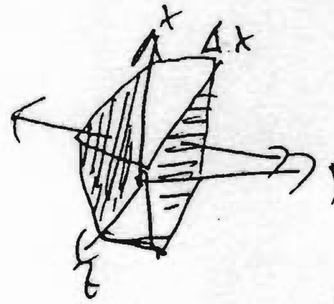
$$= -\frac{6}{2} E_0^2 e^{-2k''z}$$

$$\mathrm{div}\, \overline{\vec{S}(t)} = -\overline{P_H(t)}$$

Integration über ein Volumen V

$$\int_V \mathrm{div}\, \overline{\vec{S}(t)}\, dV \underset{\text{Gauß}}{=} \oint_{\partial V} \overline{\vec{S}(t)}\, d\vec{F} = -\int_V \overline{P_j(t)}\, dV$$



$V = \Delta x \Delta y \Delta P$

$\Delta \vec{F} = \Delta x \Delta y\, \vec{e}_z$

$$\left(\overline{\vec{S}_z(t)}\Big|_{z=\Delta z} - \overline{\vec{S}_z(t)}\Big|_{z=0} \right) \Delta x \Delta y = -\Delta x \Delta y \int_0^{\Delta z} \overline{P_j(t)}\, dz$$

$$= -P_j$$

Zusatzaufgabe I | AUFGABE 1 HOO

a) $\mathrm{rot}\, \underline{\vec{E}} = -j\omega\mu_0 \underline{\vec{H}} \rightarrow \mathrm{div}\, \underline{\vec{H}} = 0$

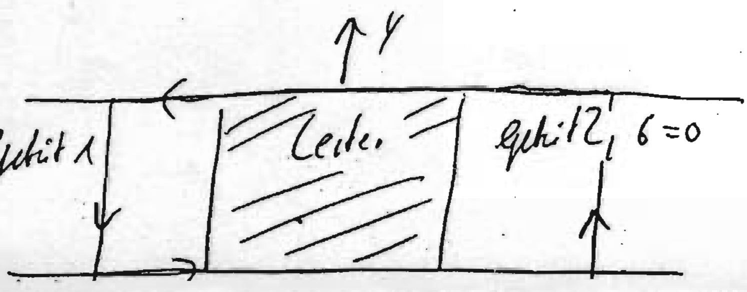
$\mathrm{rot}\, \underline{\vec{H}} = (\sigma_0 + j\omega\varepsilon_0)\, \underline{\vec{E}} = \sigma_0 \vec{E}(1 + \underbrace{\omega \frac{\varepsilon_0}{\sigma_0}}_{=\omega T}) = \sigma_0 \vec{E} (= \vec{J})$

$\mathrm{rot}\,\mathrm{rot}\, \underline{\vec{H}} = \sigma_0\, \mathrm{rot}\, \underline{\vec{E}} = -j\omega\mu_0\sigma_0 \underline{\vec{H}}$

$\mathrm{grad}\,\underbrace{\mathrm{div}\, \underline{\vec{H}}}_{=0} - \Delta \underline{\vec{H}}$

$\Delta \underline{\vec{H}} - \underline{k}^2 \underline{\vec{H}} = \vec{0}$ $\qquad \underline{k} = \sqrt{j\omega\mu_0\sigma_0} = \sqrt{\frac{\omega\mu_0\sigma_0}{2}}(1+j)$

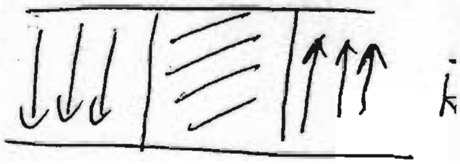
b)



$\oint_C \vec{H}\,d\vec{r} = I$ $\qquad \underline{\vec{H}} = \vec{e}_y\, \underline{H}_y(x)$

$|\underline{H}_y(-x)| = |\underline{H}_y(x)|$

$\underline{H}_y(-x) = -\underline{H}_y(x)$



$$\oint_C \underline{\vec{H}}\,d\vec{r} = h\left(\underline{H}_{1y}(x) - \underline{H}_{2y}(-x)\right)$$

$$\rightarrow \underline{H}_{1y} = \frac{I}{2h} \qquad \underline{H}_{2y} = -\frac{I}{2h}$$

zu a) $\quad \Delta \underline{\vec{H}} - \underline{k}^2 \underline{\vec{H}} = \vec{0} \qquad$ Ansatz $\underline{\vec{H}} = \underline{H}_y(x)\,\vec{e}_y$

$$\frac{d^2 \underline{H}_y}{dx^2} - \underline{k}^2 \underline{H}_y = 0$$

allg. Lösung $\quad \underline{H}_y = \underline{\alpha}\cosh(\underline{k}x) + \underline{\beta}\sinh(\underline{k}x)$

$$= \underline{\gamma}\, e^{-\underline{k}x} + \underline{\delta}\, e^{+\underline{k}x}$$

c) Betrachte Grenze $x = \pm a$; dort muss $H_{tang} = H_y$ stetig sein

$$\Rightarrow \underline{H}_y(x=+a) = \underline{\alpha}\cosh(\underline{k}a) + \underline{\beta}\sinh(\underline{k}a) = \frac{I}{2h}$$

$$\underline{H}_y(x=-a) = \underline{\alpha}\underbrace{\cosh(-\underline{k}a)}_{=\cosh(\underline{k}a)} + \underline{\beta}\underbrace{\sinh(-\underline{k}a)}_{-\sinh(\underline{k}a)} = -\frac{I}{2h}$$

$$\Rightarrow \underline{\alpha} = 0, \quad \boxed{\underline{\beta} = \frac{I}{2h}\,\frac{1}{\sinh(\underline{k}a)}}$$

$$\left(\underline{H}_y = \frac{I}{2h}\cdot\frac{\sinh(\underline{k}x)}{\sinh(\underline{k}a)} \right) \qquad \underline{k} = (1+j)\frac{1}{a_0}$$

$$a_0 = \sqrt{\frac{2}{\mu_0 \kappa_0 \omega}}$$

15/12/10

④

$$\underline{\vec{J}} = \mathrm{rot}\, \underline{\vec{H}} = \vec{e}_z \frac{\partial H_y}{\partial x} = \frac{I}{2h} \underline{k} \frac{\cosh(kx)}{\sinh(ka)}$$

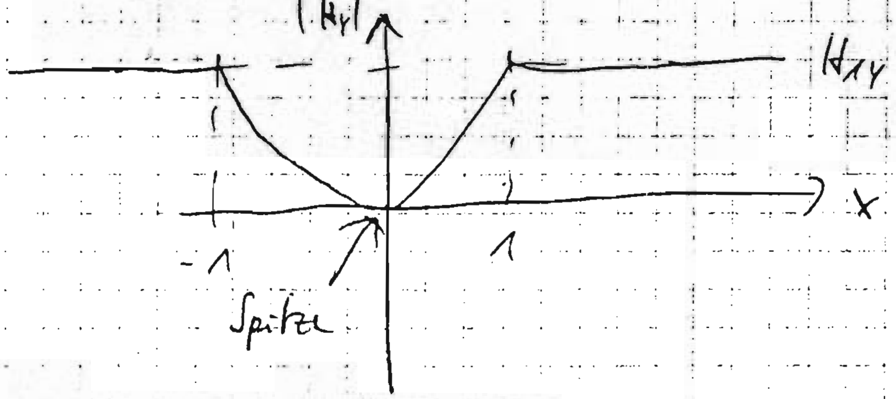
d)

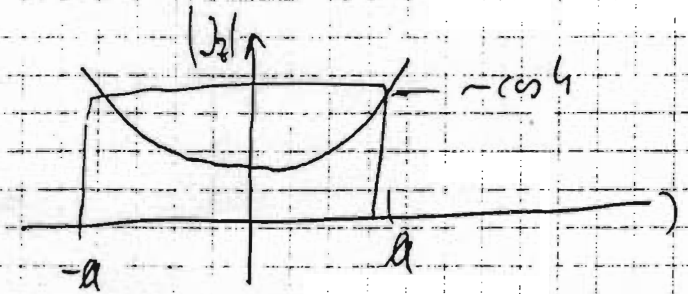
Skizzen von $|\underline{H}_y|$ $|\underline{J}_z|$

$|\sinh[(1+j)u]|$ kann in guter Näherung durch $|\sinh(u)|$ ersetzt werden; cos analog

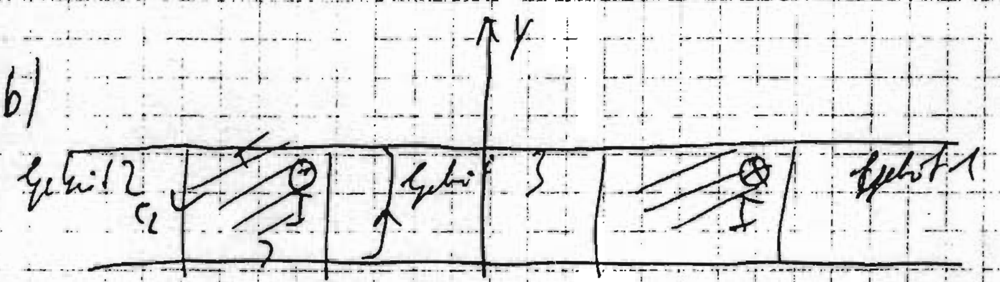
$$\Rightarrow |\underline{H}_y| = \frac{I}{2h} \frac{\sinh \frac{x}{a_0}}{\sinh \frac{a}{a_0}}$$

$$|\underline{J}_z| = \frac{I}{2h} \frac{\sqrt{2}}{a_0} \frac{\cosh(\frac{x}{a_0})}{\sinh(\frac{a}{a_0})}$$





b)



$$\oint_{c_1} \vec{H}\, d\vec{r} = [\underline{H}_y(x_2) - \underline{H}_y(x_1)]\, h = 0$$

kein Strom eingeschlossen

$$\Rightarrow \underline{H}_y(x_2) = \underline{H}_y(x_1) = \underline{H}_y(x) \qquad (x > 3a)$$

$= \ldots$

$\underline{H}_{y2}$ = Konst. analog

$\underline{H}_y \to 0$ für $x \to \infty$ $\Rightarrow$ const = 0

Konst = 0 analog

$$\oint_{C_2} \vec{H}\, d\vec{r} = I = \underline{H}_{y3}(x) \cdot h \qquad \Big(H_y(x=-3a) = \underset{\substack{\uparrow \\ \text{Stetigkeit}}}{=} H_{y2} = 0\Big)$$

$$\Rightarrow \underline{H}_{y3}(x) = \frac{I}{h} = \text{const.}$$

c) Feld im rechten Leiter:

aus a) $\underline{H}_y(x) = \underline{\alpha} \cosh(\underline{k}x) + \underline{\beta} \sinh(\underline{k}x)$

$$= \underline{\alpha}' \cosh(\underline{k}(x-3a)) + \underline{\beta}' \sinh(\underline{k}(x-3a))$$

Grenzbed. bei $x = 3a$: $\underline{H}_y(3a) = \underline{H}_{1y}(3a) = 0 \Rightarrow \underline{\alpha}' \overset{!}{=} 0$

Grenzbed. bei $x = a$

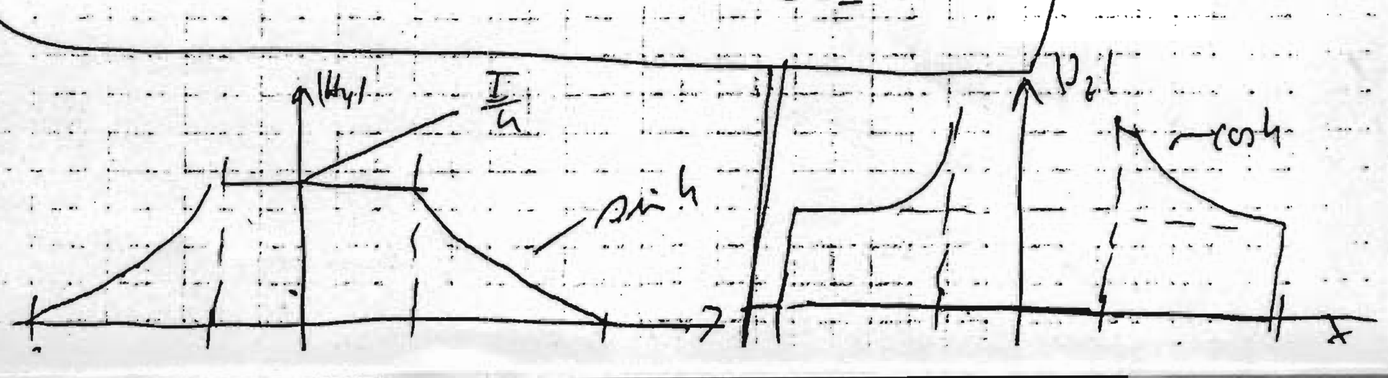
$$H_y(a) = \underline{\beta}' \underbrace{\sinh[\underline{k}(-2a)]}_{= -\sinh[\underline{k} \cdot 2a]} = \underline{H}_{3y}(a) = \frac{I}{h}$$

$$\Rightarrow \underline{H}_y(x) = -\frac{I}{h} \frac{\sinh[\underline{k}(x-3a)]}{\sinh[2\underline{k}a]}$$

$$= \frac{I}{h} \frac{\sinh[\underline{k}(3a-x)]}{\sinh(2\underline{k}a)}$$

$$\vec{J} = \vec{e}_z \frac{\partial H_y}{\partial x} = -\frac{I}{h} \underline{k} \frac{\cosh[\underline{k}(3a-x)]}{\sinh[2\underline{k}a]}$$

g)
d)



Musterlösung, ETMF2, H00, Aufgabe 2

a) TEM Welle: $\underline{\vec{H}} = \frac{1}{Z} \vec{n} \times \vec{E}$

$$\underline{\vec{H}}_e = \sqrt{\frac{\varepsilon}{2\mu_0}} \left(\vec{e}_z - \vec{e}_x\right) \underline{E}_{e0}\, e^{-jk\vec{n}_e\vec{r}}$$

$$\underline{\vec{H}}_r = \sqrt{\frac{\varepsilon}{\mu_0}} \left(\vec{e}_z \sin\varphi_r + \vec{e}_x \cos\varphi_r\right) \underline{E}_{r0}\, e^{-jk\vec{n}_r\vec{r}}$$

b) $\vec{n}_e \cdot \vec{r} = \frac{1}{\sqrt{2}}(x+z) \qquad \vec{n}_e \cdot \vec{r}\Big|_{z=0} = \frac{x}{\sqrt{2}}$

$\vec{n}_r \cdot \vec{r} = x \sin\varphi_r - z\cos\varphi_r \qquad \vec{n}_r \cdot \vec{r}\Big|_{z=0} = x \sin\varphi_r$

$z=0$: $\operatorname{Rot}\vec{E} = 0 \iff E_{tang}$ stetig $\overset{hier}{\iff}$ $\underline{E}_y$ stetig $\Rightarrow \underline{E}_y = 0$ wg. idealem Leiter

$$\Rightarrow \left(\underline{E}_{ey} + \underline{E}_{ry}\right)_{z=0} = \underline{E}_{e0}\, e^{-jk\frac{x}{\sqrt{2}}} + \underline{E}_{r0}\, e^{-jkx\sin\varphi_r} = 0 \quad \forall x$$

$\Rightarrow$ 1) $\sin\varphi_r = \frac{1}{\sqrt{2}} \Rightarrow \varphi_r = \frac{\pi}{4} \Rightarrow \cos\varphi_r = \frac{1}{\sqrt{2}}$

$\Rightarrow \vec{n}_r \cdot \vec{r} = \frac{1}{\sqrt{2}}(x-z) \qquad \vec{n}_r = \frac{1}{\sqrt{2}}\left(\vec{e}_x - \vec{e}_z\right)$

2) $\underline{E}_{r0} = -\underline{E}_{e0}$

c) $\underline{\vec{E}} = \underline{\vec{E}}_e + \underline{\vec{E}}_r = \vec{e}_y E_0 \left(e^{-jk\frac{x+z}{\sqrt{2}}} - e^{-jk\frac{x-z}{\sqrt{2}}}\right)$

$$= \vec{e}_y E_0\, e^{-jk\frac{x}{\sqrt{2}}} \underbrace{\left(e^{-jk\frac{z}{\sqrt{2}}} - e^{+jk\frac{z}{\sqrt{2}}}\right)}_{-2j\sin\left(\frac{kz}{\sqrt{2}}\right)}$$

$$\underline{\vec{E}} = -\vec{e}_y E_0\, 2j\, e^{-jk\frac{x}{\sqrt{2}}} \sin\left(\frac{kz}{\sqrt{2}}\right)$$

$$\underline{\vec{H}} = \underline{\vec{H}}_e + \underline{\vec{H}}_r = \sqrt{\frac{\varepsilon}{2\mu_0}}\, E_0 \left\{\left(\vec{e}_z - \vec{e}_x\right) e^{-jk\frac{x+z}{\sqrt{2}}} - \left(\vec{e}_z + \vec{e}_x\right) e^{-jk\frac{x-z}{\sqrt{2}}}\right\}$$

$$\vec{H} = \sqrt{\frac{\varepsilon}{2\mu_0}}\, e^{-jk\frac{x}{\sqrt{2}}} \Big(\vec{e_z} \underbrace{\left(e^{-jk\frac{z}{\sqrt{2}}} - e^{jk\frac{z}{\sqrt{2}}} \right)}_{-2j\sin\left(\frac{kz}{\sqrt{2}}\right)} - \vec{e_x} \underbrace{\left(e^{-jk\frac{z}{\sqrt{2}}} + e^{jk\frac{z}{\sqrt{2}}} \right)}_{2\cos\left(\frac{kz}{\sqrt{2}}\right)} \Big) E_0$$

$$\Rightarrow \vec{H} = -E_{e0} \sqrt{\frac{2\varepsilon}{\mu_0}} \left(\vec{e_x} \cos\left(\frac{kz}{\sqrt{2}}\right) + \vec{e_z}\, j \sin\left(\frac{kz}{\sqrt{2}}\right) \right) e^{-jk\frac{x}{\sqrt{2}}}$$

d) $\overline{\vec{S}(t)} = \frac{1}{2} \operatorname{Re}\left\{ \vec{E} \times \vec{H}^{*} \right\}$

$$\overline{\vec{S}(t)} = \frac{1}{2} \operatorname{Re}\left\{ 2j\, E_{e0}^2 \sqrt{\frac{2\varepsilon}{\mu_0}} \sin\left(\frac{kz}{\sqrt{2}}\right) \left(\vec{e_y} \times \left(\vec{e_x} \cos\left(\frac{kz}{\sqrt{2}}\right) - j\vec{e_z} \sin\left(\frac{kz}{\sqrt{2}}\right) \right) \right) \right\}$$

$$\overline{\vec{S}(t)} = E_{e0}^2 \sqrt{\frac{2\varepsilon}{\mu_0}} \sin\left(\frac{kz}{\sqrt{2}}\right) \left(-j\vec{e_z} \cos\left(\frac{kz}{\sqrt{2}}\right) + \vec{e_x} \sin\left(\frac{kz}{\sqrt{2}}\right) \right)$$

$$\overline{\vec{S}(t)} = E_{e0}^2 \sqrt{\frac{2\varepsilon}{\mu_0}} \sin^2\left(\frac{kz}{\sqrt{2}}\right) \vec{e_x}$$

Im Zeitmittel wird Energie in $+x$ – Richtung transportiert.
(stehende Welle bezüglich z – Richtung)

e) 1) $|\overline{\vec{S}(t)}| = 0$ für $\sin\left(\frac{kz}{\sqrt{2}}\right) = 0$

$$\Rightarrow \cancel{\frac{kz_l}{\sqrt{2}l}} \quad \frac{kz_l}{\sqrt{2}} = l \cdot \pi \quad ; \quad l = 0, -1, -2, \ldots$$

$$k = \frac{2\pi}{\lambda} \Rightarrow z_l = \frac{\sqrt{2}\, l\pi}{k} = \frac{\sqrt{2}\, l\pi}{2\pi} \lambda = \frac{l\lambda}{\sqrt{2}} \qquad l = 0, -1, -2, \ldots$$

2) $|\overline{\vec{S}(t)}|$ – Maximum für $\sin^2\left(\frac{kz_m}{\sqrt{2}}\right) = 1$

$$\frac{kz_m}{\sqrt{2}} = \left(m - \frac{1}{2}\right)\pi \quad ; \quad m = 0, -1, -2, \ldots$$

$$z_m = \frac{(2m-1)\lambda}{2\sqrt{2}} \qquad m = 0, -1, -2, \ldots$$

A3 HCD Felder II 4/2/01

1

Zusatzaufgabe II

② ε_0 | ① $\varepsilon \gg \varepsilon_0$ $\rightarrow \vec{n}$

$D_{n1} = D_{n2}$ Stetigkeit

$\varepsilon E_{n1} = \varepsilon_0 E_{n2}$

$$\frac{E_{n1}}{E_{n2}} = \frac{\varepsilon_0}{\varepsilon} \ll 1$$

$\Rightarrow E_{n1} \ll E_{tan,1} \Rightarrow$ Annahme (idealisiert) $\boxed{E_{n1} = 0}$

$\vec{H} \perp \vec{E}$ (aus Maxwell gl.) $\Rightarrow \underbrace{\boxed{H_{tan} = 0}}_{\hat{=} \text{ magn. Wand}}$

a) $\text{rot } \underline{\vec{E}} = -j\omega \underline{\vec{B}} = -j\omega\mu \underline{\vec{H}}$

$$\underline{\vec{H}} = \frac{j}{\omega\mu} \text{ rot } \underline{\vec{E}}$$

TE-Welle (nach Vorraus.) $\hat{=} \underline{E_z} = 0$

$$\frac{\partial \underline{E_x}}{\partial z} = -j\beta \underline{E_x} \; ; \; \frac{\partial \underline{E_y}}{\partial z} = -j\beta \underline{E_y}$$

$$\underline{H_x} = \frac{j}{\omega\mu}\Big(\underbrace{\cancel{\frac{\partial}{\partial y} \underline{E_z}}}_{=0} - \frac{\partial}{\partial z} \underline{E_y}\Big) = \frac{j}{\omega\mu}(+j\beta) E_y$$

$$\underline{H_x} = -\frac{\beta}{\omega\mu} B \cos(k_x x) \sin(k_y y) e^{-j\beta z}$$

$$\underline{H}_y = \frac{j}{\omega\mu}\left(\frac{\partial}{\partial z}\underline{E}_x - \underbrace{\cancel{\frac{\partial}{\partial x}\underline{E}_z}}_{=0}\right) = \frac{\beta}{\omega\mu}\underline{E}_x = \frac{\beta}{\omega\mu} A \sin(k_x x)\cos(k_y y) e^{-j\ldots}$$

$$\underline{H}_z = \frac{j}{\omega\mu}\left(\frac{\partial}{\partial x}\underline{E}_y - \frac{\partial}{\partial y}\underline{E}_x\right) = \frac{j}{\omega\mu}(A k_y - B k_x)\sin(k_x x)\sin(k_y y) e^{-j\ldots}$$

b) Zusammenhang zw. A, B

$$\operatorname{div}\underline{\vec{E}} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{=0}$$

$$\Rightarrow \frac{\partial \underline{E}_x}{\partial x} = -\frac{\partial \underline{E}_y}{\partial y} \quad \Rightarrow k_x A = -k_y B$$

$$B = -A\frac{k_x}{k_y}$$

alternativ $\quad \operatorname{rot}\underline{\vec{H}} = j\omega\varepsilon\underline{\vec{E}}$

$\Rightarrow \underline{\vec{E}} = \frac{1}{j\omega\varepsilon}\operatorname{rot}\underline{\vec{H}}$ muß wieder gegebenes E-Feld liefern

geschickt: mit $\underline{E}_z = 0$ anfangen

$$0 = \frac{1}{j\omega\varepsilon}\left(\frac{\partial}{\partial x}\underline{H}_y - \frac{\partial}{\partial y}\underline{H}_x\right)$$

$$\Rightarrow A k_x \cos(k_x x)\cos(k_y y) := -B k_y \cos(k_x x)\cos(k_y y)$$

$$\Rightarrow A k_x = -B k_y$$

c) G_1, G_2 : $E_{tan} = 0 \quad H_{norm} = 0$ (elektr. Wand)

G_1 : $0 \le x \le a, \; y = b$

$E_x = 0 \quad (E_z = 0$ erfüllt$)$; $H_y = 0$

$E_x \doteq 0$ für $y = b \rightarrow \cos(k_y b) = 0$

$H_y = 0$ " "

$$k_y b = \frac{2l-1}{2}\pi \quad k_y = \frac{2l-1}{2}\frac{\pi}{b};\ l = 1,2,3.$$

G_2: $x = a$, $0 \le y \le b$

$$\underbrace{E_y = 0 \quad (E_z = 0 \text{ erfüllt}) \quad H_x = 0}$$

$$\cos(k_x a) = 0$$

$$k_x = \frac{2m-1}{2}\frac{\pi}{a};\ m = 1,2,3\ldots$$

G_3, G_4: $E_{norm} = 0$, $H_{tan} = 0$ (magn. Wand)

G_3: $E_y = 0$, $H_x = 0$, $H_z = 0$ erfüllt
$\hookrightarrow y = 0$

$G_1 \hat{=}\ x = 0$ $E_x = 0$, $H_y = 0$, $H_z = 0$ erfüllt

d) $k^2 = k_x^2 + k_y^2 + \cancel{k_z^2}\,\beta^2$, $k^2 = \omega^2 \varepsilon \mu$

Es gilt [folgt aus Maxwell gl.]

$$\Delta \underline{\vec{E}} + k^2 \underline{\vec{E}} = \vec{0} \quad \Delta \underline{\vec{H}} + k^2 \underline{\vec{H}} = 0$$

also auch z.B.

$$\Delta \underline{E}_x + k^2 \underline{E}_x = 0$$

$$\underbrace{\frac{\partial^2}{\partial x^2}\underline{E}_x} + \frac{\partial^2}{\partial y^2}\underline{E}_x + \frac{\partial^2}{\partial z^2}\underline{E}_x + k^2 \underline{E}_x = 0$$

$-k_x^2 E$ ... [illegible]