26. NOV. 2010

TI GÜ 6

A1.)

a.) $$f_{|h_k|}(|h_k|) = \frac{2|h_k|}{\sigma_h^2} \cdot \exp\left(-\frac{|h_k|^2}{\sigma_h^2}\right) \cdot \mathbb{I}_{[0,\infty)}(|h_k|)$$

ges.:

$$f_{|h_k|^2}(|h_k|^2) \qquad x = |h_k|$$

$$|h_k|^2 = T(|h_k|) \qquad T(x) = x^2 = y$$

$$x = T^{-1}(y) = \sqrt{y}$$

$$\frac{\partial T(x)}{\partial x} = 2x$$

$$f_{|h_k|^2}(y) = \frac{1}{\left|\frac{\partial T}{\partial x}\Big|_{x=T^{-1}(y)}\right|} f_{|h_k|}(T^{-1}(y))$$

$$= \frac{1}{\left|2x|_{x=\sqrt{y}}\right|} \cdot f_{|h_k|}(\sqrt{y})$$

$$= \frac{1}{2\sqrt{y}} \cdot \frac{2\sqrt{y}}{\sigma_h^2} \cdot \exp\left(-\frac{(\sqrt{y})^2}{\sigma_h^2}\right) \mathbb{I}_{[0,\infty)}(\sqrt{y})$$

- Exponentialverteilung mit Parameter $\frac{1}{\sigma_h^2}$

b.) $$SNR = \frac{E[|h_k x_k|^2]}{E[|n_k|^2]}$$

$$E[|n_k|^2] = E[(n_{R_k} + i n_{ik}) \cdot (n_{R_k} - i n_{ik})]$$

$$= E[n_{R_k}^2 - \cancel{i n_{R_k} n_{ik}} + \cancel{i n_{R_k} n_{ik}} + n_{ik}^2]$$

$$= E[n_{R_k}^2] + E[n_{ik}^2]$$

$$f_{n_{Rk}}(z) = f_{n_{Ik}}(z) = \frac{1}{\sqrt{\pi \sigma_n^2}} \cdot \exp\left(-\frac{z^2}{\sigma_n^2}\right)$$

$$\left(= \frac{1}{\sqrt{2\pi \sigma^2}} \cdot \exp\left(-\frac{z^2}{2\sigma^2}\right) \right)$$

$$\sim \mathcal{N}\left(0, \frac{\sigma_n^2}{2}\right)$$

$$\Rightarrow E[|n_k|^2] = \frac{\sigma_n^2}{2} + \frac{\sigma_n^2}{2} = \sigma_n^2$$

$$E[|h_k \cdot x_k|^2] = E[|h_k|^2 \cdot |x_k|^2]$$

$$= E[|h_k|^2] \cdot E[|x_k|^2]$$

stochastische Unabhängigkeit von h_k und x_k

~~Analogie der Verteilung von $|h_k|^2$ und $|n_k|^2$~~

Analogie der Verteilung von $|h_k|^2$ und $|n_k|^2$.

$$E[|h_k|^2] = \sigma_h^2$$

$$\Rightarrow E[|h_k \cdot x_k|^2] = \sigma_h^2 \cdot \underbrace{E[|x_k|^2]}_{= \sigma_x^2}$$

$$= \sigma_h^2 \cdot \sigma_x^2$$

$= \sigma_x^2$ da die mittlere Leistung von x_k σ_x^2 ist

$$SNR = \frac{\sigma_h^2 \cdot \sigma_x^2}{\sigma_n^2}$$

c.) weitere Annahme $|x_k|^2 = \sigma_x^2$
Sendesymbole konstante Leistung

gilt: $$\frac{E[|h_k \cdot x_k|^2]}{E[|n_k|^2]} = E\left[\frac{|h_k x_k|^2}{|n_k|^2}\right]$$

$$E\left[\frac{|h_k x_k|^2}{|n_k|^2}\right] = E[|h_k \cdot x_k|^2] \cdot E\left[\frac{1}{|n_k|^2}\right]$$
$$= \sigma_h^2 \cdot \sigma_x^2 \cdot E\left[\frac{1}{|n_k|^2}\right]$$

Analogie $|n_k|^2$ und $|h_k|^2$

$$f_{|n_k|^2}(z) = \frac{1}{\sigma_n^2} \exp\left(-\frac{z}{\sigma_n^2}\right) \mathbb{I}_{[0,\infty)}(z)$$

$$E\left[\frac{1}{|n_k|^2}\right] = \int_{-\infty}^{\infty} \frac{1}{z} \cdot f_{|n_k|^2}(z)\, dz$$
$$= \int_0^{\infty} \frac{1}{z} \cdot \frac{1}{\sigma_n^2} \exp\left(-\frac{z}{\sigma_n^2}\right) dz$$

$$y = \frac{z}{\sigma_n^2} \qquad z = y \cdot \sigma_n^2$$
$$\frac{dy}{dz} = \frac{1}{\sigma_n^2} \qquad dz = \sigma_n^2\, dy$$

$$= \int_0^{\infty} \frac{1}{y\sigma_n^2} \cdot \frac{1}{\sigma_n^2} \cdot \exp(-y)\, \sigma_n^2\, dy$$
$$= \frac{1}{\sigma_n^2} \int_0^{\infty} \frac{1}{y} \cdot \exp(-y)\, dy$$

$$= \frac{1}{\sigma_n^2} \lim_{x \to 0} \int_x^\infty \frac{1}{y} \exp(-y)\, dy$$

$$= -\frac{1}{\sigma_n^2} \cdot \lim_{x \to 0} Ei(-x)$$

Def.: $Ei(x) = -\int_{-t}^{\infty} \frac{e^{-t}}{t}\, dt$

$$\Rightarrow E\left[\frac{|h_k x_k|^2}{|n_k|^2}\right] = \infty$$

$$\neq \frac{E[|h_k x_k|^2]}{E[|n_k|^2]} = \frac{\sigma_h^2 \sigma_x^2}{\sigma_n^2}$$

d.)

$$h_k = h_{Rk} + i \cdot h_{ik}$$

$$E[h_{Rk} \cdot h_{Rk+1}] = E[h_{ik} \cdot h_{ik+1}] = c$$

$$|x_k| = \sigma_x$$

Eingangs~~verteile~~symbole x_k
die unabhängige Phasen haben

$$\underline{y} = \begin{pmatrix} y_{R1} \\ y_{i1} \\ y_{R2} \\ y_{i2} \end{pmatrix}$$

sind die Elemente von $\underline{y}$ gemeinsam normalverteilt?

$$y_k = x_k \cdot h_k + n_k = (x_{Rk} + i \cdot x_{ik}) \cdot (h_{Rk} + i h_{ik})$$

$$+ n_{Rk} + i n_{ik}$$

$$= x_{Rk} \cdot h_{Rk} + i \cdot x_{Rk} \cdot h_{ik} + i \cdot x_{ik} \cdot h_{Rk}$$

$$- x_{ik} h_{ik} + n_{Rk} + i n_{ik}$$

$$= (x_{Rk} \cdot h_{Rk} - x_{ik} \cdot h_{ik} + n_{Rk}) \qquad \leftarrow Re\{y_k\} = y_{Rk}$$

$$+ i \cdot (x_{Rk} h_{ik} + x_{ik} h_{Rk} + n_{ik}) \qquad \leftarrow Im\{y_k\} = y_{ik}$$