H09

Aufgabe 1

a)

$$\vec{k} = \omega \sqrt{\varepsilon_0 \mu_0} \cdot \vec{e}_z \quad ; \quad k = |\vec{k}|$$

$$\vec{\underline{E}} = \underline{E}_0 \cdot \exp(-jkz) = E_{re} \cdot e^{j\varphi} \cdot e^{-jkz} \cdot \vec{e}_x$$

$$= E_{re} \cdot [\cos\varphi + j\sin\varphi] \cdot \vec{e}_x \cdot e^{-jkz}$$

Randbedingungen:

$$\mathrm{Re}\{\vec{\underline{E}} \cdot e^{j\omega t}\}\Big|_{\substack{t=0\\z=0}} \overset{!}{=} 0 \quad \Longrightarrow \varphi = \frac{\pi}{2};\ \frac{3\pi}{2};\ \frac{5\pi}{2};\ \ldots$$

$$\Rightarrow \cos\varphi = 0 \qquad e^{j\varphi} = j, \text{ da } \sin\frac{\pi}{2} = 1$$

Wahl von $\varphi = \frac{\pi}{2}$

$$\Longrightarrow \vec{\underline{E}} = j \cdot E_{re} \cdot \vec{e}_x \cdot e^{-jkz} \quad ; \quad \vec{E} = -E_{re} \cdot \vec{e}_x \cdot \sin(\omega t - kz)$$

b)

$$\vec{\underline{H}} = \frac{1}{Z} \cdot (\vec{e}_z \times \vec{\underline{E}}) \quad \text{da TEM-Welle}$$

$$\Rightarrow \vec{\underline{H}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot j \cdot E_{re} \cdot e^{-jkz} \cdot \vec{e}_y$$

$$\vec{H} = \mathrm{Re}\{\vec{\underline{H}} \cdot e^{+j\omega t}\} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot E_{re} \cdot \vec{e}_y \cdot \mathrm{Re}\{j\, e^{-jkz} \cdot e^{j\omega t}\}$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot E_{re} \cdot \vec{e}_y \cdot (-\sin(\omega t - kz))$$

c)

$$\Phi_L(t) = \int_{F_L} \mu_0 \vec{H}\, d\vec{F} \qquad d\vec{F} = dx\,dz \cdot \vec{e}_y$$

$$= a \cdot \mu_0 \int_a^{2a} \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot E_{re} \cdot (-\sin(\omega t - k \cdot z))\, dz$$

$$= -a \cdot \sqrt{\varepsilon_0 \mu_0} \cdot E_{re} \cdot \int_a^{2a} \sin(\omega t - kz)\, dz$$

: Substitution : $u = \omega t - k \cdot z$ etc.

$$= +a \cdot \sqrt{\varepsilon_0 \mu_0} \cdot \frac{1}{\omega \sqrt{\varepsilon_0 \mu_0}} \cdot \Big[-\cos(u)\Big]_{\omega t - k \cdot a}^{\omega t - 2ka} \cdot E_{re}$$

$$= -\frac{a}{\omega} \cdot \Big[\cos(\omega t - 2k \cdot a) - \cos(\omega t - k \cdot a)\Big] \cdot E_{re}$$

d) $\oint_L \vec{B}_{ind} \cdot d\vec{s} = -\dot{\phi}_L(t)$

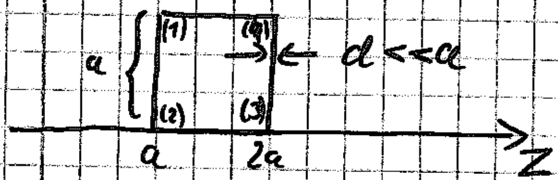
$$= -E_{re} \cdot \frac{a}{\omega} \cdot \omega \cdot \left[\sin(\omega t - 2ka) - \sin(\omega t - ka)\right]$$

$$= -a \cdot E_{re} \cdot \left[\sin(\omega t - 2ka) - \sin(\omega t - k \cdot a)\right]$$

e) $\vec{E} = -E_{re} \cdot \vec{e}_x \cdot \sin(\omega t - kz)$

$$\oint_L \vec{E}\, d\vec{s} = a \cdot (-E_{re}) \cdot \left[\sin(\omega t - k \cdot 2a) - \sin(\omega t - k \cdot a)\right]$$

da das eingeprägte $\vec{E}$-Feld nur eine x-Komponente hat



Beiträge nur auf
(1) → (2) und (3) → (4)
Math. Drehsinn beachten!
(1) → (2) mit neg. Vorzeichen

Ergebnisse d) und e) müssen gleich sein, da beide aus den Maxwell-Gl. folgen.

f) $\oint_L \vec{B}_{ind}\, d\vec{s}$ verschwindet wenn beide sin-Argumente $n \cdot 2\pi$ Phasendifferenz haben

$$\Rightarrow k \cdot a = n \cdot 2\pi$$

$$\Leftrightarrow \omega\sqrt{\varepsilon_0\mu_0} \cdot a = n \cdot 2\pi$$

$$\Rightarrow \omega = \frac{n \cdot 2\pi}{a\sqrt{\varepsilon_0\mu_0}} \quad ; \quad n = 1, 2, 3, 4, \ldots \qquad (n \neq 0,\ \text{da } \omega > 0)$$

$$\Rightarrow \lambda = \frac{c}{f} = \frac{2\pi}{\omega} \cdot \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{2\pi a\sqrt{\varepsilon_0\mu_0}}{n \cdot 2\pi\sqrt{\varepsilon_0\mu_0}} = \frac{a}{n}$$

H0912 ①

a) linear polarisiert ; $|\vec{k}_e| = \omega\sqrt{\varepsilon_0\mu_0} \longrightarrow a = \frac{\omega}{\sqrt{2}}\sqrt{\varepsilon_0\mu_0}$

b) Ansatz 1 (allgemein gültig): $\vec{\nabla} \times \underline{\vec{H}} = j\omega\varepsilon\underline{\vec{E}}$

Ansatz 2 (nur für TEM-Fall gültig): $\underline{\vec{E}}_e = -Z_0(\vec{n}_e \times \underline{\vec{H}}_e)$; $\underline{\vec{E}}_r = -Z_0(\vec{n}_r \times \underline{\vec{H}}_r)$

$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$; $\vec{n}_r = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

$$\Rightarrow \underline{\vec{E}}_e = -\vec{e}_z \frac{2H_{0e} \cdot a}{\sqrt{2}\,\omega\varepsilon_0} e^{-j\vec{k}_e\vec{r}} = -\vec{e}_z Z_0 H_{0e} e^{-j\vec{k}_e\vec{r}}$$

$$\underline{\vec{E}}_r = \vec{e}_z \frac{2\underline{H}_{0r}\, a}{\omega\varepsilon_0} e^{-j\vec{k}_r\vec{r}} = \vec{e}_z \sqrt{2}\,\underline{H}_{0r} Z_0 e^{-j\vec{k}_r\vec{r}}$$

<u>Grenzbed</u>: $H_n = 0$ und $E_{tan} = 0$

$$\Rightarrow \underline{H}_{0r} = \frac{H_{0e}}{\sqrt{2}}$$

c) an Grenzfläche $(x=0)$ gilt: Halbraum $\hat{=}$ HR

$$\underline{\vec{J}}_F = \text{Rot}\,\underline{\vec{H}} = \vec{e}_x \times \left(\underline{\vec{H}}_{Halbraum 2} - \underline{\vec{H}}_{Halbraum 1}\right) = \vec{e}_x \times (-\underline{\vec{H}}_{HR1})$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \frac{H_0}{\sqrt{2}} \left(e^{-j\vec{k}_e\vec{r}} + e^{-j\vec{k}_r\vec{r}}\right)$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} \frac{H_0}{\sqrt{2}} e^{jay} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \frac{H_0}{\sqrt{2}} e^{jay}$$

$$\vec{J}_F(\vec{r},t) = \text{Re}\left\{\underline{\vec{J}}_F e^{j\omega t}\right\} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \frac{H_0}{\sqrt{2}} \cos(ay - \omega t)$$

$\boxed{H0912}$ ②

d) $$\underline{\vec{H}} = \underline{\vec{H}}_e + \underline{\vec{H}}_r = \frac{H_{0e}}{\sqrt{2}} \begin{pmatrix} e^{-j\vec{k}_e \vec{r}} - e^{-j\vec{k}_r \vec{r}} \\ e^{-j\vec{k}_e \vec{r}} + e^{-j\vec{k}_r \vec{r}} \\ 0 \end{pmatrix} = \frac{H_{0e}}{\sqrt{2}} e^{jay} \begin{pmatrix} -2j\sin(ax) \\ 2\cos(ax) \\ 0 \end{pmatrix}$$

$$\underline{\vec{E}} = \underline{\vec{E}}_e + \underline{\vec{E}}_r = H_{0e} Z_0 \begin{pmatrix} 0 \\ 0 \\ -e^{-j\vec{k}_e \vec{r}} + e^{-j\vec{k}_r \vec{r}} \end{pmatrix} = H_{0e} Z_0 e^{jay} \begin{pmatrix} 0 \\ 0 \\ 2j\sin(ax) \end{pmatrix}$$

$$\underline{\vec{E}} \times \underline{\vec{H}}^* = \frac{H_{0e}^2}{\sqrt{2}} Z_0 \begin{pmatrix} -4j\sin(ax)\cos(ax) \\ -4\sin^2(ax) \\ 0 \end{pmatrix}$$

$$\frac{1}{2} \mathrm{Re}\{\underline{\vec{E}} \times \underline{\vec{H}}^*\} = \sqrt{2}\, H_0^2 Z_0 \begin{pmatrix} 0 \\ -\sin^2(ax) \\ 0 \end{pmatrix}$$

e) • Richtung d. Zeitmittelwertes d. Poynting-Vektors : $(-\vec{e}_y)$

• Überlagerung ergibt TE_y - Welle

H09

Aufgabe 3

TE_x - Welle . $\left\{ \begin{array}{l} \vec{\underline{G}}_i = \vec{0} \\ \vec{\underline{E}}_i = \underline{f}_i(x,z) \cdot \vec{e}_x = \tilde{\underline{f}}_i(x)\, e^{-j\beta z} \cdot \vec{e}_x \end{array} \right.$

mit $\beta^2 = \omega^2 \varepsilon_{eff} \mu_0$

$E_x = 0$, $\varepsilon_2 \le \varepsilon_{eff} \le \varepsilon_1$

a) $\Delta \underline{f}_i + h_i^2 \underline{f}_i = 0$ Helmholtz - Gleichung

$p_1^2 \ge 0$

$p_2^2 \le 0$

$\alpha_2 \ge 0$

$$\boxed{\tilde{\underline{f}}_i'' + p_i^2 \cdot \tilde{\underline{f}}_i = 0}$$ mit $p_i^2 = h_i^2 - \beta^2$ oder $p_i^2 = (\varepsilon_{ri} - \varepsilon_{reff}) \cdot h_0^2$

Mit $\alpha_2^2 = -p_i^2$, folgt:

$$\boxed{\begin{array}{l} \tilde{\underline{f}}_1'' + p_1^2 \cdot \tilde{\underline{f}}_1 = 0 \\ \tilde{\underline{f}}_2'' - \alpha_2^2 \cdot \tilde{\underline{f}}_2 = 0 \end{array}}$$

$$\tilde{\underline{f}}_1(x) = \cancel{A_1 \cdot \cos(p_1 x)} + B_1 \cdot \sin(p_1 x)$$
$$\tilde{\underline{f}}_2(x) = \cancel{A_2 \cdot e^{\alpha_2 x}} + B_2 \cdot e^{-\alpha_2 x}$$

b) $$\vec{\underline{E}}_i = -\nabla \underline{f}_i \times \vec{e}_x = \left(0, -\frac{\partial \underline{f}_i}{\partial z}, 0\right)$$

$$\vec{\underline{H}}_i = \frac{1}{j\omega\mu_0}\left(h_i^2 \cdot \underline{f}_i \cdot \vec{e}_x + \nabla\left(\frac{\partial \underline{f}_i}{\partial x}\right)\right)$$
$$= \left(\frac{1}{j\omega\mu_0}\left(h_i^2 + \frac{\partial^2}{\partial x^2}\right)\underline{f}_i ,\ 0 ,\ \frac{1}{j\omega\mu_0} \cdot \frac{\partial^2 \underline{f}_i}{\partial x \partial z}\right)$$

mit $h_i^2 = \omega^2 \varepsilon_i \mu_0 = \varepsilon_{ri} \cdot h_0^2$

$h_0^2 = \omega^2 \varepsilon_0 \mu_0$

$$\underline{E}_{yi} = -\frac{\partial \underline{f}_i}{\partial z} = j\beta \underline{f}_i = j\beta \tilde{\underline{f}}_i \cdot e^{-j\beta z}$$

$$\underline{E}_{y1} = j\beta \cdot \left(A_1 \cdot \cos(p_1 x) + B_1 \cdot \sin(p_1 x)\right) \cdot e^{-j\beta z}$$

$$\underline{E}_{y2} = j\beta \left(A_2 \cdot e^{\alpha_2 x} + B_2 \cdot e^{-\alpha_2 x}\right) \cdot e^{-j\beta z}$$

$k^2 f_i + f_i''$

$k^2 \cdot f_i + (-p^2) f_i$

$(k^2 - p_i^2) \cdot f_i$

$\beta^2 \cdot f_i$

$$\underline{H}_{xi} = \frac{\beta^2}{j\omega\mu_0} \cdot \underline{f}_i = \frac{\beta^2}{j\omega\mu_0} \cdot \tilde{\underline{f}}_i \cdot e^{-j\beta z}$$

$$\underline{H}_{x1} = \frac{\beta^2}{j\omega\mu_0} \left(A_1 \cdot \cos(p_1 x) + B_1 \cdot \sin(p_1 x)\right) \cdot e^{-j\beta z}$$

$$\underline{H}_{x2} = \frac{\beta^2}{j\omega\mu_0} \left(A_2 \cdot e^{\alpha_2 x} + B_2 \cdot e^{-\alpha_2 x}\right) \cdot e^{-j\beta z}$$

$$\underline{H}_{zi} = \frac{1}{j\omega\mu_0} \cdot \frac{\partial^2}{\partial x \partial z} \underline{f}_i = -\frac{\beta}{\omega\mu_0} \cdot \frac{\partial \underline{f}_i}{\partial x}$$

$$\underline{H}_{z1} = \frac{\beta \cdot p_1}{\omega\mu_0} \left(A_1 \cdot \sin(p_1 x) - B_1 \cdot \cos(p_1 x)\right) \cdot e^{-j\beta z}$$

$$\underline{H}_{z2} = -\frac{\beta\alpha_2}{\omega\mu_0} \left(A_2 \cdot e^{\alpha_2 x} - B_2 \cdot e^{-\alpha_2 x}\right) \cdot e^{-j\beta z}$$

c) elektrische Wand bei $x = 0$:

$\rightarrow \vec{\underline{E}}_{tan} = \vec{0}$ und $\vec{\underline{H}}_{norm} = \vec{0}$ bzw. $\frac{\partial}{\partial n} \vec{\underline{H}}_{tan} = \vec{0}$

$$\left.\begin{array}{l} \underline{E}_{y1}(x=0) = 0 \\ \underline{H}_{x1}(x=0) = 0 \\ \frac{\partial \underline{H}_{z1}}{\partial x}(x=0) = 0 \end{array}\right\} \boxed{A_1 = 0}$$

Bedingung für $x \to \infty$:

$$\left.\begin{array}{l} E_{y2}(x \to \infty) = 0 \\ H_{x2}(x \to \infty) = 0 \\ H_{z2}(x \to \infty) = 0 \end{array}\right\} \boxed{A_2 = 0}$$

d) Grenzfläche zweier Dielektrika bei $x=h$:

$\vec{\underline{E}}_{tan}$ stetig und $\vec{\underline{H}}_{tan}$ stetig

$$\underline{E}_{y1}(x=h) = \underline{E}_{y2}(x=h)$$
$$\underline{H}_{z1}(x=h) = \underline{H}_{z2}(x=h)$$

$$B_1 \cdot \sin(p_1 h) = B_2 \cdot e^{-\alpha_2 h}$$
$$-p_1 B_1 \cdot \cos(p_1 h) = \alpha_2 \cdot B_2 \cdot e^{-\alpha_2 h}$$

$$\Rightarrow \boxed{\tan(p_1 h) = -\frac{p_1}{\alpha_2}}$$ charakteristische Gleichung

e) $\tan(p_1 h) = -\sqrt{3}$ $\qquad \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$$\Rightarrow p_1 h \underset{\text{kleinster Eigenwert } p_1}{=} \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\Rightarrow p_1 = \frac{2\pi}{3h} \quad \text{und} \quad \alpha_2 \underset{\text{charakteristische Gleichung}}{=} \frac{p_1}{\sqrt{3}} = \frac{2\pi}{3\sqrt{3}\cdot h}$$

$$p_1^2 = k_1^2 - \beta^2 = \omega^2 \mu_0 (\varepsilon_1 - \varepsilon_{eff}) \geq 0$$
$$\alpha_2^2 = \beta^2 - k_2^2 = \omega^2 \mu_0 (\varepsilon_{eff} - \varepsilon_2) \geq 0$$

$$\Rightarrow p_1^2 + \alpha_2^2 = \omega^2 \mu_0 (\varepsilon_1 - \varepsilon_2)$$

$$\Rightarrow \boxed{\omega^2 = \frac{p_1^2 + \alpha_2^2}{\mu_0 (\varepsilon_1 - \varepsilon_2)}}$$

$$\Rightarrow \alpha_2^2 - p_1^2 = \omega^2 \mu_0 (2\varepsilon_{eff} - \varepsilon_1 - \varepsilon_2)$$

$$\Rightarrow \varepsilon_{eff} = \frac{\alpha_2^2 - p_1^2}{2\omega^2 \mu_0} + \frac{\varepsilon_1 + \varepsilon_2}{2}$$

ω^2 einsetzen:

$$\boxed{\varepsilon_{eff} = \underbrace{\frac{\alpha_2^2 - p_1^2}{\alpha_2^2 + p_1^2}} \cdot \frac{\varepsilon_1 - \varepsilon_2}{2} + \frac{\varepsilon_1 + \varepsilon_2}{2}}$$

$$-1 \leq \quad \leq +1 \quad \Rightarrow \boxed{\varepsilon_2 \leq \varepsilon_{eff} \leq \varepsilon_1}$$

$$\omega^2 = \frac{4\pi^2}{9h^2}\left(1 + \frac{1}{3}\right) \cdot \frac{1}{\mu_0(\varepsilon_1 - \varepsilon_2)}$$

$$\boxed{\omega^2 = \frac{16\pi^2}{27h^2} \cdot \frac{1}{\mu_0(\varepsilon_1 - \varepsilon_2)}}$$

$$\varepsilon_{eff} = \frac{1/3 - 1}{1/3 + 1} \cdot \frac{\varepsilon_1 - \varepsilon_2}{2} + \frac{\varepsilon_1 + \varepsilon_2}{2}$$

$$\boxed{\Rightarrow \varepsilon_{eff} = \frac{\varepsilon_1 + 3\varepsilon_2}{4} = \varepsilon_2 + \frac{1}{4}(\varepsilon_1 - \varepsilon_2)}$$

f) Grenzfrequenz für $\alpha_2 = 0$ bzw. $\varepsilon_{eff} = \varepsilon_2$
(Felder klingen in x-Richtung nicht mehr ab)

$\Rightarrow \tan(p_1 h) = -\infty$ charakt. Gleichung

$$\Rightarrow p_1 \cdot h = \frac{\pi}{2}$$

kleinster Eigenwert ~~für~~ p_1

$$\Rightarrow p_1 = \frac{\pi}{2h} \quad \Rightarrow \varepsilon_{eff} = -\frac{\varepsilon_1 - \varepsilon_2}{2} + \frac{\varepsilon_1 + \varepsilon_2}{2} = \varepsilon_2$$

$$\Rightarrow \omega_c^2 = \frac{\pi^2}{4h^2} \cdot \frac{1}{\mu_0(\varepsilon_1 - \varepsilon_2)}$$

$$\Leftrightarrow \omega_c = \frac{\pi}{2h} \frac{1}{\sqrt{\mu_0(\varepsilon_1 - \varepsilon_2)}}$$

$$\Rightarrow \boxed{f_c = \frac{1}{4h} \cdot \frac{1}{\sqrt{\mu_0(\varepsilon_1 - \varepsilon_2)}}}$$

H08|1 Koaxialleiter ①

a) $l_z = 10\,m$; $\varepsilon_r = 4$; $\omega = 2\pi \cdot 10^7 \frac{1}{s}$

elektr. Länge $\frac{l_z}{\lambda} = \frac{l_z f}{c} = \frac{l_z}{c_0}\sqrt{\varepsilon_r} \cdot \frac{\omega}{2\pi}$

$$= \frac{10m}{3 \cdot 10^8 \frac{m}{s}} \cdot 2 \cdot 10^7 \tfrac{1}{s} = \frac{2}{3} \approx 1$$

$$\vec{k} = \vec{e_z} \cdot \frac{2\pi}{\lambda}$$

b) $\underline{\vec{E}}(\rho,\varphi,z) = \underline{\vec{E}}_A(\rho,\varphi)\, e^{-j\vec{k}\cdot\vec{e_z}\cdot z} = \underline{\vec{E}}_A(\rho,\varphi) \cdot e^{-j\frac{2\pi}{\lambda}\cdot z}$

c) $= \underline{\vec{E}}_A(\rho,\varphi) \cdot e^{-j\frac{2\pi}{c_0}\sqrt{\varepsilon_r}\frac{\omega}{2\pi}\cdot z}$

$= \underline{\vec{E}}_A(\rho,\varphi) \cdot e^{-j\frac{\omega\sqrt{\varepsilon_r}}{c_0}\cdot z}$

d) $\Delta_{2D}\, \Phi(\rho,\varphi, z=0, t) = 0$

$$\Rightarrow \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) + \frac{1}{\rho^2}\cancel{\frac{\partial^2\Phi}{\partial\varphi^2}} = 0$$

0 wegen TEM-Welle (rotationssymmetrisch)

Ansatz $\Phi(\rho,\varphi,z=0,t) = const_1 + const_2 \ln(\rho)\, h(t)$

ist Lsg. der DGL

e) $\underline{\vec{E}}_A(\rho,\varphi)$ gesucht

$\Phi(\rho,\varphi,z=0,t) = const_2 \ln(\rho)\cdot h(t) + const_1$

Es gilt bei $z=0$:

$$\Phi(\rho=r_i;\varphi;z=0,t) - \Phi(\rho=r_a,\varphi,z=0,t) = U_0 \cdot \sin(\omega t) \quad (*)$$

$$\mathrm{Re}\left\{\underline{\vec{E}}_A(\rho,\varphi)\cdot e^{j\omega t}\right\} = -\nabla\Phi(\rho,\varphi,z=0,t)$$

$$(*) \quad const_2 \cdot \ln\left(\frac{r_i}{r_a}\right) \cdot h(t) = U_0 \sin(\omega t)$$

$$\Rightarrow const_2 = \frac{U_0}{\ln(r_i/r_a)} \quad \text{und} \quad h(t) = \sin(\omega t)$$

$$\Rightarrow \Phi = -\frac{U_0}{\ln(r_a/r_i)} \cdot \ln(\rho) \cdot \sin(\omega t)$$

$$\mathrm{Re}\left\{ \underline{\vec{E}}_A(\rho,\varphi) \cdot e^{j\omega t} \right\} = -\frac{\partial \Phi}{\partial \rho} \vec{e}_\rho = \frac{U_0}{\ln\left(\frac{r_a}{r_i}\right)} \frac{1}{\rho} \vec{e}_\rho \sin(\omega t)$$

$$\Rightarrow \underline{\vec{E}}_A(\rho,\varphi) = -j \frac{U_0}{\ln\left(\frac{r_a}{r_i}\right)} \frac{1}{\rho} \vec{e}_\rho$$

$$\underline{\vec{E}} = \underline{\vec{E}}_A \cdot e^{-j \frac{\omega}{c_0} \sqrt{\varepsilon_r} \cdot z}$$

f) $l_z = 1 \cdot m \quad ; \quad \omega = 2\pi \cdot 10^5 \frac{1}{s}$

$$\frac{l_z}{\lambda} = \frac{l_z}{c_0} \sqrt{\varepsilon_r} \frac{\omega}{2\pi} = \frac{1m}{3 \cdot 10^8 \frac{m}{s}} \cdot 2 \cdot 10^5 s^{-1} = \frac{2}{3} \cdot 10^{-3} \ll 1$$

$$\vec{E}(\rho,\varphi,z,t) = \mathrm{Re}\left\{ \underline{\vec{E}} \cdot e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left[-j \frac{U_0}{\ln\left(\frac{r_a}{r_i}\right)} \frac{1}{\rho} \vec{e}_\rho \cdot e^{-jkz + j\omega t} \right\}$$

$$= \frac{U_0}{\ln\left(\frac{r_a}{r_i}\right)} \cdot \frac{1}{\rho} \vec{e}_\rho \cdot \mathrm{Re}\left\{ -j\left(\cos(\omega t - kz) + j \sin(\omega t - kz) \right) \right\}$$

$$= \frac{U_0}{\ln\left(\frac{r_a}{r_i}\right)} \frac{1}{\rho} \vec{e}_\rho \cdot \sin(\omega t - \underbrace{kz}_{\ll 1,\ \text{da}\ \frac{l_z}{\lambda} \ll 1})$$

$$\approx \frac{U_0}{\ln\left(\frac{r_a}{r_i}\right)} \cdot \frac{1}{\rho} \vec{e}_\rho \cdot \sin(\omega t)$$

H08

Aufgabe 2

a) $\vec{E}(z) = \frac{U_0}{h} \cdot \exp(-jkz) \cdot \vec{e}_x \quad , \quad k = \omega\sqrt{\varepsilon\mu}$

$$\underline{\vec{H}}(z) = \frac{1}{Z_F} \cdot (\vec{n} \times \vec{E}) = \sqrt{\frac{\varepsilon}{\mu}} \cdot (\vec{n} \times \vec{E})$$

$$= \sqrt{\frac{\varepsilon}{\mu}} \cdot (\vec{e}_z \times \vec{e}_x) \cdot \frac{U_0}{h} \cdot e^{-jkz}$$

$$= \sqrt{\frac{\varepsilon}{\mu}} \cdot \frac{U_0}{h} \cdot e^{-jkz} \cdot \vec{e}_y$$

b) $u_1^h(z) = \int_{P_1}^{P_2} \vec{E}(z)\, d\vec{r} \Rightarrow u_1^h(z) = U_0 \cdot e^{-jkz}$

$$i_1^h(z) = \oint_C \underline{\vec{H}}\, d\vec{\ell} \Rightarrow \underline{i}_1^h = b \cdot \underline{H}(z)$$

$$\Rightarrow \underline{i}_1^h = b \cdot \sqrt{\frac{\varepsilon}{\mu}} \cdot \frac{U_0}{h} \cdot e^{-jkz}$$

$$Z_L = \frac{\underline{u}_1^h}{\underline{i}_1^h} = \frac{h}{b} \cdot \sqrt{\frac{\mu}{\varepsilon}} = Z_F \cdot \frac{h}{b}$$

c) $\underline{z=0}$: $\underline{u}_1^h + \underline{u}_1^r = \underline{u}_2^h + \underline{u}_2^r$ (1)

$$-\underline{i}_1^r + \underline{i}_1^h = \underline{i}_2^h - \underline{i}_2^r + \underline{i}_R = \underline{i}_2^h - \underline{i}_2^r + \frac{\underline{u}_2^h + \underline{u}_2^r}{R}$$

$$\Rightarrow -\frac{\underline{u}_1^r}{Z_L} + \frac{\underline{u}_1^h}{Z_L} = \frac{\underline{u}_2^h}{Z_L} - \frac{\underline{u}_2^r}{Z_L} + \frac{\underline{u}_2^h + \underline{u}_2^r}{R} \quad (2)$$

$\underline{z=\ell}$: $\underline{u}_2^h \cdot e^{-jk\ell} = -\underline{u}_2^r \cdot e^{+jk\ell}$ (3)

Einschub (nicht zuzuordnen):

$$e^{-jk\frac{\lambda}{4}} = e^{-j\frac{2\pi}{4}} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$$

$$e^{jk\frac{\lambda}{4}} = j$$

d) $l_{min} = \frac{\lambda}{4}$ $\quad \underline{U}_1^r = 0$

$$\Rightarrow \underline{U}_1^h = \underline{U}_2^h + \underline{U}_2^r \quad (1)$$

$$\Rightarrow \frac{\underline{U}_1^h}{Z_L} = \frac{\underline{U}_2^h}{Z_L} - \frac{\underline{U}_2^r}{Z_L} + \frac{\underline{U}_2^h + \underline{U}_2^r}{R} \quad (2)$$

$$\Rightarrow \underline{U}_2^h \cdot (-j) = -\underline{U}_2^r \cdot j \Leftrightarrow \underline{U}_2^h = +\underline{U}_2^r \quad (3)$$

$$\underline{U}_1^h = \underline{U}_2^h + \underline{U}_2^r \Rightarrow \underline{U}_1^h = U_0 = 2\underline{U}_2^h \quad (1) + (3)$$

$$(2) \quad \frac{\underline{U}_1^h}{Z_L} = \frac{U_0}{Z_L} = \frac{\underline{U}_2^h}{Z_L} - \frac{\underline{U}_2^r}{Z_L} + \frac{\underline{U}_2^h + \underline{U}_2^r}{R}$$

$$\Rightarrow \frac{U_0}{Z_L} = \frac{\underline{U}_2^h}{Z_L} - \frac{\underline{U}_2^h}{Z_L} + \frac{\underline{U}_2^h + \underline{U}_2^h}{R}$$

$$\Rightarrow U_0 = \frac{2\underline{U}_2^h}{R} \cdot Z_L$$

mit (1) folgt: $\underline{R = Z_L}$

$$\frac{l}{\lambda} \approx 1$$

$$\underline{i}_1^{Ges}\Big|_{z=0} = \underline{i}_1^h - \underline{i}_{1a} = \underline{i}_1^h = \frac{U_0}{Z_L}$$

$$\underline{i}_R = \frac{U_0}{Z_L} = \underline{i}_1^{Ges}$$

$$\underline{i}_2^{Ges}\Big|_{z=0} = 0, \quad \text{da } \underline{i}_1^{Ges} = \underline{i}_R + \underline{i}_2^{Ges}$$

$$i_u = \underline{i}_2^{Ges}\Big|_{z=l} = \underline{i}_2^h \cdot e^{-jkl} + \underline{i}_2^r \cdot e^{+jkl}$$

$$= \frac{\underline{U}_2^h}{Z_L} \cdot e^{-jkl} + \frac{-\underline{U}_2^r}{Z_L} \cdot e^{+jkl} = \frac{U_0}{2Z_L} e^{-jkl} + \frac{-U_0}{2Z_L} e^{+jkl}$$

$$= -j \frac{U_0}{2Z_L} - j \frac{U_0}{2Z_L} = -j \frac{U_0}{Z_L}$$

e) $\underline{i}_R = 0$ (kurzgeschlossen)

$$\underline{i}_2^{ges}(z=0) = \frac{2U_0}{Z_L}$$

$$\underline{i}_K = \frac{2U_0}{Z_L}$$

$\boxed{H0813}$

a) TE_x - Hybridwelle

$(\underline{E}_x \equiv 0)$

$$\vec{\underline{F}}_i = \underline{f}_i(x,z)\,\vec{e}_x = \tilde{\underline{f}}_i(x)\cdot e^{-j\beta z}\,\vec{e}_x \qquad i = 1,2$$

$$\vec{\underline{G}}_i \equiv 0 \quad \Rightarrow \quad \underline{E}_x = 0 \qquad \text{mit } \beta^2 = \omega^2 \varepsilon_{eff}(\omega)\mu_0$$

$$k_i^2 = \omega^2 \varepsilon_i \mu_0$$

Helmholtz-Glg.:

$$\left(\frac{d^2}{dx^2} - \beta^2 + k_i^2\right)\tilde{\underline{f}}_i(x) = 0$$

$$\left(\frac{d^2}{dx^2} + \underline{k}_{xi}^2\right)\tilde{\underline{f}}_i(x) = 0 \qquad \text{mit } \underline{k}_{xi}^2 = k_i^2 - \beta^2$$

$$\tilde{\underline{f}}_1(x) = \cancel{\underline{A}_1 \cos(\underline{k}_{x1} x)} + \underline{B}_1 \sin(\underline{k}_{x1}\cdot x)$$

$$\tilde{\underline{f}}_2(x) = \cancel{\underline{A}_2 \cos(\underline{k}_{x2}(2h-x))} + \underline{B}_2 \sin(\underline{k}_{x2}(2h-x))$$

b) Gebiet 1:

Randbed.: $\vec{\underline{E}}_{tan1} = 0$ und $\underline{H}_{n1} = 0$

$\underline{E}_{x1}(y=0) = 0$ ✓ $\qquad$ $\underline{E}_{x1}(y=b) = 0$ ✓

$\underline{E}_{z1}(y=0) = 0$ ✓ $\qquad$ $\underline{E}_{z1}(y=b) = 0$ ✓

$\underline{H}_{y1}(y=0) = 0$ ✓ $\qquad$ $\underline{H}_{y1}(y=b) = 0$ ✓

$\underline{E}_{y1}(x=0) = 0 \quad \Rightarrow \quad \boxed{\underline{A}_1 = 0}$

$\underline{E}_{z1}(x=0) = 0$ ✓

$\underline{H}_{x1}(x=0) = 0 \quad \Rightarrow \quad \underline{A}_1 = 0$ ✓

Gebiet ②

Randbed.: $\vec{\underline{E}}_{tan2} = 0$ u. $\underline{H}_{n2} = 0$

$\underline{E}_{x2}(y=0) = 0$ ✓ $\quad$ $\underline{E}_{x2}(y=b) = 0$ ✓

$\underline{E}_{z2}(y=0) = 0$ ✓ $\quad$ $\underline{E}_{z2}(y=b) = 0$ ✓

$\underline{H}_{y2}(y=0) = 0$ ✓ $\quad$ $\underline{H}_{y2}(y=b) = 0$ ✓

$\underline{E}_{y2}(x=2a) = 0 \Rightarrow \boxed{\underline{A}_2 = 0}$

$\underline{E}_{z2}(x=2a) = 0$ ✓

$\underline{H}_{x2}(x=2a) = 0 \Rightarrow \underline{A}_2 = 0$ ✓

Gebiet ①:

$\underline{E}_{x1} \equiv 0$ da $\underline{g}_1 \equiv 0$

$$\underline{E}_{y1} = -\frac{\partial \underline{f}_1}{\partial z} = j\beta \underline{f}_1$$

$$\underline{E}_{y1} = j\beta\left(\underline{B}_1 \sin(\underline{k}_{x1} x)\right) e^{-j\beta z}$$

$$\underline{E}_{z1} = \frac{\partial \underline{f}_1}{\partial y} \equiv 0 \quad \text{da } \underline{f}_1(x,z)$$

$$\underline{H}_{x1} = \frac{\beta^2}{j\omega\mu_0} \underline{f}_1$$

$$\underline{H}_{x1} = \frac{\beta^2}{j\omega\mu_0}\left(\underline{B}_1 \sin(\underline{k}_{x1} x)\right) e^{-j\beta z}$$

$$\underline{H}_{y1} = \frac{1}{j\omega\mu_0} \frac{\partial^2 \underline{f}_1}{\partial x \partial y} \equiv 0 \quad \text{da } \underline{f}_1(x,z)$$

$$\underline{H}_{z1} = \frac{1}{j\omega\mu_0} \frac{\partial^2 \underline{f}_1}{\partial x \partial z} = -\frac{\beta}{\omega\mu_0} \frac{\partial \underline{f}_1}{\partial x}$$

$$\underline{H}_{z1} = -\frac{\beta \underline{k}_{x1}}{\omega\mu_0}\left(\underline{B}_1 \cos(\underline{k}_{x1} x)\right) \cdot e^{-j\beta z}$$

Gebiet ②:

$\underline{E}_{x2} \equiv 0$

$\underline{E}_{y2} = j\beta (\underline{B}_2 \sin(\underline{k}_{x2}(2h - x))) e^{-j\beta z}$

$\underline{E}_{z2} \equiv 0$

$\underline{H}_{x2} = \frac{\beta^2}{j\omega\mu_0} \left(\underline{B}_2 \sin(\underline{k}_{x2}(2h-x)) \right) e^{-j\beta z}$

$\underline{H}_{y2} \equiv 0$

$\underline{H}_{z2} = - \frac{\beta \underline{k}_{x2}}{\omega \mu_0} \left(- \underline{B}_2 \cos(\underline{k}_{x2} \cdot (2h - x)) \right) \cdot e^{-j\beta z}$

c) Stetigkeit bei $x = h$ $\quad \vec{\underline{E}}_{tan 1} = \vec{\underline{E}}_{tan 2}$ und $\vec{\underline{H}}_{tan 1} = \vec{\underline{H}}_{tan 2}$

$\underline{E}_{y1}(x=h) = \underline{E}_{y2}(x=h)$

$\Rightarrow \underline{B}_1 \sin(\underline{k}_{x1} h) = \underline{B}_2 \sin(\underline{k}_{x2} h)$

$\underline{E}_{z1}(x=h) = \underline{E}_{z2}(x=h) \equiv 0$ ✓

$\underline{H}_{y1}(x=h) = \underline{H}_{y2}(x=0) \equiv 0$ ✓

$\underline{H}_{z1}(x=h) = \underline{H}_{z2}(x=h)$

$\Rightarrow -\underline{k}_{x1} \cdot \underline{B}_1 \cos(\underline{k}_{x1} \cdot h) = \underline{k}_{x2} \cdot \underline{B}_2 \cdot \cos(\underline{k}_{x2} \cdot h)$

char. Glg.:

$$\boxed{\underline{k}_{x1} \cot(\underline{k}_{x1} h) = -\underline{k}_{x2} \cdot \cot(\underline{k}_{x2} \cdot h)}$$

mit $\underline{k}_{xi}^2 = k_i^2 - \beta^2$

oder

$\underline{k}_{x2} \tan(\underline{k}_{x1} h) = -\underline{k}_{x1} \tan(\underline{k}_{x2} h)$

mit $\underline{k}_{xi}^2 = k_i^2 - \beta^2$

d) Fall: $\varepsilon_1 = \varepsilon_2$; $k_1 = k_2$; $k_{x1} = k_{x2} \neq 0$

char. Gl:

$$k_{x1} \cot(k_{x1} h) = -\underbrace{k_{x2}}_{=k_{x1}} \cot(\underbrace{k_{x2}}_{=k_{x1}} \cdot h)$$

$\Rightarrow k_{x1} \cot(k_{x1} \cdot h) = 0$

$\Rightarrow \cot(k_{x1} h) = 0$ da $k_{x1} \neq 0$ (Aufg'stellung)

$\Rightarrow k_{x1} = \pm \frac{\pi}{2h}$; $\pm \frac{3\pi}{2h}$; $\pm \frac{5\pi}{2h}$; ...

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die andere char. Gl.:

$$k_{x2} \cdot \tan(k_{x1} \cdot h) = -k_{x1} \tan(k_{x2} \cdot h)$$

$\Rightarrow \tan(k_{x1} \cdot h) = 0$ (analog wie oben)

$\Rightarrow k_{x1} = 0$; $\pm \frac{\pi}{h}$; $\pm \frac{2\pi}{h}$; $\pm \frac{3\pi}{h}$; ...

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für TE_x - Grundwelle:
(TE_{x10}-Welle)

$$\boxed{k_{x1} = k_{x2} = \frac{\pi}{2h}}$$

Grenzfrequenz ω_c mit $\beta(\omega_c) = 0$:

$$k_{x1}^2 = k_1^2 - \cancel{\beta^2} = k_1^2 = \omega_c^2 \varepsilon_1 \mu_0$$

$$\boxed{\omega_c = \frac{1}{\sqrt{\varepsilon_1 \mu_0}} \frac{\pi}{2h}}$$

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a) $\operatorname{rot} \underline{\vec{H}} = j\omega \underline{\vec{D}} = j\omega \varepsilon_0 \underline{\vec{E}} \Rightarrow \underline{\vec{E}} = -\frac{1}{\omega\varepsilon_0} \operatorname{rot} \underline{\vec{H}}$

$$\underline{\vec{E}}_1 = -\frac{j}{\omega\varepsilon_0}\left(-\frac{\partial H_y}{\partial z}\vec{e}_x + \frac{\partial H_y}{\partial x}\vec{e}_z\right)$$

$$\underline{\vec{E}}_1 = -\frac{j}{\omega\varepsilon_0}\left(h_1(x)\, j\omega\sqrt{\mu_{eff}\,\varepsilon_0}\, \exp\left(-j\omega\sqrt{\mu_{eff}\,\varepsilon_0}\, z\right)\vec{e}_x + \frac{d h_1(x)}{dx}\exp\left(-j\omega\sqrt{\mu_{eff}\,\varepsilon_0}\, z\right)\vec{e}_z\right)$$

$$\underline{\vec{E}}_2 = -\frac{j}{\omega\varepsilon_0}\left(h_2(x)\, j\omega\sqrt{\mu_{eff}\,\varepsilon_0}\, \exp\left(-j\omega\sqrt{\mu_{eff}\,\varepsilon_0}\, z\right)\vec{e}_x + \frac{d h_2(x)}{dx}\exp\left(-j\omega\sqrt{\mu_{eff}\,\varepsilon_0}\, z\right)\vec{e}_z\right)$$

b) G.B. für $\underline{\vec{E}}$ bei $x=0$ und $x=2d$

Rot $\underline{\vec{E}} = \vec{0} \Rightarrow E_{tan} = 0 \Rightarrow E_z = 0$

$$\Rightarrow \frac{d h_1(0)}{dx} = 0 \qquad (1)$$

und $$\frac{d h_2(2d)}{dx} = 0 \qquad (2)$$

c) Aufstellen d. Helmholtz-Gl. für G1 und G2

G1: $\Delta \underline{\vec{H}}_1 + \omega^2 \mu_1 \varepsilon_0 \underline{\vec{H}}_1 = 0$

$$\frac{d^2 h_1(x)}{dx^2} - \omega^2 \mu_{eff}\,\varepsilon_0\, h_1(x) + \omega^2 \mu_1 \varepsilon_0 h_1(x) = 0$$

$$\frac{d^2 h_1(x)}{dx^2} + \underbrace{\omega^2 \varepsilon_0 (\mu_1 - \mu_{eff})}_{p_1^2} h_1(x) = 0$$

LSg:

$h_1(x) = A_1 \cos(p_1 x) + B_1 \sin(p_1 x)$

mit (1) $\Rightarrow \boxed{B_1 = 0}$

$\Rightarrow h_1(x) = A_1 \cos(p_1 x)$

G2: $\Delta \underline{\vec{H}}_2 + \omega^2 \mu_2 \varepsilon_0 \underline{\vec{H}}_2 = 0$

$$\frac{d^2 h_2(x)}{dx^2} - \omega^2 \mu_{eff}\,\varepsilon_0\, h_2(x) + \omega^2 \mu_2 \varepsilon_0 h_2(x) = 0$$

$$\frac{d^2 h_2(x)}{dx^2} + \underbrace{\omega^2 \varepsilon_0 (\mu_2 - \mu_{eff})}_{p_2^2} h_2(x) = 0$$

$h_2(x) = A_2 \cos(p_2(x-2d)) + B_2 \sin(p_2(x-2d))$

mit (2) $\Rightarrow \boxed{B_2 = 0}$

$\Rightarrow h_2(x) = A_2 \cos(p_2(x-2d))$

d) G.B. in der $(x=d)$ Ebene

$\mathrm{Rot}\,\underline{\vec{H}} = \vec{0}$ und $\mathrm{Rot}\,\underline{\vec{E}} = \vec{0}$

$\mathrm{Rot}\,\underline{\vec{E}} = 0 \Rightarrow -A_1 p_1 \sin(p_1 d) = A_2 p_2 \sin(p_2 d)$

$\mathrm{Rot}\,\underline{\vec{H}} = 0 \Rightarrow A_1 \cos(p_1 d) = A_2 \cos(p_2 d)$

$\Rightarrow -p_1 \tan(p_1 d) = p_2 \tan(p_2 d)$

e) mit $\omega \to 0 \Rightarrow |p_1| d \ll 1$ u. $|p_2| d \ll 1$ gilt: $\tan(p_1 d) \approx p_1 d$

und $\tan(p_2 d) \approx p_2 d$

$-p_1^2 = p_2^2$

$-\omega^2 (\mu_1 - \mu_{eff}) \varepsilon_0 = \omega^2 (\mu_2 - \mu_{eff}) \varepsilon_0$

$-\mu_1 + \mu_{eff} = \mu_2 - \mu_{eff} \Rightarrow \mu_{eff} = \frac{1}{2} (\mu_1 + \mu_2)$

Phasengeschw v:

$$\omega \sqrt{\mu_{eff} \varepsilon_0} = k = \frac{2\pi}{\lambda} = \frac{\omega}{v} \Rightarrow v = \frac{1}{\sqrt{\mu_{eff} \varepsilon_0}}$$

$$= \frac{\sqrt{2}}{\sqrt{(\mu_1 + \mu_2)\varepsilon_0}} \neq f(\omega)$$