TI1 GÜ 9

21. JAN. 2011

A1.)
Eingang: weißes Rauschen $\{u(t)\}$

AKF: $R_{uu}(t) = 2\,\delta(t)$

Leistungsdichtespektrum $S_u(f) = \int_{-\infty}^{\infty} R_{uu}(t) \cdot e^{-i2\pi f t}\,dt$

$= \int_{-\infty}^{\infty} 2 \cdot \delta(t)\, e^{-i2\pi f t}\,dt \underline{= 2}$

Nach Theorem 2.7.13 gilt

$S_{yy}(f) = |H(f)|^2 \cdot S_u(f)$

$= \left| \frac{1}{2B} \Pi_{[-B,B]}(f) \right|^2 \cdot 2$

$= \frac{1}{4B^2} \Pi_{[-B,B]}(f) \cdot 2$

AKF: $R_{yy}(t) = \int_{-\infty}^{\infty} S_{yy}(f) \cdot e^{i2\pi f t}\,df$

$= \int_{-\infty}^{\infty} \frac{1}{2B^2} \Pi_{[-B,B]}(f)\, e^{i2\pi f t}\,df$

$= \frac{1}{2B^2} \int_{-B}^{B} e^{i2\pi f t}\,df = \frac{1}{2B^2} \frac{1}{i2\pi t} \left[e^{i2\pi f t} \right]_{-B}^{B}$

$= \frac{1}{2B^2} \cdot \frac{1}{i2\pi t} \left(e^{i2\pi B t} - e^{-i2\pi B t} \right)$

$= \frac{1}{2B^2} \frac{1}{\cancel{i}2\pi t} \cdot 2\cancel{i} \sin(2\pi B t)$

$= \frac{1}{B} \frac{\sin(2\pi B t)}{2B\pi t} = \frac{1}{B} si(2\pi B t)$

Erwartete Momentanleistung (Prop. 2.7.12.a)

$$E(|Y(t)|^2) = R_{YY}(0) = \frac{1}{B} \underbrace{si(0)}_{1} = \underline{\underline{\frac{1}{B}}}$$

<u>Aufgabe 2)</u>

<u>a.)</u> $$S_{WW}(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-i2\pi ft}\, dt = 1$$

(wie bei Aufg. 1)

$$S_{NN}(f) = |H(f)|^2 \underbrace{S_{WW}(f)}_{1}$$

$$= \left| \frac{1}{1+i\sqrt{2}\,f - f^2} \right|^2$$

$$= \left| \frac{1-f^2 - i\sqrt{2}\,f}{(1-f)^2 + 2f^2} \right|^2$$

$$= \frac{(1-f^2)^2 + 2f^2}{((1-f^2)^2 + 2f^2)^2}$$

$$= \frac{1}{(1-f^2)^2 + 2f^2} = \frac{1}{1+f^4}$$

$z = a+ib$

$|z| = \sqrt{a^2+b^2}$

$|z| = \sqrt{z \cdot \bar{z}}$

<u>b.)</u> $$E(|N(t)|^2) = R_{NN}(0)$$

$$= \int_{-\infty}^{\infty} S_{NN}(f) \cdot e^{i2\pi f0}\, df$$

$$= \int_{-\infty}^{\infty} S_{NN}(f)\, df = \int_{-\infty}^{\infty} \frac{1}{1+f^4}\, df = \underline{\underline{\frac{\pi}{\sqrt{2}}}}$$

<u>Aufg. 3.)</u>

$$h(t) = \begin{cases} e^{-\alpha t} & t \geq 0 \\ 0 & \text{sonst} \end{cases} \qquad \alpha > 0$$

$X(t)$ schwach stationär

T1.1 Güs

Nach Theorem 2.7.9 gilt:

$$R_{yy}(t) = h(t) * h^*(-t) * R_{xx}(t)$$

$$h(t) * h^*(-t) = \int_{-\infty}^{\infty} h(u) \cdot h^*(u-t)\, du$$

$$= \int_{-\infty}^{\infty} e^{-\alpha u}\, \mathbb{1}_{[0,\infty]}(u) \cdot e^{-\alpha(u-t)}\, \mathbb{1}_{[0,\infty)}(u-t)\, du$$

$$= e^{\alpha t} \int_{-\infty}^{\infty} e^{-2\alpha u}\, \underbrace{\mathbb{1}_{[0,\infty)}(u) \cdot \mathbb{1}_{[0,\infty)}(u-t)}_{=1 \text{ falls } u \geq \max\{0,t\}}\, du$$

$$= e^{\alpha t} \int_{\max\{0,t\}}^{\infty} e^{-2\alpha u}\, du$$

$$= e^{\alpha t} \left[-\frac{1}{2\alpha} e^{-2\alpha u} \right]_{\max\{0,t\}}^{\infty}$$

$$= \begin{cases} e^{\alpha t} \dfrac{e^{-2\alpha t}}{2\alpha} = \dfrac{1}{2\alpha} e^{-\alpha t}, & t \geq 0 \\[2ex] e^{\alpha t} \dfrac{1}{2\alpha} = \dfrac{1}{2\alpha} e^{\alpha t} & t < 0 \end{cases}$$

$$\Rightarrow h(t) * u^*(-t) = \frac{1}{2\alpha} e^{-\alpha|t|}$$

$$\Rightarrow R_{yy}(t) = \frac{1}{2\alpha} e^{-\alpha|t|} * R_{xx}(t)$$

$$S_{yy}(f) = |H(f)|^2 \cdot S_{xx}(f)$$

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-i2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-\alpha t} e^{-i2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-(2\pi i f + \alpha)t}\, dt$$

$$= \frac{1}{2\pi i f + \alpha}$$

gesucht: $$|H(f)|^2 = \left|\frac{1}{2\pi i f + \alpha}\right|^2 = \left|\frac{\alpha - i2\pi f}{\alpha^2 + 4\pi^2 f^2}\right|^2$$

$$= \frac{\alpha - i2\pi f}{\alpha^2 + 4\pi^2 f^2} \cdot \frac{\alpha + i2\pi f}{\alpha^2 + 4\pi^2 f^2}$$

$$= \frac{\alpha^2 \cancel{- i2\pi \alpha} + \cancel{i2\pi f \alpha} + 4\pi^2 f^2}{(\alpha^2 + 4\pi^2 f^2)^2}$$

$$= \frac{1}{\alpha^2 + 4\pi^2 f^2}$$

$$\Rightarrow S_{yy}(f) = \frac{1}{\alpha^2 + 4\pi^2 f^2} \cdot S_{xx}(f)$$