KT GÜ6

4.3.1

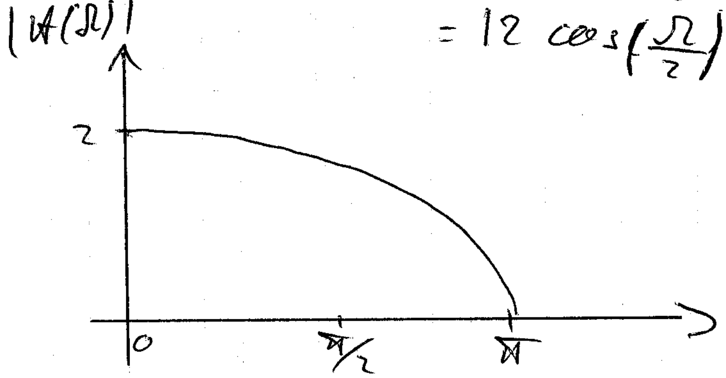
b.) $|H(\Omega)| = \sqrt{1 - 2a\cos(\Omega) + a^2}$

$a = -1 \Rightarrow \; = \sqrt{2 + 2\cos(\Omega)}$

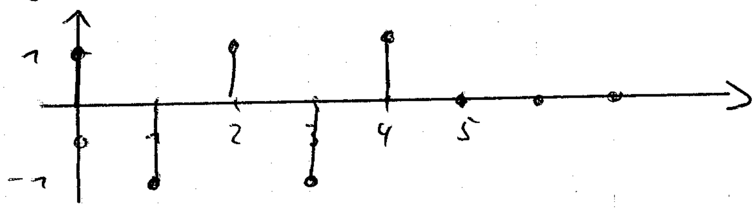
$= \sqrt{4\cos^2\left(\frac{\Omega}{2}\right)}$

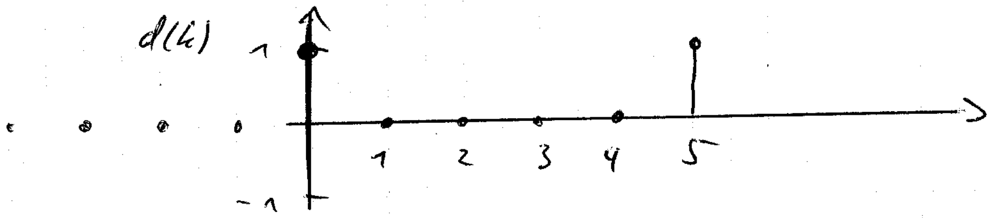
$= \left|2\cos\left(\frac{\Omega}{2}\right)\right|$

$1 + \cos(2x) = 2\cos^2(x)$

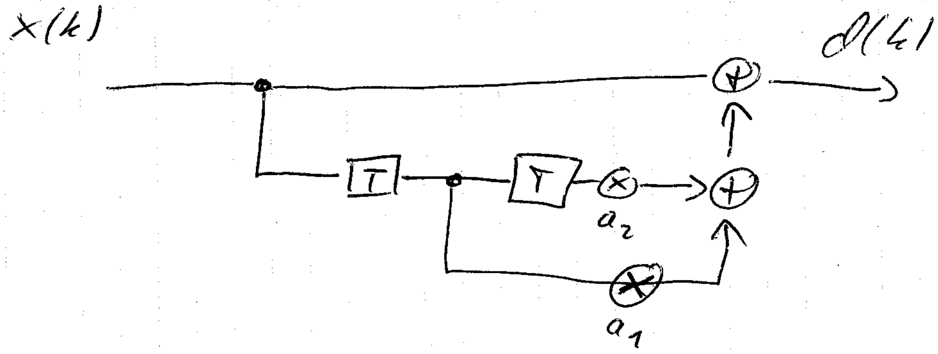


c.) $x_2(k)$





d.)



Normalengleichungen

$$\begin{pmatrix} \varphi_{x_1x_1}(1) \\ \varphi_{x_1x_1}(2) \end{pmatrix} = \begin{pmatrix} \varphi_{xx}(0) & \varphi_{xx}(1) \\ \varphi_{xx}(1) & \varphi_{xx}(0) \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Rightarrow \begin{array}{l} a_1 - a_2 = -1 \\ -a_1 + a_2 = 1 \end{array} \quad \Rightarrow \quad a_2 = a_1 + 1$$

a_1 <u>beliebig</u>

z. B. $a_1 = -1$
$a_2 = 0$

$a_1 = 0$
$a_2 = 1$

$$A(\Omega) = 1 - a_1 e^{-j\Omega} - a_2 e^{-j2\Omega}$$
$$= 1 - a_1 e^{-j\Omega} - e^{-j2\Omega} - a_1 e^{-j2\Omega}$$

$$\Rightarrow |A(\Omega=0)| = |1 - a_1 - 1 - a_1| = 2|a_1|$$
$$|A(\Omega=\pi)| = |1 + a_1 - 1 - a_1| = 0$$

<u>e.)</u>

$$\begin{pmatrix} \varphi_{x_2x_2}(1) \\ \varphi_{x_2x_2}(2) \end{pmatrix} = \begin{pmatrix} \varphi_{x_2x_2}(0) & \varphi_{x_2x_2}(-1) \\ \varphi_{x_2x_2}(1) & \varphi_{x_2x_2}(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$\Rightarrow a_1 + a_2 = 1$

aus d.) $a_2 = a_1 + 1 \quad \Rightarrow \quad a_1 = 0$
$a_2 = 1$

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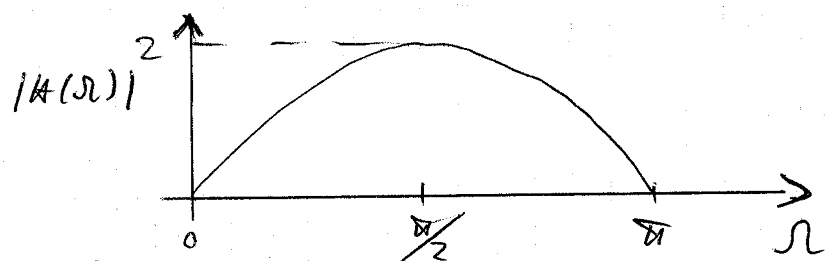
6.) $H(\Omega) = 1 - e^{-2j\Omega}$

$= e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$

$\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$

$\Rightarrow |H(\Omega)| = |2 \cdot \sin(\Omega)|$



4.4.)

a.) Gesucht a_{opt}

$$E\{d^2(k)\} = E\{(x(k) - a \cdot x(k-1))^2\}$$
$$= \varphi_{xx}(0) - 2a\,\varphi_{xx}(1) + a^2 \varphi_{xx}(0)$$

$$\frac{d\,E\{d^2(k)\}}{da} = 0 - 2\varphi_{xx}(1) + 2a\,\varphi_{xx}(0) \stackrel{!}{=} 0$$

$$\Rightarrow a_{opt} = \frac{\varphi_{xx}(1)}{\varphi_{xx}(0)}$$

$$\frac{d^2 E\{d^2(k)\}}{da^2} = 2\varphi_{xx}(0) > 0 \Rightarrow \text{Tiefpunkt}$$

$$\Rightarrow a_{opt} = \frac{2\,\mathrm{si}(\frac{\pi}{2} \cdot 1)}{2 \cdot \mathrm{si}(\frac{\pi}{2} \cdot 0)} = \frac{\sin(\pi/2)}{\pi/2} \cdot \frac{1}{1}$$
$$= \frac{2}{\pi} = \underline{\underline{0{,}637}}$$

b.) Gesucht: Prädiktionsgewinn

$$G_p = \frac{E\{x^2\}}{E\{d^2\}}$$

$$G_p = \frac{\varphi_{xx}(0)}{\varphi_{xx}(0) - 2a\,\varphi_{xx}(1) + a^2\,\varphi_{xx}(0)}$$

<u>hier</u> $a = a_{opt}$

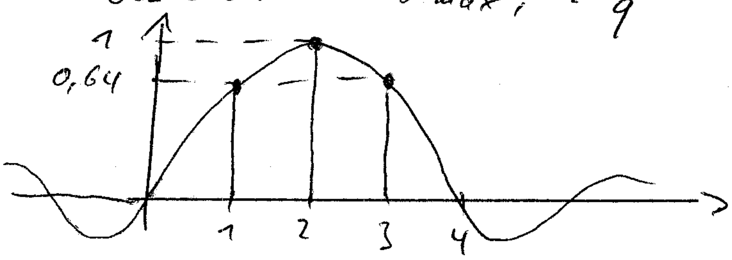
$$\Rightarrow G_p = \frac{1}{1 - 2\,\frac{\varphi_{xx}(1)}{\varphi_{xx}(0)} \cdot \frac{\varphi_{xx}(1)}{\varphi_{xx}(0)} + \frac{\varphi_{xx}^2(1)}{\varphi_{xx}^2(0)}}$$

$$= \frac{1}{1 - a_{opt}^2}$$

$$G_{p\,dB} = 10 \log_{10}\left(\frac{1}{1-a_{opt}^2}\right) = \underline{\underline{2{,}26\,dB}}$$

<u>c)</u>

Gesucht: d_{max}, N_q



$d(1) = x(1) - a_{opt} \cdot x(0)$
$= 0{,}64$

$d(2) = x(2) - a_{opt}(x(1))$
$= 0{,}59$

$d(3) = x(3) - a_{opt} \cdot x(2)$
$= 0$

$d_{max} = 0{,}64$

5 bit Quantisierung
$\Rightarrow w = 5$

$$\Rightarrow \Delta d = \frac{2\,d_{max}}{2^w} = \frac{0{,}64 \cdot 2}{2^5} = 0{,}04$$

$$N_q = \frac{\Delta d^2}{12} = 1{,}33 \cdot 10^{-4}$$

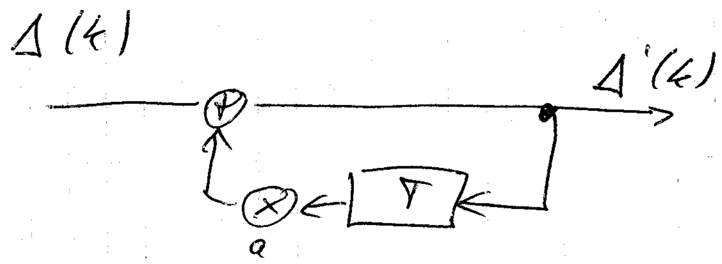
d.) Gesucht: $N_{out} = \varphi_{zz}(0)$

Wiener-Lee-Beziehung

$$\varphi_{zz}(0) = \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} S_{\Delta\Delta}(\Omega) \cdot |G(\Omega)|^2 \, d\Omega$$

$\Delta(k)$: gleichverteilt, weiß

$\Rightarrow S_{\Delta\Delta}(\Omega) = N$ f. alle Ω



$$\Delta'(k) = \Delta(k) + a\,\Delta'(k-1)$$

$$\Delta'(z) = \Delta(z) + a \cdot \Delta'(z) \cdot z^{-1}$$

$$G(z) = \frac{\Delta'(z)}{\Delta(z)} = \frac{1}{1 - a z^{-1}}$$

$$\Rightarrow |G(\Omega)| = \frac{1}{\sqrt{(1 + a\cos(\Omega))^2 + a^2 \sin^2(\Omega)}}$$

$$= \frac{1}{\sqrt{1 - 2a\cos(\Omega) + a^2}}$$

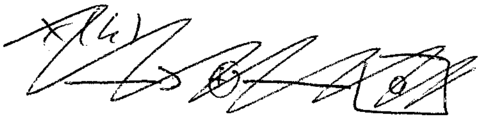
$$\Rightarrow \varphi_{zz}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} N \cdot \frac{1}{1 + a^2 - 2a\cos(\Omega)}$$

mit $b = (1+a^2)$ und $c = 2a$

$$\Rightarrow \varphi_{zz}(0) = \frac{N}{\sqrt{(1+a^2)^2 - 4a^2}} = \frac{N}{\sqrt{1-2a^2+a^4}}$$

$$= \frac{N}{1-a^2} = \underline{\underline{2{,}22 \cdot 10^{-4}}}$$

<u>e.)</u> Rückwärts prediktion



$\Rightarrow$ Quantisierungsrauschen am Empfänger identisch mit Q-Rauschen am Sender

$\Rightarrow \varphi_{zz}(0) = N_q = 1{,}33 \cdot 10^{-4}$