac{\beta}{\omega\mu_0} p_2 (\underline{A}_2 \sin(p_2(2a-x)) - \cancel{B_2 \cos(p_2(2a-x))}) e^{-j\beta z}$

c) Stetigkeitsbedingungen bei $x = a$

$\vec{E}_{1tan} = \vec{E}_{2tan}$ $\quad \vec{H}_{1tan} = \vec{H}_{2tan}$

$\underline{E}_{1y}(x=a) = \underline{E}_{2y}(x=a)$

$\underline{H}_{1z}(x=a) = \underline{H}_{2z}(x=a)$

$\underline{A}_1 \cos(p_1 a) = \underline{A}_2 \cos(p_2 a)$

$p_1 \underline{A}_1 \sin(p_1 a) = -p_2 \underline{A}_2 \sin(p_2 a)$

$$\boxed{p_1 \tan(p_1 a) = -p_2 \tan(p_2 a)}$$

d) $\omega = v_{phase} \cdot \beta = \frac{1}{\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_{r,eff}}} \cdot \frac{2\pi}{\lambda_\beta}$

$p_i \cdot a = \omega \sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_{ri} - \varepsilon_{reff}}\, a = 2\pi \sqrt{\frac{\varepsilon_{ri}}{\varepsilon_{r,eff}} - 1}\ \frac{a}{\lambda_\beta}$

Näherung: ω klein

$p_1^2 = -p_2^2$ mit $p_i^2 = (\varepsilon_{ri} - \varepsilon_{r,eff})\, k_0^2$

$\varepsilon_{r1} - \varepsilon_{reff} = -(\varepsilon_{r2} - \varepsilon_{reff})$

$$\Rightarrow \boxed{\varepsilon_{r,eff} = \frac{1}{2}(\varepsilon_{r1} + \varepsilon_{r2})}$$

Quasi TEM Welle

$H_{iz} \sim \frac{\beta}{\omega} p_i = \frac{\cancel{\omega}\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_{r,eff}}}{\cancel{\omega}} \cdot \omega \sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_i - \varepsilon_{r,eff}}$

$H_i \sim \omega$

also $H_{iz} \to 0$ für $\omega \to 0$

$E_{iz} = 0$

$v_{phase} = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_{r,eff}}} = \frac{c_0}{\sqrt{\varepsilon_{r,eff}}}$

v_{phase} ist unabhängig von ω (Grenzfrequenz $\omega_c = 0$)

Grenzfrequenz $\beta(\omega_c) = 0 \Rightarrow \omega_c = 0$

$v_g = \frac{d\omega}{d\beta} = \left(\frac{d\beta}{d\omega}\right)^{-1} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_{r,eff}}} = \frac{c_0}{\sqrt{\varepsilon_{r,eff}}} = v_{phase}$

$\beta = \omega\sqrt{\mu_0\, \varepsilon_{eff}} \qquad \frac{d\beta}{d\omega} = \sqrt{\mu_0\, \varepsilon_{eff}}$

e) $\underline{\vec{S}} = \underline{\vec{E}} \times \underline{\vec{H}}^* = \underline{E}_y \vec{e}_y \times (\underline{H}_x^* \vec{e}_x + \underline{H}_z^* \vec{e}_z) = \underline{E}_y \underline{H}_x^* (-\vec{e}_z) + \underline{E}_y \underline{H}_z^* \vec{e}_x$

$S_x = E_y \cdot H_z^*$

Bereich 1: $E_{1y} = j\beta \underline{A}_1 \cos(p_1 x) e^{-j\beta z}$

$H_{1z}^* = \left(\frac{\beta}{\omega\mu} \underline{A}_1^* p_1 \sin(p_1 x) e^{j\beta z}\right)$

$\Rightarrow \underline{S}_{1x} = j \frac{\beta^2}{\omega\mu} |\underline{A}_1|^2 p_1 \sin(p_1 x)\cos(p_1 x) \Rightarrow \overline{S}_{1x} = \frac{1}{2} \mathrm{Re}\{\underline{S}_{1x}\} = 0$

Bei der betrachteten TE_x Welle findet im zeitlichen Mittel im Bereich 1 kein Leistungstransport in x-Richtung statt. Durch die mag. Wand bei $x = 0$ kann wegen $H_{tan} = 0$ keine Leistung hindurchtreten.

Klausur FO9

Aufgabe 4

a) Ports 2 and 3 are terminated with Z_0 and an RF source with internal impedance Z_0 is connected at port 1. The impedance seen at port 1 is:

$$Z_{in} = \frac{1}{3}Z_0 + \frac{1}{2}\left(\frac{1}{3}Z_0 + Z_0\right) = Z_0$$

Therefore: $\underline{S}_{11} = 0$

b) Remember that $a_i = \frac{1}{2}\left(\frac{\underline{U}_i}{\sqrt{Z_0}} + \sqrt{Z_0}\,\underline{I}_i\right)$

and $b_i = \frac{1}{2}\left(\frac{\underline{U}_i}{\sqrt{Z_0}} - \sqrt{Z_0}\,\underline{I}_i\right)$

$$\underline{S}_{21} = \frac{\underline{b}_2}{\underline{a}_1} = \frac{\underline{U}_2 - Z_0\underline{I}_2}{\underline{U}_1 + Z_0\underline{I}_1} = \frac{\underline{I}_2}{\underline{I}_1}\cdot\frac{\frac{\underline{U}_2}{\underline{I}_2} - Z_0}{\frac{\underline{U}_1}{\underline{I}_1} - Z_0} = -\frac{\underline{I}_2}{\underline{I}_1}$$

da $-\frac{\underline{U}_2}{\underline{I}_2} = Z_0$ und $\frac{\underline{U}_1}{\underline{I}_1} = Z_0$

The current $\underline{I}_2$ can be calculated by means of current division or just by symmetry.

$$-\underline{I}_2 = \underline{I}_1\frac{Z_0 + \frac{1}{3}Z_0}{2(Z_0 + \frac{1}{3}Z_0)} = \frac{1}{2}\underline{I}_1$$

Therefore $\underline{S}_{21} = \frac{1}{2}$

Second way of solving the problem:

$\underline{S}_{21} = \frac{\underline{U}_2^-}{\underline{U}_1^+}$ where $\underline{U}_1 = \underline{U}_1^+ + \underline{U}_1^-$ and $\underline{U}_2 = \underline{U}_2^+ + \underline{U}_2^-$

Since the divider is matched at all ports $\underline{U}_1^- = 0$ and $\underline{U}_2^- = 0$. Therefore $\underline{S}_{21} = \frac{\underline{U}_2}{\underline{U}_1}$.

By voltage division:

$\underline{U}_p = \frac{2}{3}\underline{U}_1$ and $\underline{U}_2 = \frac{3}{4}\underline{U}_p \Rightarrow \underline{U}_2 = \frac{1}{2}\underline{U}_1$

Therefore $\underline{S}_{21} = \frac{1}{2}$

c) $P_{RP1} = 1\,W \mathrel{\hat{=}} 30\,dBm$

The transmission loss of the divider is -6dB (a quarter of the power that enters port 1 gets out of port 2).

$P_2 = |\underline{S}_{21}|^2 P_1$

$P_{RP3} = 30\,dBm - 6\,dB + G_{amp} = 50\,dBm$

$\Rightarrow G_{amp} = 26\,dB$

also $G = \frac{100W}{0{,}25W} = 400$

e) S-parameters of transmission line of length l_x:

$$\underline{S} = \begin{bmatrix} 0 & e^{-j\theta_x} \\ e^{-j\theta_x} & 0 \end{bmatrix} \quad \text{with } \theta_x = \beta l_x$$

f) S-Parameters of combiner and transmission line:

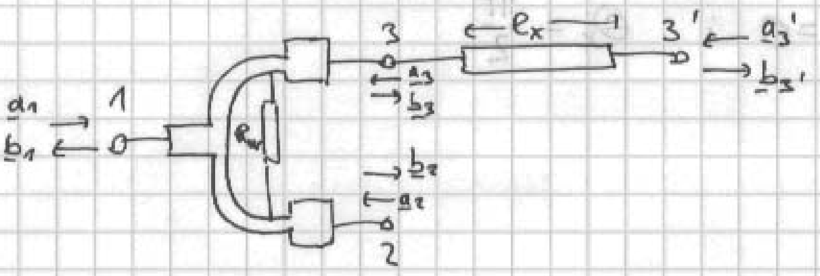
Let's write the following two relations:

$$\underline{a}_3 = \underline{a}_{3'}\, e^{-j\Theta_x} \quad \text{and} \quad \underline{b}_{3'} = \underline{b}_3\, e^{-j\Theta_x} = -\frac{j}{\sqrt{2}}\, \underline{a}_1\, e^{-j\Theta_x}$$

Knowing this, it's possible to write the full matrix as follow:

$$\begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \\ \underline{b}_3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -j & -je^{-j\Theta_x} \\ -j & 0 & 0 \\ -je^{-j\Theta_x} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \underline{a}_1 \\ \underline{a}_2 \\ \underline{a}_3 \end{bmatrix}$$

$$-je^{-j\Theta_x} = e^{-j(\Theta_x + \frac{\pi}{2})}$$



g) Reflected waves $\underline{a}_1$ and $\underline{a}_3$:

$$\underline{b}_2 = -\frac{j}{\sqrt{2}}\underline{a}_1 \Rightarrow \underline{a}_2 = -\frac{j}{\sqrt{2}}\underline{a}_1 \underline{\Gamma}_x$$

$$\underline{b}_3 = -\frac{j}{\sqrt{2}}\underline{a}_1 \quad \text{and} \quad \underline{b}_{3'} = \underline{b}_3\, e^{-j\Theta_x} = \frac{j}{\sqrt{2}}\underline{a}_1\, e^{-j\Theta_x}$$

$$\Rightarrow \underline{a}_{3'} = \underline{b}_{3'} \underline{\Gamma}_x = -\frac{j}{\sqrt{2}}\underline{a}_1\, e^{-j\Theta_x} \underline{\Gamma}_x$$

but $\underline{a}_3 = \underline{a}_{3'}\, e^{-j\Theta_x}$

therefore $\underline{a}_3 = -\frac{j}{\sqrt{2}}\underline{a}_1\, e^{-j2\Theta_x}\underline{\Gamma}_x$

h) Reflected wave $\underline{b}_1$:

$$\underline{b}_1 = -\frac{j}{\sqrt{2}}(\underline{a}_2 + \underline{a}_5) = -\frac{1}{2}\left(1 + e^{-j2\theta_x}\right)\underline{a}_1 \underline{\Gamma}_x$$

i) Electrical length of the line θ for which $\underline{b}_1 = 0$:

Making $1 + e^{-j2\theta_x} = 0$ or $e^{-j2\theta_x} = -1$.

This can be written:

$$\cos(2\theta_x) - j\sin(2\theta_x) = -1 + j\,0$$

This is fulfilled when:

$$2\theta_x = \pm(2n+1)\pi \Leftrightarrow \theta_x = \pm\frac{\pi}{2}(2n+1) \quad , n = 0, 1, 2 \ldots$$

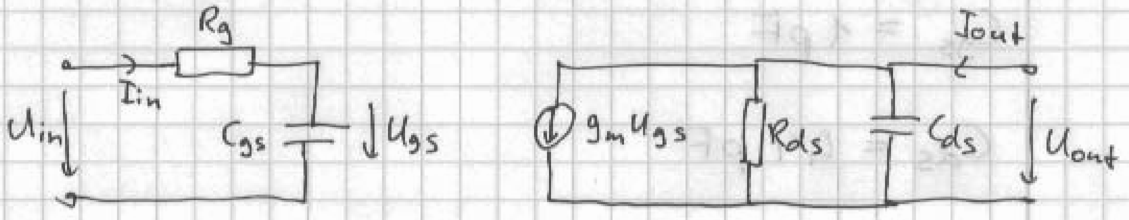
e.g. for $n = 0$: $\theta_x = \frac{\pi}{2}$

Klausur F09

Aufgabe 5

(ohne Smith-Chart!)

a)



$S_{11} \Rightarrow z_{11} = 0{,}1 - j2$

$\Rightarrow Z_{1n} = 5\Omega - j2\Omega$

$\Rightarrow R_g = 50\Omega \cdot 0{,}1 = 5\Omega$

$\Rightarrow C_{gs} = \frac{1}{10 \cdot 10^9 \cdot 2 \cdot 50} = 1\,pF$

$S_{22} \Rightarrow \cancel{Y}_{22} = 0{,}1 + j0{,}2$

$\Rightarrow \frac{1}{Z_{out}} = 2\,mS + j4\,mS$

$\Rightarrow R_{ds} = 500\Omega$

$\Rightarrow C_{ds} = 0{,}4\,pF$

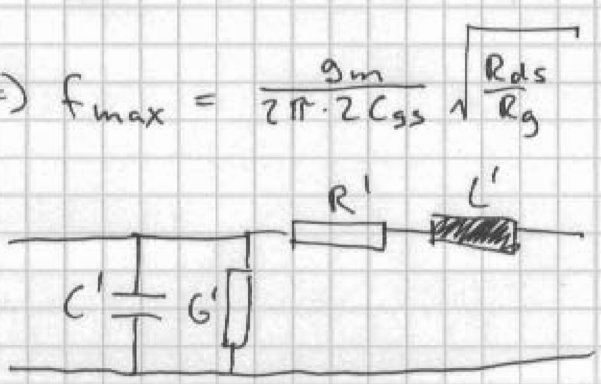
b) $H_{21} = \left|\frac{I_{out}}{I_{in}}\right| = \frac{U_{gs} \cdot g_m}{|I_{in}|} = \frac{g_m}{\omega C_{gs}} = \frac{0{,}05 \cdot 2\pi}{2\pi f_T \cdot 1pF} = 1$

$\Rightarrow f_T = \frac{0{,}05\,S}{1\,pF} = 50\ GHz$

c) $U = \frac{P_{out}}{P_{in}} = \frac{(\frac{1}{2} g_m U_{gs})^2 R_{ds}}{R_g (U_{gs} \omega C_{gs})^2} \Rightarrow \cancel{f_{max}}$

$\Rightarrow f_{max} = \frac{g_m}{2\pi \cdot 2 C_{gs}} \sqrt{\frac{R_{ds}}{R_g}} = \frac{2\pi \cdot 0{,}05}{2\pi \cdot 2 \cdot 1pF} \sqrt{\frac{500}{5}} = 250\ GHz$

d)



Tiefpassverhalten

e) $Z_0 = \sqrt{\frac{L}{C}}$

f) Verlustlos $\Rightarrow$ keine Widerstände

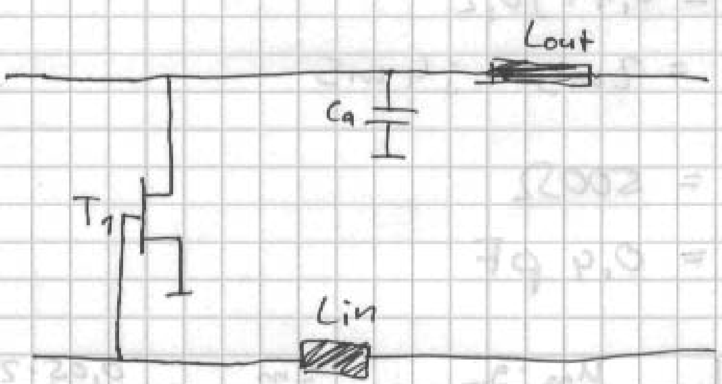
$C_{gs} = 1\,pF$

$C_{ds} = 0{,}4\,pF$

g) $L_g = Z_0^2 C_{gs} = 2500 \cdot 1\,pF = 2{,}5\,nH$

h) Größere Kapazität erforderlich, d.h. zusätzlicher Kondensator C_a parallel zum Transistorausgang.

$C_a = C_{gs} - C_{ds} = 1\,pF - 0{,}4\,pF = 0{,}6\,pF$



1. Stufe

i) Sie müssen die gleiche Phasengeschwindigkeit haben, damit sich die Wellen am Ausgang der Transistoren konstruktiv überlagern.

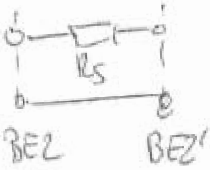
EMF-F09

Aufgabe 6:

a) $\underline{Z}_{22} = (1{,}8 - j0{,}3) \cdot 75\,\Omega = (135 - j22{,}5)\,\Omega$

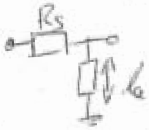
$$\underline{S}_{22} = \frac{\underline{Z}_{22} - Z_0}{\underline{Z}_{22} + Z_0}$$

b) Konz. Bauelement: Serienwiderstand



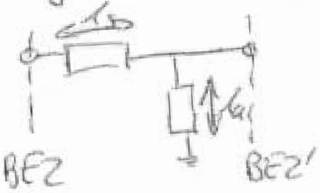
Anpassung: 1. Möglichkeit: $l_{a1} = 0{,}065\lambda$, $R_{S1} = 0{,}9 \cdot 75\,\Omega = 67{,}5$.
2. " : $l_{a2} = 0{,}0315\lambda$, $R_{S2} = 1{,}7 \cdot 75\,\Omega = 127{,}5\,\Omega$

Abschluss: KS



c) Nachteile: verlustbehaftet und mehr Rauschen

Anpassung mit verlustloser Serienleitung mit Wellenwiderstand $Z_{01} = 75\,\Omega$ ist möglich.



d) Kurzgeschl. Stichleitung der Länge $l_y = \frac{\lambda}{4}$ bei $f = f_0$ keinen Einfluss. Leitung d. Länge $l_x = \frac{\lambda}{8}$ bewirkt Phasendrehung

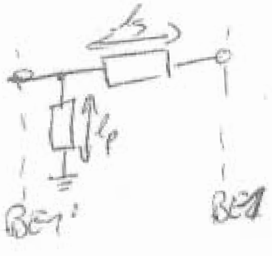
$$\underline{\Gamma}_Q' = \underline{\Gamma}_Q \cdot e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{13}} = \underline{\Gamma}_Q \cdot e^{-j\frac{4\pi}{3}}$$

Hier: $\underline{\Gamma}_Q = 0 \Rightarrow \underline{\Gamma}_Q' = 0$

e) Bei $f = 2f_0$ wirkt die Leitung als Kurzschluss da $l = \frac{\lambda}{2}$.

Bei $f = 0$ " " " " "

f) $l_s = 0{,}2515\lambda$, $l_p = 0{,}1095\lambda$



g) $l_s = 0$, $l_p = 0{,}125\lambda$

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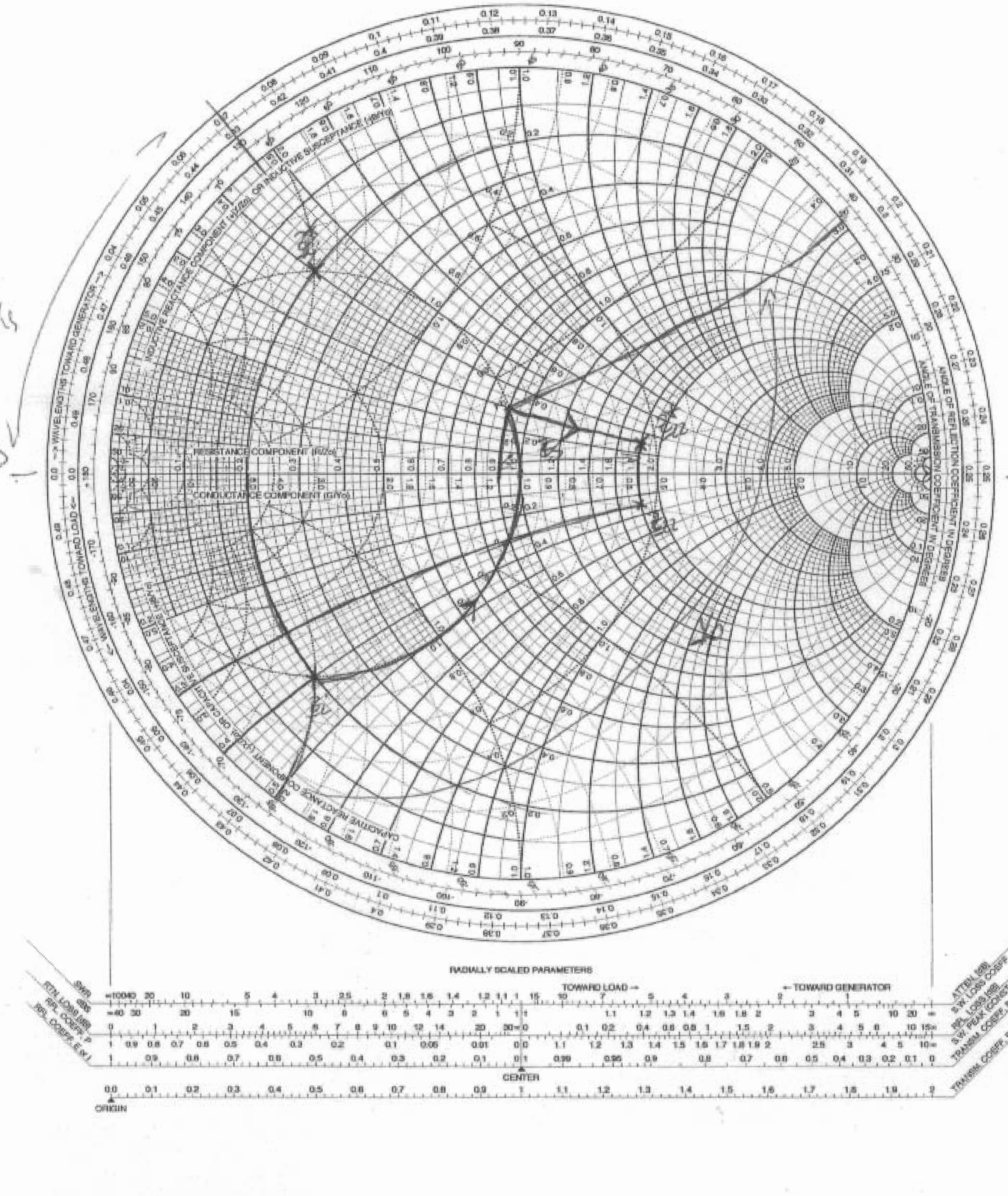
Frühjahr 2009

KLAUSURAUFGABEN "ELEKTROMAGNETISCHE FELDER 2 (EE)"

Hilfsblatt 1 zu Aufgabe 6, Unterpunkt b)

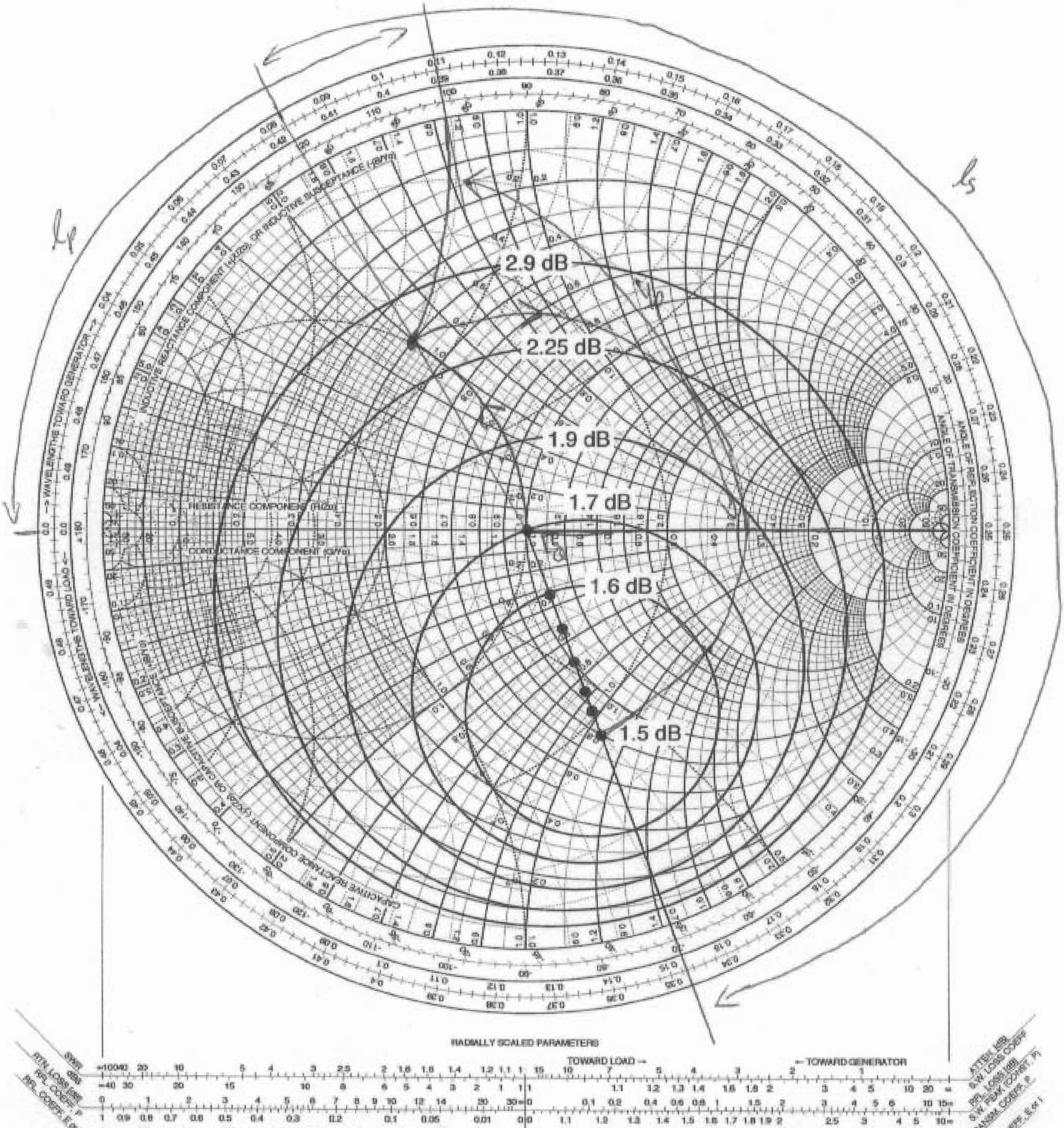
Name: ____________________ **Matr.Nr.:** __________



KLAUSURAUFGABEN "ELEKTROMAGNETISCHE FELDER 2 (EE)"

Hilfsblatt 2 zu Aufgabe 6, Unterpunkt f)

Name: ____________________ **Matr.Nr.:** __________



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Frühjahr 2009

KLAUSURAUFGABEN "ELEKTROMAGNETISCHE FELDER 2 (EE)"

Hilfsblatt 3 zu Aufgabe 6, Unterpunkt g)

Name: ____________________ **Matr.Nr.:** __________