Lösung (H07)

A1

a) Gesamtladung des Zylinders $\stackrel{!}{=} 0$

$$\rho_+ \cdot 2 \cdot l \cdot \hat{a} \cdot r_i^2 = -\rho_- \; 2 \cdot l \cdot \hat{a} \left(r_a^2 - (r_a - d)^2\right)$$

$$\Rightarrow \hat{a} \cdot r_i^2 = \hat{a} \cdot r_a^2 - \hat{a}\left(r_a^2 - 2 r_a d + d^2\right)$$

$$\Rightarrow r_i^2 = 2 r_a \cdot d - d^2$$

$$\Rightarrow d^2 - 2 r_a \cdot d + r_a^2 = -r_i^2 + r_a^2$$

$$\Rightarrow \cancel{(d - r_a^2) = r_a^2 - r_i^2} \qquad d = r_a \pm \sqrt{r_a^2 - r_i^2}$$ sinnvoll nur: $d < r_a$

$$d = r_a - \sqrt{r_a^2 - r_i^2}$$

b) $\operatorname{rot} \vec{H} = \vec{J} + \dot{\vec{D}}$ $\quad \boxed{\operatorname{div} \operatorname{rot} \vec{A} = 0}$

$$\operatorname{div} \operatorname{rot} \vec{H} = 0 = \operatorname{div} \vec{J} + \operatorname{div} \dot{\vec{D}}$$

$$\operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad \text{da } \operatorname{div} \vec{D} = \rho$$

c) ohm'sches Gesetz $\vec{J} = \sigma_{Zy} \vec{E}$

$$\operatorname{div} \vec{J} = \operatorname{div}(\sigma_{Zy} \cdot \vec{E}) = \sigma_{Zy} \cdot \operatorname{div} \vec{E}$$

$$\sigma_{Zy} \cdot \frac{\rho}{\varepsilon_0 \varepsilon_r} + \dot{\rho} = 0$$

$$\Rightarrow \frac{\dot{\rho}}{\rho} = -\frac{\sigma_{Zy}}{\varepsilon_0 \varepsilon_r}$$

Lösung für $\underline{0 \le r \le r_i}$:

$$\rho(t) = \rho_+ \cdot e^{-\frac{\sigma_{Zy}}{\varepsilon_0 \varepsilon_r} \cdot t}$$

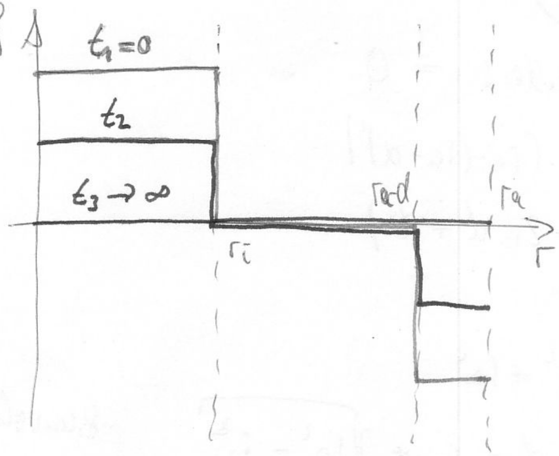
$\underline{r_i \le r \le r_a - d}$:

$$\rho(t) \equiv 0$$

$\underline{r_a - d \le r \le r_a}$:

$$\rho(t) = \rho_- \cdot e^{-\frac{\sigma_{Zy} \cdot t}{\varepsilon_0 \varepsilon_r}}$$

d)



e) $I' = -\dot{\varrho}(t) \cdot \pi \cdot r_i^2$ $\quad\quad \varrho = \text{Lösung für } 0 \leq r \leq r_i$

$$= +\frac{\sigma_{z_4}}{\varepsilon_0 \varepsilon_r} \cdot \varrho_+ \cdot e^{-\frac{\sigma_{z_4} \cdot t}{\varepsilon_0 \varepsilon_r}} \cdot \pi \cdot r_i^2$$

$$\boxed{I'(t) = \frac{I(t)}{\Delta l} = \frac{\dot{\varrho}(t) \cdot \Delta V}{\Delta l} = \frac{\dot{\varrho}(t) \Delta A \cancel{\Delta l}}{\cancel{\Delta l}}}$$

f) $D_r(r,t) = \dfrac{2 \cdot l \cdot \varrho(t) \cdot \pi \cdot r_i^2}{2 \cdot l \cdot 2 \cdot \pi \cdot r_i \pi} = \varrho_+ \cdot e^{-\frac{\sigma_{z_4} \cdot t}{\varepsilon_0 \varepsilon_r}} \cdot \dfrac{r_i}{2}$

eingeschl. Ladung

Oberfläche

allgemein: $D_r(r,t) = \dfrac{\varrho_+ \cdot e^{-\frac{\sigma_{z_4}}{\varepsilon_0 \varepsilon_r} t} \cdot r_i^2}{2 \cdot r}$ $\quad$ für $r_i < r < r_a - d$

$$p = \vec{J} \cdot \vec{E} = \vec{J} \cdot \frac{\vec{D}}{\varepsilon_0 \varepsilon_r} = J_r \frac{D_r}{\varepsilon_0 \varepsilon_r}$$

$$|\vec{J}| = J_r = \frac{I'}{2\pi r} = \varrho_+ \frac{\sigma_{z_4}}{\varepsilon_0 \varepsilon_r} \cdot e^{-\frac{\sigma_{z_4}}{\varepsilon_0 \varepsilon_r}} \cdot \frac{\pi \cdot r_i^2}{2\pi r}$$

$$p = \frac{1}{\varepsilon_0 \varepsilon_r} \varrho_+ \frac{\sigma_{z_4}}{\varepsilon_0 \varepsilon_r} \cdot e^{-\frac{\sigma_{z_4} \cdot t}{\varepsilon_0 \varepsilon_r}} \frac{\pi r_i^2}{2\pi r} \circ \frac{\varrho_+ \; e^{-\frac{\sigma_{z_4} \cdot t}{\varepsilon_0 \varepsilon_r}} r_i^2}{2r} =$$

$$= \frac{1}{(\varepsilon_0 \varepsilon_r)^2} \sigma_{z_4} \cdot \varrho_+^2 \cdot e^{-\frac{2\sigma_{z_4} \cdot t}{\varepsilon_0 \varepsilon_r}} \frac{\cancel{\pi} r_i^4}{4 \cancel{\pi} r^2}$$

Lösung (H07)

a) $\vec{n} = \frac{\vec{S}}{|\vec{S}|} = \frac{1}{\sqrt{1^2+2^2}}(-\vec{e}_y + 2\vec{e}_z) = \frac{1}{\sqrt{5}}(-\vec{e}_y + 2\vec{e}_z)$

$Z = \frac{|\vec{E}|}{|\vec{H}|}$ in $(0,0,0)$ oder $\vec{H} = \frac{1}{Z}(\vec{n} \times \vec{E})$ bzw. $\vec{E} = -Z(\vec{n} \times \vec{H})$

$\underline{\vec{E}}(0,0,0)$ über $\vec{S}(0,0,0) = \underline{\vec{E}}(0,0,0) \times \underline{\vec{H}}^*(0,0,0)$

$$\underline{\vec{E}} \times \underline{\vec{H}}^* = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \underline{E}_x & \underline{E}_y & \underline{E}_z \\ -j\frac{8E_0}{Z_0} & 0 & 0 \end{vmatrix} = \vec{S}(0,0,0)$$

Koeffizienten Vergleich:

$\underline{E}_z = -j\frac{E_0}{\cancel{2}}$; $\underline{E}_y = -j E_0$

weiterhin gilt: $\underline{\vec{E}} \cdot \underline{\vec{H}} \stackrel{!}{=} 0$

$\Rightarrow \underline{E}_x = 0$

$Z = \frac{|\vec{E}|}{|\vec{H}|} = \frac{\sqrt{5}\, E_0 \cdot Z_0}{2 \cdot 8 \cdot E_0} = \frac{\sqrt{5}}{16} Z_0$

nicht so einfach

$\underline{\vec{E}} \times \underline{\vec{H}}^* = -Z(\vec{n} \times \underline{\vec{H}}) \times \underline{\vec{H}}^*$

$\vec{S} = Z \cdot \underline{\vec{H}}^* \times (\vec{n} \times \underline{\vec{H}})$

$\vec{S} = Z \cdot [\vec{n} \cdot (\underline{\vec{H}} \cdot \underline{\vec{H}}^*) - \underbrace{(\vec{n} \cdot \underline{\vec{H}}^*)}_{=0} \cdot \underline{\vec{H}}]$

$= Z \cdot \vec{n} (\underline{\vec{H}} \cdot \underline{\vec{H}}^*)$

$\frac{4E_0^2}{Z_0} \vec{n} \sqrt{5} = Z \cdot \frac{64 E_0^2}{\cancel{\ }Z_0^2} \vec{n}$

$\Rightarrow Z = \frac{\sqrt{5}}{16} Z_0$

$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \frac{1}{\sqrt{\varepsilon_r}} Z_0$

$\frac{1}{\sqrt{\varepsilon_r}} = \frac{\sqrt{5}}{16} \Rightarrow \varepsilon_r = \frac{16^2}{5} = 51{,}2$

$\vec{k} = \frac{2\pi \cdot \vec{n}}{\lambda} = \frac{\omega}{c} \cdot \vec{n} = \omega\sqrt{\varepsilon_r \varepsilon_0 \mu_0} \cdot \vec{n} = k_0 \sqrt{\varepsilon_r} \cdot \vec{n} = \frac{16}{\sqrt{5}} k_0 \cdot \vec{n}$

b) $\underline{\vec{E}}(x,y,z) = \underline{\vec{E}}(0,0,0) \cdot e^{-j\vec{k}\vec{r}}$

$\vec{k} \cdot \vec{r} = k_0 \frac{16}{\sqrt{5}} \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = k_0 \frac{16}{5}(-y + 2z)$

$\Rightarrow \underline{\vec{E}} = -j \frac{E_0}{2}(2\vec{e}_y + \vec{e}_z) \cdot e^{-j k_0 \frac{16}{5}(-y+2z)}$

analog: $\underline{\vec{H}} = j \frac{8E_0}{Z_0} \vec{e}_x \cdot e^{-j k_0 \frac{16}{5}(-y+2z)}$

c) $\vec{\underline{E}}(x,y,z,t) = \vec{\underline{E}}(x,y,z)\cdot e^{-j\omega t} = -j\frac{E_0}{2}(2\cdot\vec{e}_y+\vec{e}_z)\cdot e^{-j[k_0\frac{16}{5}(-y+2z)\cancel{\omega}-\omega t}$

$$= \frac{E_0}{2}(2\vec{e}_y+\vec{e}_z)\cdot e^{-j[k_0\frac{16}{5}(-y+2z)-\omega t+\frac{\pi}{2}]}$$

$$\vec{E}(x,y,z,t) = \mathrm{Re}\{\vec{\underline{E}}(x,y,z,t)\}$$

$$\vec{\underline{E}}(x,y,z,t) = \frac{E_0}{2}(2\vec{e}_y+\vec{e}_z)\cdot\left[\cos\left(k_0\frac{16}{5}(-y+2z)-\omega t+\frac{\pi}{2}\right) - j\sin\left(k_0\frac{16}{5}(-y+2z)-\omega t+\frac{\pi}{2}\right)\right]$$

$\vec{E}(x,y,z,t) = \frac{E_0}{2}(2\cdot\vec{e}_y+\vec{e}_z)\cdot\left\{-\sin\left[k_0\frac{16}{5}(-y+2z)-\omega t\right]\right\}$ da: $\cos(x+\frac{\pi}{2}) = \sin(x)$

linear polarisierte Welle

d) allg.: $\vec{\underline{E}}_{tan} = 0$ an ideal leitender Wand

hier: $\left.\left(\vec{\underline{E}}_{h,tan} + \vec{\underline{E}}_{r,tan}\right)\right|_{\substack{x=0\\y=0\\z=0}} = 0$ h: hinlaufend r: rücklaufend

e) stehende Welle

Die Welle wird vollständig reflektiert, keine Leistung wird im Zeitmittel transportiert: $\overline{\vec{S}} = 0$

Lösung (H07)

...ellengleichung: $\Delta \underline{g}(x,y,z) + k^2 \underline{g}(x,y,z) = 0$ *

Separationsansatz: $\underline{g}(x,y,z) = \underline{u}(x) \cdot \underline{v}(y)\, e^{-j\beta z}$

allg. Lsg.: $\underline{u}(x) = A \cdot \cos(k_x \cdot x) + B \cdot \sin(k_x \cdot x)$

$\underline{v}(y) = C \cdot \cos(k_y \cdot y) + D \cdot \sin(k_y \cdot y)$

$* \Rightarrow \quad 0 = -k_x^2 - k_y^2 - \beta^2 + k^2 \Rightarrow \beta(\omega) = \sqrt{\omega^2 \varepsilon \mu - k_x^2 - k_y^2}$

c) $\underline{G_1}$ & $\underline{G_3}$: ideal elektrisch leitende Wände

$\vec{\underline{E}}_{tan} = \vec{0}$ d.h. $0 = \underline{E}_x(x,0,z) = \underline{E}_x(x,b,z)$

$0 = \underline{E}_z(x,0,z) = \underline{E}_z(x,b,z)$

$\underline{G_2}$ & $\underline{G_4}$: ideal magnetische Wände:

$\vec{\underline{H}}_{tan} = \vec{0}$ d.h. $0 = \underline{H}_y(0,y,z) = \underline{H}_y(a,y,z)$

($H_z = 0$, da TM-Welle)

$\underline{H}_y = \left(\vec{\nabla} \times \underline{g}(x,y,z) \cdot \vec{e}_z\right)_y = -\frac{\partial}{\partial x} \underline{g}(x,y,z)$

$x = 0$: $0 = H_y(0,y,z) = -\underline{u}'(0)\, \underline{v}(y)\, e^{-j\beta z}$

$\Rightarrow \quad 0 = 0 + B k_x \Rightarrow B = 0$

$x = a$: $0 = H_y(a,y,z) = -\underline{u}'(a)\, \underline{v}(y) \cdot e^{-j\beta z}$

$\Rightarrow k_x = \frac{c_1 \cdot \pi}{a} \qquad c_1 = 1,2,3$

$\underline{G_1}$ & $\underline{G_3}$:

$y = 0$: $0 = \frac{\partial}{\partial x} \underline{g}(x,y,z)\Big|_{y=0} = \underline{u}'(x) \cdot \underline{v}(0) \cdot e^{-j\beta z}$

$v(0) = 0 \Rightarrow C = 0$

$y = b$: $v(b) = 0 \Rightarrow D \cdot \sin(k_y \cdot b) = 0$

$\Rightarrow k_y = \frac{c_2 \cdot \pi}{b} \qquad c_2 = 1,2,3$

Somit ist $\underline{g}(x,y,z) = A \cdot \cos\left(\frac{c_1 \cdot \pi}{a} \cdot x\right) \sin\left(\frac{c_2 \cdot \pi}{b}\right) e^{-j\beta z}$ A=const c_1, c_2

d) $$\underline{\vec{H}} = \vec{\nabla} \times \underline{g}(x,y,z) = \begin{pmatrix} \frac{\partial}{\partial y} \underline{g}(x,y,z) \\ -\frac{\partial}{\partial x} \underline{g}(x,y,z) \\ 0 \end{pmatrix}$$

$b = a$: $k_x = \frac{c_1 \cdot \pi}{a}$ $\quad k_y = \frac{c_2 \cdot \pi}{a}$

$$\Rightarrow \underline{H}_x = \frac{\partial}{\partial y} \underline{g}(x,y,z) = \frac{c_2 \cdot \pi}{a} \cdot A \cdot \cos\left(\frac{c_1 \cdot \pi}{a} x\right) \cdot \cos\left(\frac{c_2 \cdot \pi}{a} \cdot y\right) \cdot e^{-j\beta z}$$

$$\underline{H}_y = -\frac{\partial}{\partial x} (\underline{g}(x,y,z)) = \frac{c_1 \cdot \pi}{a} \cdot A \cdot \sin\left(\frac{c_1 \cdot \pi}{a} \cdot x\right) \cdot \sin\left(\frac{c_2 \cdot \pi}{a} \cdot y\right) \cdot e^{-j\beta z}$$

mit $c_1 = m$, $c_2 = n$ und $A_{mn} = \frac{c_2 \cdot \pi}{a} \cdot A$

folgt: $\underline{H}_x = A_{mn} \cos\left(\frac{m \cdot \pi}{a} x\right) \cos\left(\frac{n \cdot \pi}{a} \cdot y\right) e^{-j\beta z}$

$\underline{H}_y = \frac{m}{n} A_{mn} \sin\left(\frac{m \cdot \pi}{a} x\right) \sin\left(\frac{n \cdot \pi}{a} \cdot y\right) \cdot e^{-j\beta z}$

e) C_5: $\underline{\vec{H}}_{tan} = 0$ $\quad H_z \equiv 0$ $\quad$ (TM-Welle)

$\underline{H}_y = \left(\frac{3}{4} \cdot a, y, z\right) = 0$

C_6: $\underline{\vec{E}}_{tan} = 0$ $\quad \underline{E}_z\left(x, \frac{1}{2} a, z\right) = 0$

$\underline{E}_x\left(x, \frac{1}{2} a, z\right) = 0$

f) C_5: $H_y\left(\frac{3}{7} a, y, z\right) = 0$

$\Rightarrow 0 = \sin\left(k_x \frac{3}{7} a\right) = \sin\left(\frac{m \cdot \pi}{a} \frac{3}{7} a\right) = \sin\left(3 \frac{m \cdot \pi}{7}\right)$ ist erfüllt für $\frac{m}{7} = 1, 2, 3, \ldots$

C_6: $\underline{E}_z\left(x, \frac{1}{2} a, z\right) = 0$

$\Rightarrow 0 = \sin\left(k_y \frac{1}{2} a\right) = \sin\left(\frac{n}{2} \cdot \pi\right)$ ist erfüllt für $n = 2, 4, 6$

$\omega^2 \cdot \varepsilon \mu = \beta^2 + k_x^2 + k_y^2$

$$\omega_{mn} = \sqrt{\frac{\beta^2 + \left(\frac{m}{a} \cdot \pi\right)^2 + \left(\frac{n}{a} \cdot \pi\right)^2}{\varepsilon \cdot \mu}}$$

$$\omega_{min} = \sqrt{\frac{0 + \left(\frac{7}{a} \cdot \pi\right)^2 + \left(\frac{2}{a} \cdot \pi\right)^2}{\varepsilon \mu}} = \frac{\sqrt{53}\, \pi}{\sqrt{\varepsilon\mu} \cdot a}$$

$$f_{min} = \frac{\omega_{min}}{2\pi} = \frac{\sqrt{53}}{2a\sqrt{\varepsilon\mu}}$$

Lsg. #07

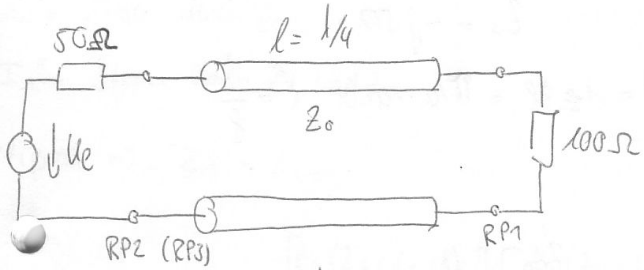
a)
$$\underline{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

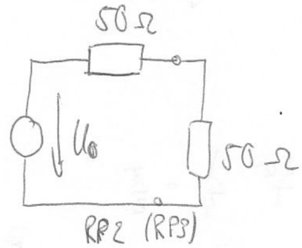
b) Reziprozität: $S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$

Symmetrie: $S_{22} = S_{33}$, $S_{21} = S_{31}$

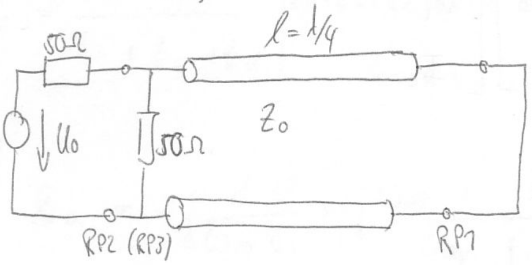
unabhängige S-Parameter: $S_{11}, S_{12}, S_{23}, S_{22}$

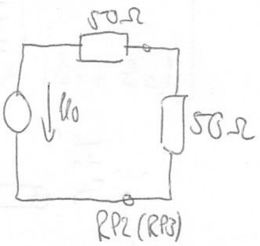
c)





Even





Odd

d) Spannung am Tor 3: $\underline{U}_{g3} = 2V = 1V + 1V = \underline{U}_{g3}^{e} + \underline{U}_{g3}^{o}$ (1)

Spannung am Tor 2: $\underline{U}_{g2} = 0V = 1V - 1V = \underline{U}_{g2}^{e} + \underline{U}_{g2}^{o}$ (2)

$\underline{U}_2 = \underline{U}_2^{e} + \underline{U}_2^{o}$

even mode circuit $\Rightarrow$ $\underline{U}_2^{e} = 0{,}5V$

odd mode circuit $\Rightarrow$ $\underline{U}_2^{o} = -0{,}5V$

$\Big\} \Rightarrow \underline{U}_2 = 0V$

e) Z_x: Impedanz am Eingang der offenen 50Ω Leitung der Länge l_0

Die Schaltung ist symmetrisch, Einspeisung an Tor 1 ruft die gleichen Spannungen an Tor 2 und 3 hervor. Deswegen kann der 100Ω Widerstand vernachlässigt werden.

$Z_{in} = \frac{1}{2}\frac{Z_0^2}{Z_x}$ und $\underline{S}_{11} = \frac{Z_{in} - 50}{Z_{in} + 50}$ $\underline{S}_{11} = 1 \angle 90° = j$

Der Smith Chart kann benutzt werden, um den Wert der Eingangsimpedanz zu bestimmen um den Reflektionskoeffizient von $\underline{S}_{11} = 1 \angle 90° = j$ bzw. $Z_{in} = j\,50\,\Omega$ zu erreichen.

Ohne Smith Chart:

$$Z_{in} = 50 \cdot \frac{1+S_{11}}{1-S_{11}} = 50 \cdot \frac{1+j}{1-j} = 50 \frac{\sqrt{2}\,\angle 45°}{\sqrt{2}\,\angle -45°} = 50 \angle 90° - j50\,\Omega$$

von (1) $Z_x = \frac{1}{2}\frac{Z_0^2}{Z_{in}} = \frac{1}{2}\frac{(\sqrt{2}\,50)^2}{j50} = -j50\,\Omega \Rightarrow Z_x^{norm} = -j$

Bestimmung von l_0 ohne Smith Chart:

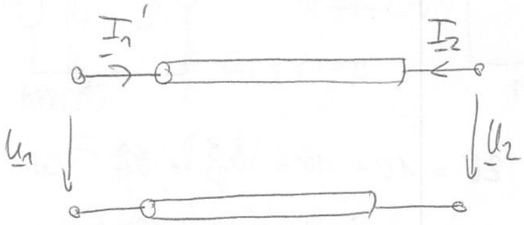
$$Z_x = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$ mit $Z_L = \infty$ $Z_x = -jZ_0\cot(\beta l) = -j50\cot(\beta l)$

$Z_x = -j50$

$\Rightarrow \cot(\beta l) = 1 \Leftrightarrow \beta l = \pi/4$ und $l = \frac{\lambda}{8}$

f) $\beta \cdot l = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1' \end{bmatrix} = \begin{bmatrix} 0 & jZ_{01} \\ j/Z_{01} & 0 \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ -\underline{I}_2 \end{bmatrix}$$



$\underline{U}_1 = jZ_{01}(-\underline{I}_2)$

$\underline{U}_2 = Z_{ref_2}(-\underline{I}_2)$

$$\frac{\underline{U}_1}{\underline{U}_2} = j\left(\frac{Z_{01}}{Z_{ref2}}\right)$$

$$\underline{U}_2 = -j\left(\frac{Z_{ref2}}{Z_{01}}\right)\cdot \underline{U}_1 = -j\left(\frac{50/\sqrt{3}}{75/\sqrt{3}}\right)\cdot \underline{U}_1 = -j\left(\frac{50}{75}\right)\underline{U}_1$$

$$\underline{U}_3 = -j\left(\frac{50}{75}\right)\cdot \underline{U}_1$$

$P_{RP2} = \frac{1}{2}\frac{|\underline{U}_2|^2}{Z_{ref_2}}$ $\quad P_{RP3} = \frac{1}{2}\frac{|\underline{U}_3|^2}{Z_{ref_3}}$ $\quad \underline{U}_3 = \underline{U}_2$

$$\frac{P_{RP2}}{P_{RP3}} = \frac{50\cdot\sqrt{3}}{50/\sqrt{3}} = 3$$

LSG. #07

Das Netzwerk ist reziprok. Daraus folgt: $\underline{Z}_{12} = \underline{Z}_{21}$

$\underline{Z}_{11}$ und $\underline{Z}_{22}$: Spannungsteiler $\underline{Z}_{11} = \underline{Z}_a + \underline{Z}_c \,\|\, (\underline{Z}_b + \underline{Z}_d)$

$$\underline{Z}_{22} = \underline{Z}_c \,\|\, (\underline{Z}_b + \underline{Z}_d)$$

$$\underline{Z}_{21} = \frac{U_2}{I_1}\bigg|_{I_2=0} = \frac{\underline{Z}_d \cdot I_1''}{I_1' + I_1''} = \frac{\underline{Z}_d \cdot I_1''}{I_1'' + I_1''\left(\frac{\underline{Z}_b+\underline{Z}_d}{\underline{Z}_c}\right)} = \frac{\underline{Z}_c \cdot \underline{Z}_d}{\underline{Z}_b+\underline{Z}_c+\underline{Z}_d}$$

I_1' = Strom durch $\underline{Z}_c$

I_1'' = Strom durch $\underline{Z}_b$ und $\underline{Z}_d$

reziprok $\Rightarrow$ $\underline{Z}_{12} = \underline{Z}_{21}$

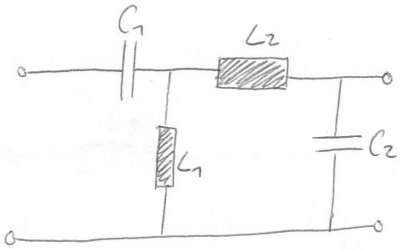
$$\underline{Z} = \frac{1}{\underline{Z}_b + \underline{Z}_c + \underline{Z}_d}\begin{bmatrix} \underline{Z}_a(\underline{Z}_b+\underline{Z}_c+\underline{Z}_d) + \underline{Z}_c(\underline{Z}_b+\underline{Z}_d) & \underline{Z}_c \cdot \underline{Z}_d \\ \underline{Z}_c \cdot \underline{Z}_d & \underline{Z}_d(\underline{Z}_b + \underline{Z}_c) \end{bmatrix}$$

c)

$$\underline{Z}_a = -j\frac{1}{\omega_{AP} C_1} + j\frac{\omega_{AP} \cdot 1}{\omega_{AP}^2 \cdot C_1} = 0$$

$$\Rightarrow \underline{Z}_{AP} = \frac{1}{\underline{Z}_b + \underline{Z}_c + \underline{Z}_d}\begin{bmatrix} \underline{Z}_c(\underline{Z}_b+\underline{Z}_d) & \underline{Z}_c \cdot \underline{Z}_d \\ \underline{Z}_c \cdot \underline{Z}_d & \underline{Z}_d(\underline{Z}_b + \underline{Z}_c) \end{bmatrix}$$

d)



$$\underline{Z}_d = -j\frac{1}{\omega C_2} \quad \underline{Z}_c = j\omega L_1$$

$$\underline{Z}_c - \underline{Z}_d = +j\frac{1}{\omega C_2} + j\omega L_1 \Rightarrow \mathrm{Im}\{\underline{Z}_c - \underline{Z}_d\} > 0$$

e) $Z_0 = 10\,\Omega$

$Z_{Transistor} = 1 - j1$

$\underline{Z}_a = 0 \quad \underline{Z}_b = j\,0{,}6 \quad \underline{Z}_c = j\,0{,}5 \quad \underline{Z}_d = -j2$

$$\underline{z}_{in} = \underline{z}_e // (\underline{z}_s + \underline{z}_d // \underline{z}_{Transistor}) = 0{,}4 + j\,0{,}2$$

$$S_{11} = \frac{0{,}4 + j0{,}2 - 1}{0{,}4 + j0{,}2 + 1} = -0{,}4 + j\,0{,}2$$

$$\underline{z}_{a,ideal} = -j\,0{,}2 \Rightarrow \underline{Z}_{a,ideal} = -j\,2\,\Omega$$

$$z_q = 0{,}4 \Rightarrow Z_q = 4\,\Omega = 0{,}4\,Z_0$$

LSG. 407

$$\underline{S}_{21} = \frac{b_2}{a_1} = \frac{b_2' \, e^{j\beta la}}{a_1' \, e^{-j\beta le}} = \underline{S}_{21}' \cdot e^{j\beta(le+la)}$$

$$\underline{S}_{22} = \frac{b_2}{a_2} = \frac{b_2' \cdot e^{j\beta la}}{a_2' \, e^{-j\beta la}} = \underline{S}_{22}' \cdot e^{j2\beta la}$$

b)

$\underline{S}_{11} = \underline{S}_{11}' = 0{,}72 \measuredangle -43°$ $\quad \lambda/2$ Transformation

$\underline{S}_{21} = 0$

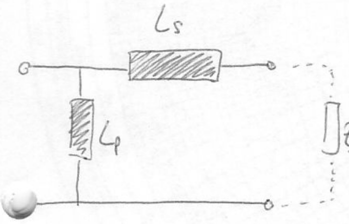
$\underline{S}_{22} = \underline{S}_{22}' + 90° = 0{,}55 \measuredangle -114°$ $\quad$ Drehung um $\lambda/4$ (+90°) (entgegen dem Uhrzeigersinn)

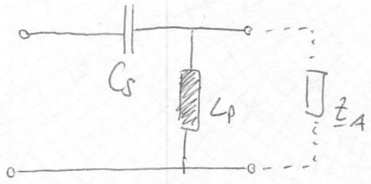
c)

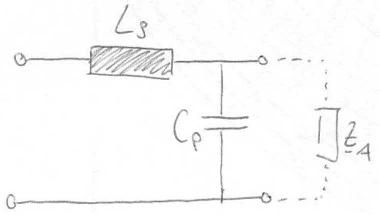
$\underline{Z}_{11} = 1{,}1 - j\,2{,}1$ $\qquad R_{gs} = 1{,}1 \qquad \omega C_{gs} = \frac{1}{2{,}1}$

$\underline{Y}_{22} = 0{,}83 + j\,1{,}18$ $\qquad R_{ds} = \frac{1}{0{,}83} \qquad \omega C_{ds} = 1{,}18$

d)





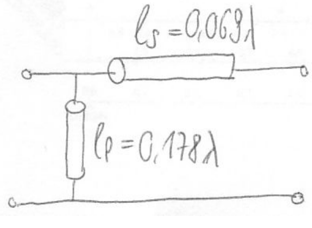


$\underline{Z}_{Ls} = j\,1{,}4$

$\underline{Y}_{Cp} = j\,0{,}26$

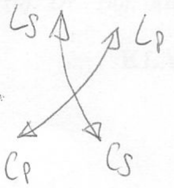
e)

ohne Eingangs-Anpassung $\qquad F = 1{,}73\ dB$



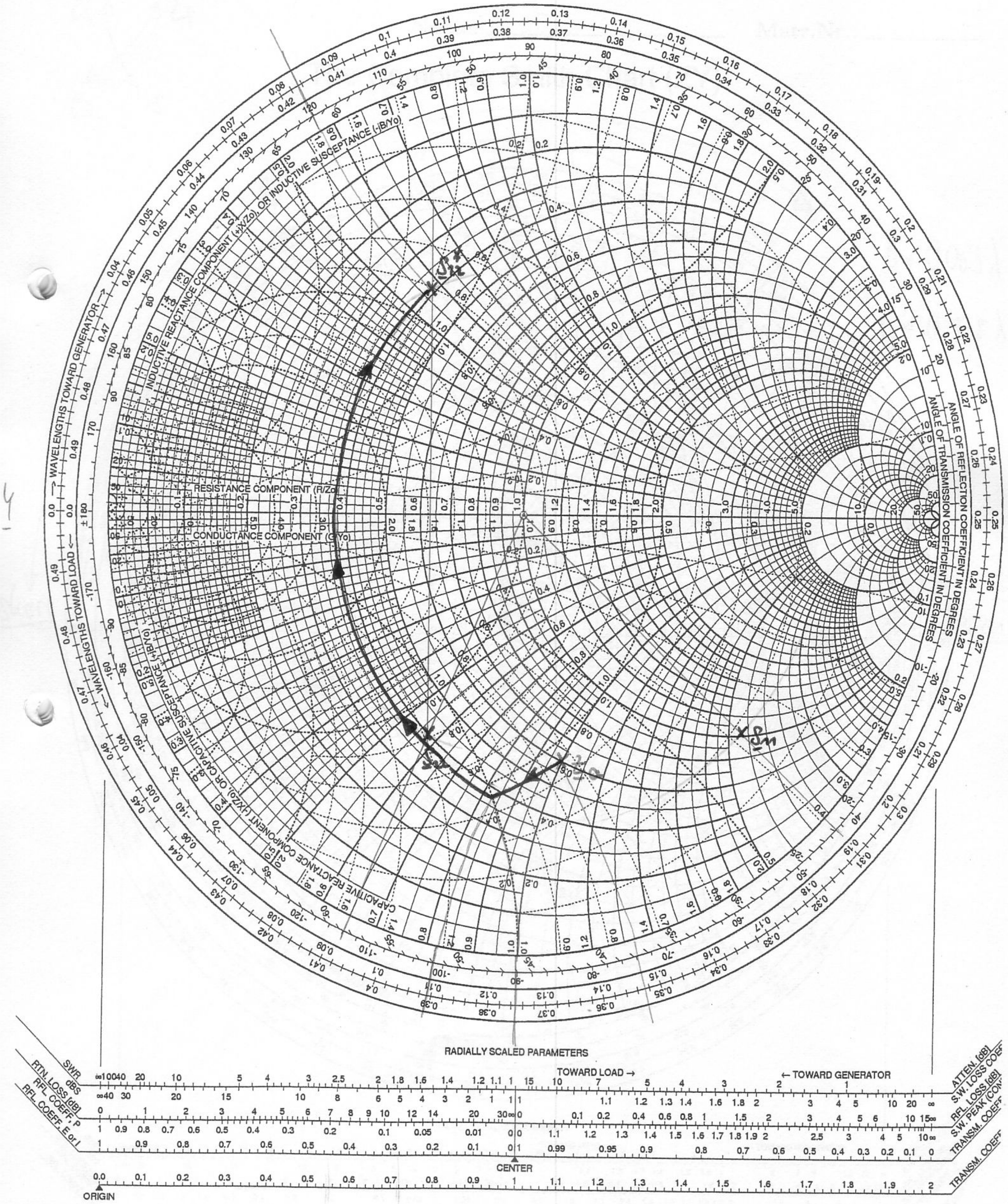
Abschluss der Stichleitung: short
(verhält sich wie parallele Induktivität)

LSG. H07 A:6



The Complete Smith Chart (ZY)

y



z

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LSG. H07

Herbst 2007

KLAUSURAUFGABEN "ELEKTROMAGNETISCHE FELDER 2 (EE)"

Hilfsblatt 3 zu Aufgabe 6, Unterpunkt e)

Name: ____________________ **Matr.Nr.:** ____________

Ls Lp Cp Cs

The Complete Smith Chart (ZY)

$l_S = 0{,}069\,\lambda$

$l_P = 0{,}178\,\lambda$