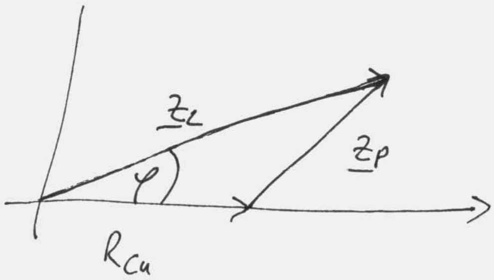
ÜG 8 reale/techn. Spule Seminar Claßen (1) 04.06.09

zu a) <u>Cosinussatz</u>

$$z_p^2 = z_L^2 + R_{Cu}^2 - 2\, z_L R_{Cu} \cdot \cos\varphi$$

$$\cos\varphi = \ldots = 0{,}61$$

Widerstandsdreieck



$$\varphi = \arccos 0{,}61 = 52{,}6^\circ$$

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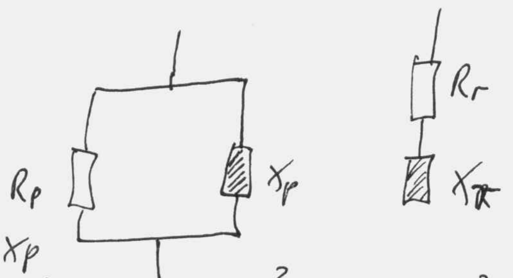
b) $\varphi + \delta = 90^\circ \rightarrow \delta = \ldots 37{,}4^\circ$

<u>Güte:</u> $Q = \frac{1}{\tan\delta} \approx \frac{1}{0{,}76} \approx 1{,}31$ (normalerweise Q ≙ Blindleistung)

<u>ÜG 9</u> es soll $\underline{z}_p = \underline{z}_r$

$\left.\begin{matrix} L_r \\ C_r \\ R_r \end{matrix}\right\}$ als Funktion von $\left\{\begin{matrix} L_p \\ C_p \\ R_p \end{matrix}\right\}$ als gegeben voraussetzen

a) b) zu Beginn gleiche Rechnung



$$\underline{z}_p = R_p \| jX_p = \frac{R_p \cdot jX_p}{R_p + jX_p} \cdot \frac{R_p - jX_p}{R_p - jX_p} = \frac{R_p X_p^2}{R_p^2 + X_p^2} + j\frac{R_p^2 X_p}{R_p^2 + X_p^2} \mathrel{\hat{=}} a + jb$$

$$\underline{z}_r = R_r + jX_r \mathrel{\hat{=}} c + jd$$

es soll $\underline{z}_p = \underline{z}_r$ d.h. $a + jb = c + jd \Leftrightarrow a = c \wedge b = d$

zwei kompl. Zahlen gleich ⇔ Real- und Imaginärteile gleich

$$\frac{R_p X_p^2}{X_p^2 + R_p^2} + j\frac{R_p^2 X_p}{R_p^2 + X_p^2} = R_r + jX_r \rightarrow R_r = \frac{R_p X_p^2}{R_p^2 + X_p^2} \quad X_r = \frac{R_p^2 X_p}{R_p^2 + X_p^2}$$

a) $X_p = \omega L_p \rightarrow R_r = \dfrac{R_p\,(\omega L_p)^2}{R_p^2 + (\omega L_p)^2}$

$$X_r = \omega L_r \qquad \cancel{\omega} L_r = \frac{R_p^2 \cdot \cancel{\omega} \cdot L_p}{R_p^2 + (\omega L_p)^2}$$

$L_r = \ldots$ s. Lsg.

b) $X_p = -\dfrac{1}{\omega C_p} \qquad R_r = \dfrac{R_p \cdot \left(-\dfrac{1}{\omega C_p}\right)^2}{R_p^2 + \left(-\dfrac{1}{\omega C_p}\right)^2}$

$X_r = -\dfrac{1}{\omega C_r}$

$$\Leftrightarrow \quad -\frac{1}{\omega C_r} = \frac{R_p^2 \left(-\dfrac{1}{\omega C_p}\right)^{\cancel{2}}}{R_p^2 + \left(-\dfrac{1}{\omega C_p}\right)^2}$$

$$\Leftrightarrow \quad C_r = \frac{1}{\omega} \, \frac{R_p^2 + \left(-\dfrac{1}{\omega C_p}\right)^2}{R_p^2 \cdot \left(\dfrac{1}{\omega C_p}\right)}$$

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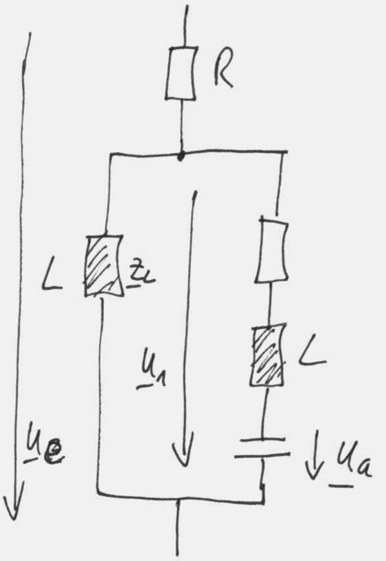
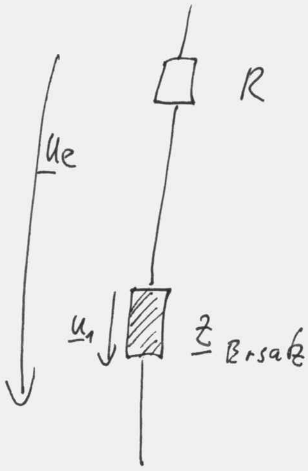
③



$$H(j\omega) = \frac{\underline{U}_a}{\underline{U}_e} = \underbrace{\frac{\underline{U}_a}{\underline{U}_1}} \cdot \underbrace{\frac{\underline{U}_1}{\underline{U}_e}}$$

je ein Spannungsteiler



mit $$\frac{\underline{U}_a}{\underline{U}_1} = \frac{\underline{Z}_C}{R + \underline{Z}_L + \underline{Z}_C}$$

$$\frac{\underline{U}_1}{\underline{U}_e} = \frac{\underline{Z}_L \,||\, (R + \underline{Z}_L + \underline{Z}_C)}{R + (\underline{Z}_L \,||\, (R + \underline{Z}_L + \underline{Z}_C)}$$

$$= \frac{\dfrac{\underline{Z}_L \cdot (R + \underline{Z}_L + \underline{Z}_C)}{\underline{Z}_L + R + \underline{Z}_L + \underline{Z}_C}}{R + \dfrac{\underline{Z}_L \cdot (R + \underline{Z}_L + \underline{Z}_C)}{\underline{Z}_L + R + \underline{Z}_L + \underline{Z}_C}}$$

$$H(j\omega) = \frac{\underline{Z}_C \cdot \underline{Z}_L}{R(R + \underline{Z}_C + 2\underline{Z}_L) + \underline{Z}_L(R + \underline{Z}_L + \underline{Z}_C)}$$

mit $\underline{Z}_C = jX_C$

$\underline{Z}_L = jX_L$

konjugiert komplex erweitern

$$H(j\omega) = \frac{-X_C X_L}{(-X_L^2 + R^2 - X_C X_L) + j(3RX_L + RX_C)} \cdot \frac{[(\dots) - j(\dots)]}{[(\dots) - j(\dots)]}$$

$$= \frac{-X_L X_C (R^2 - X_L X_C - X_L^2) + j X_L X_C (3RX_L + RX_C)}{\text{Nenner aus Muster Lsg.}}$$

b) $\underline{U}_a$, $\underline{U}_e$ - in Phase

- haben gleichen Phasenwinkel

$$\underline{H}(j\omega) = \frac{\underline{U}_a}{\underline{U}_e} = \frac{U_a\, e^{j\varphi_{u_a}}}{U_e\, e^{j\varphi_{u_e}}}$$ d.h. $\underline{H}(j\omega) = \frac{U_a}{U_e}$ also rein reell

mit $\varphi_{u_a} = \varphi_{u_e}$, da in Phase

also $\mathrm{Im}\{\underline{H}(j\omega)\} = 0$

also: $$\underbrace{X_L X_C}_{\neq 0} \underbrace{(3 R X_L + R X_C)}_{=0} \stackrel{!}{=} 0$$

$$3 R \omega L + R\left(-\frac{1}{\omega C}\right) = 0 \rightarrow L = \frac{1}{3} \frac{1}{\omega^2 C}$$

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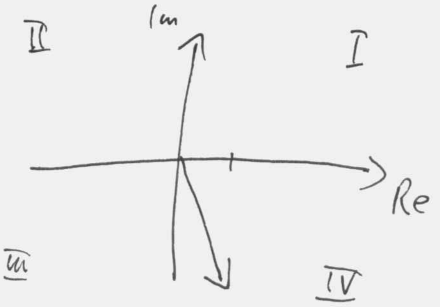
$$= R + j\left(\omega L - \frac{1}{\omega C}\right) \text{ mit } \omega = 2\pi f$$

$$= 50\Omega + j\Big(\underbrace{\underbrace{(84{,}55\Omega)}_{X_L} + \underbrace{(-630{,}82\Omega)}_{X_C}}\Big)$$

$$= 50\Omega + j(-546{,}27\Omega) = \underline{50\Omega - j\,546{,}27\Omega}$$

IV Quadrant



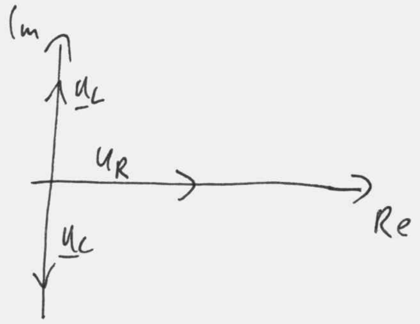
$$= 548{,}55\Omega\, e^{-84{,}77°}$$

$|\underline{Z}| = 548{,}55\Omega$

$\varphi = -84{,}77°$

(5)

$$I = \frac{U}{|Z|} = \frac{50\,V}{548{,}55\,\Omega} = 91{,}15\,mA$$



hiermit: $|\underline{U}_R|$, $|\underline{U}_L|$, $|\underline{U}_C|$

Vorgabe: $\underline{U}_R$ auf reeller Achse

$$U_R = I \cdot R = 91{,}15\,mA \cdot 50\,\Omega = 4{,}56\,V$$

$$|\underline{U}_L| = |\underline{I}| \cdot |\underline{Z}_L| = |\underline{I}| \cdot |jX_L| = I \underbrace{|j|}_{1} |X_L| = I \cdot X_L$$

$$= 91{,}15\,mA \cdot 84{,}55\,\Omega$$
$$= 7{,}71\,V$$

$$|\underline{U}_C| = I\,|X_C| = I \cdot \left|-\frac{1}{\omega C}\right|$$

$$= 91{,}15\,mA\,|-630{,}82\,\Omega| = 57{,}5\,V$$

Winkel:

$\underline{U}_R = U_R\, e^{j\varphi_{U_R}}$ " $\varphi_{U_R} = 0°$

$\underline{U}_L = U_L\, e^{j\varphi_{U_L}}$ mit $\varphi_{U_L} = \varphi_{U_R} + 90°$

$\underline{U}_C = U_C\, e^{j\varphi_{U_C}}$ " $\varphi_{U_C} = \varphi_{U_R} - 90°$

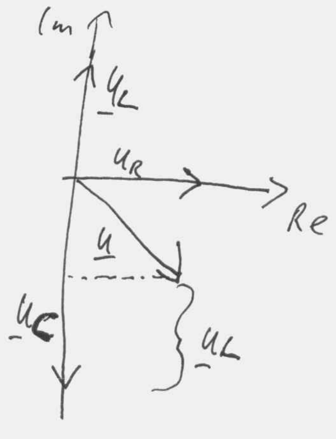
$\underline{I} = 91{,}15\,mA\, e^{j0°} = I$

$\underline{U}_R = 4{,}56\,V e^{j0°}$

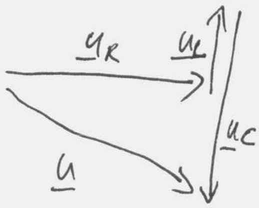
$\underline{U}_L = 7{,}71\,V e^{j90°}$

$\underline{U}_C = 57{,}5\,V e^{-j90°}$

$\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C$



s. auch Musterlsg im Netz

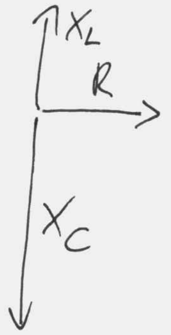


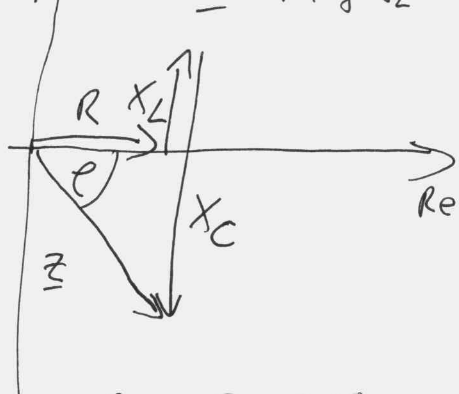
b) es war $X_L = 84{,}55\,\Omega$

$X_C = -630{,}82\,\Omega$

$|X_L| < |X_C|$ bzw. $X_L + X_C < 0$ → kapazitives Verhalten

$\underline{Z} = R + jX_L + jX_C$





$\varphi = -84{,}77°$

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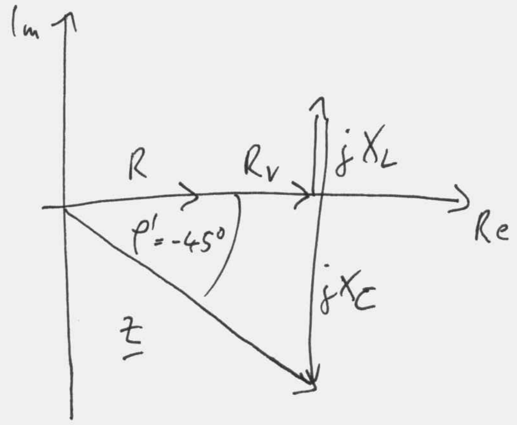
c) gefordert: φ von $-84{,}77°$ auf $-45°$



$jX_L + jX_C$

$j(X_L + X_C)$

$|X_L + X_C| = \left|\omega L + \left(-\frac{1}{\omega C}\right)\right|$

$\tan \varphi' = \dfrac{|X_L + X_C|}{R + R_V}$ mit $|\varphi'| = 45°$

d.h. $\tan |\varphi'| = 1$

$\Rightarrow R + R_V = |X_L + X_C|$

$R_V = |X_L + X_C| - R = |84{,}55\,\Omega - 630{,}82\,\Omega| - 50\,\Omega$

$= 496{,}27\,\Omega$

$\underline{Z} = R + R_V + j(X_L + X_C) = 772{,}54\,\Omega \cdot e^{-j45°}$

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d) Resonanzfrequenz:

$\underline{Z} = R + \cancel{(R_V)} + j(X_L + X_C)$

$|\underline{I}| = \frac{|U|}{|\underline{Z}|} = \frac{U}{R}$ da $Im\{\underline{Z}\} = 0$

$= \frac{50V}{50\Omega} = 1A$

bei Resonanz: $Im\{\underline{Z}\} = 0$

also $X_L = -X_C$ bzw. $|X_L| = |X_C|$ $\omega L = \frac{1}{\omega C}$

$\omega^2 = \frac{1}{LC}$ $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$

$\rightarrow f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$