TI GÜ 6

A 1.)

a.) Fortsetzung

$$y_k = \underbrace{x_{Rk} \cdot h_{Rk} - x_{ik} \cdot h_{ik}}_{Re(y_k)} + i \underbrace{(x_{ik} \cdot h_{Rk} + x_{Rk} \cdot h_{ik} + n_{ik})}_{Im(y_k)}$$

$$\underline{y} = \begin{pmatrix} y_{R1} \\ y_{i1} \\ y_{R2} \\ y_{i2} \end{pmatrix} = \begin{pmatrix} x_{R1} h_{R1} - x_{i1} h_{i1} + n_{R1} \\ x_{i1} h_{R1} + x_{R1} h_{i1} + n_{i1} \\ x_{R2} h_{R2} + x_{i2} h_{i2} + n_{R2} \\ x_{i2} h_{R2} + x_{R2} h_{i2} + n_{i2} \end{pmatrix}$$

x_k und h_k sind stoch. unabh.

$$E[x_k] = E[h_k] = E[n_k] = 0$$

$$\Rightarrow E[\underline{y}_k] = 0$$

$$Cov[\underline{y}] = E[\underline{y}\,\underline{y}^T] = E\left[\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & \ddots & & \vdots \\ a_{31} & & \ddots & \vdots \\ a_{41} & - & - & a_{44} \end{pmatrix} \right]$$

$$E[a_{11}] = E\{(x_{R1} h_{R1} - x_{i1} h_{i1} + n_{R1})^2]$$

$$= E[\, x_{R1}^2 h_{R1}^2 - x_{R1} h_{R1} x_{i1} h_{i1} + x_{R1} h_{R1} n_{R1} - x_{i1} h_{i1} x_{R1} h_{R1} + x_{i1}^2 h_{i1}^2 - x_{i1} h_{i1} n_{R1} + n_{R1} x_{R1} h_{R1} - n_{R1} x_{i1} h_{R1} + n_{R1}^2 \,]$$

h_{Rk} s.u. von h_{ik}

$$= E[x_{R1}^2 h_{R1}^2 + x_{i1}^2 h_{i1}^2 + n_{R1}^2]$$

$$= E[x_{R1}^2]\underbrace{E[h_{R1}^2]}_{\frac{\sigma_h^2}{2}} + E[x_{i1}^2]\underbrace{E[h_{i1}^2]}_{\frac{\sigma_h^2}{2}} + \underbrace{E[n_{R1}^2]}_{\frac{\sigma_n^2}{2}}$$

$$h_{R1} \sim h_{i1} \sim \mathcal{N}(0, \frac{\sigma_h^2}{2})$$

$$= \{E[x_{R1}^2] + E[x_{i1}^2]\} \cdot \frac{\sigma_h^2}{2} + \frac{\sigma_n^2}{2}$$

$$= \{E[x_{R1}^2 + x_{i1}^2]\} \cdot \frac{\sigma_h^2}{2} + \frac{\sigma_n^2}{2}$$

$$= E[|x_1|^2] \cdot \frac{\sigma_h^2}{2} + \frac{\sigma_n^2}{2}$$

$$= \sigma_x^2 \cdot \frac{\sigma_h^2}{2} + \frac{\sigma_n^2}{2}$$

$$E[a_{12}] = E[(x_{R1}h_{R1} - h_{i1}x_{i1} + n_{R1}) \cdot (x_{i1}h_{R1} + x_{R1}h_{i1} + n_{i1})]$$

$$= E[x_{R1}x_{i1} \cdot h_{R1}^2 - x_{i1}x_{R1}h_{i1}^2]$$

$$= \frac{\sigma_h^2}{2} \cdot E[x_{R1}x_{i1} - x_{i1}x_{R1}]$$

$$= 0$$

$$E[a_{13}] = E[(x_{R1}h_{R1} - x_{i1}h_{i1} + n_{R1})(x_{R2}h_{R2} - x_{i2}h_{i2} + n_{R2})]$$

$$= E[x_{R1}h_{R1} \cdot x_{R2}h_{R2} + x_{i1}h_{i1} \cdot x_{i2}h_{i2}]$$

$$E[x_{R1} \cdot x_{R2}] = 0$$

$$\underline{\underline{= 0}}$$

TI GÜ6

$$E[a_{14}] = E[(x_{R1} h_{R1} - x_{i1} h_{i1} + n_{R1}) \cdot (x_{i2} h_{R2} + x_{R2} h_{i2} + n_{i2})]$$

$$= 0$$

$$E[a_{21}] = E[a_{12}]$$

$$E[a_{22}] = E[(x_{i1} h_{R1} + x_{R1} h_{i1} + n_{i2}) \cdot (x_{i1} h_{R1} + x_{R1} \cdot h_{i1} + n_{i1})]$$

$$= E[x_{i1}^2 h_{R1}^2 + x_{R1}^2 h_{i1}^2 + n_{i1}^2]$$

$$= E[x_{i1}^2] \underbrace{E[h_{R1}^2]}_{\frac{\sigma_h^2}{2}} + E[x_{R1}^2] \cdot \underbrace{E[h_{i1}^2]}_{\frac{\sigma_h^2}{2}} + \underbrace{E[n_{i1}^2]}_{\frac{\sigma_n^2}{2}}$$

$$= \frac{\sigma_h^2}{2} E[x_{i1}^2 + x_{R1}^2] + \frac{\sigma_n^2}{2}$$

$$= \frac{\sigma_h^2}{2} E[|x_1|^2] + \frac{\sigma_n^2}{2}$$

$$= \frac{\sigma_h^2}{2} \sigma_x^2 + \frac{\sigma_n^2}{2}$$

$$E[a_{23}] = E[(x_{i1} h_{R1} + x_{R1} h_{i1} + n_{i1}) \cdot (x_{R2} h_{R2} - x_{i2} h_{i2} + n_{R2})]$$

$$\underline{\underline{= 0}}$$

$$E[a_{24}] = E[(x_{i1} h_{R1} + x_{R1} h_{i1} + n_{i1}) \cdot (x_{R2} h_{R2} + x_{R2} h_{i2} + n_{i2})]$$

$$\underline{\underline{= 0}}$$

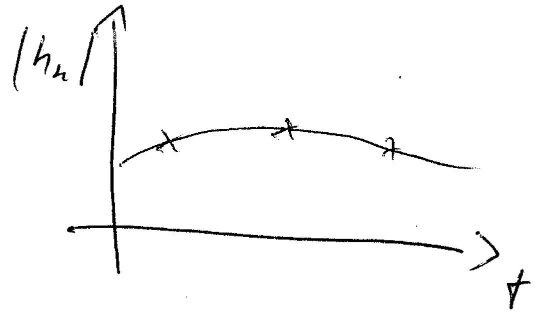
$$E[a_{31}] = E[a_{13}]$$
$$E[a_{32}] = E[a_{23}]$$
$$E[a_{33}] = E[a_{11}]$$
$$E[a_{34}] = E[(x_{R2} h_{R2} - x_{i2} h_{i2} + n_{R2}) \cdot (x_{i2} h_{R2} + x_{R2} h_{i2} + n_{i2})]$$
$$= E[x_{R2} x_{i2} h_{R1}^2 - x_{i2} x_{R2} h_{i2}^2]$$
$$= \frac{\sigma_h^2}{2} E[x_{R2} x_{i2} - x_{i2} x_{R2}]$$
$$\underline{= 0}$$

$$E[a_{41}] = E[a_{14}]$$
$$E[a_{42}] = E[a_{24}]$$
$$E[a_{43}] = E[a_{34}]$$
$$E[a_{44}] = E[a_{22}]$$

$$\mathrm{Cov}[\underline{y}] = \left(\frac{\sigma_x^2 \sigma_h^2}{2} + \frac{\sigma_n^2}{2}\right) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

wenn $\underline{y}$ gemeinsam normalverteilt wäre, dann würde eine diagonale Kovarianzmatrix implizieren, dass y_1 und y_2 stoch. unabh. wären.

$$y_k = x_k \cdot h_k + n_k$$
$$|x_k| = \sigma_x$$



TI GÜ 6 03. DEZ. 2010

Annahme: $\sigma_n^2 = 0$

$$E[h_{Rk} \cdot h_{Rk+1}] = E[h_{ik}\, h_{ik+1}] = c = \frac{\sigma_h^2}{2}$$

=> Kanal sich zwischen Zeitpunkt 1 und 2 nicht ändert.

für diesen Spezialfall $\operatorname{cov}[|y_1|^2 |y_2|^2]$

$$E[|y_k|^2] = E[|x_k \cdot h_k|^2] = E[|x_k|^2] \cdot E[|h_k|^2] = \sigma_x^2 \cdot \sigma_h^2$$

$$\operatorname{cov}[|y_1|^2 |y_2|^2] = E[|y_1|^2 |y_2|^2] - E[|y_1|^2] \cdot E[|y_2|^2]$$

$$= E[|x_1|^2 |h_1|^2 |x_2|^2 |h_2|^2] - \sigma_x^4 \sigma_h^4$$

$$= \sigma_x^4 \cdot E[|h_1|^2 |h_2|^2] - \sigma_x^4 \sigma_h^4$$

$$= \sigma_x^4 \cdot E[(h_{R1}^2 + h_{i1}^2)(h_{R2}^2 + h_{i2}^2)] - \sigma_x^4 \sigma_h^4$$

$$= \sigma_x^4\, E[h_{R1}^2 h_{R2}^2 + h_{R1}^2 h_{i2}^2 + h_{i1}^2 h_{R2}^2 + h_{i1}^2 h_{i2}^2] - \sigma_x^4 \sigma_h^4$$

$$= \sigma_x^4 \cdot E[h_{R1}^4 + h_{R1}^2 h_{i1}^2 + h_{i1}^2 h_{R1}^2 + h_{i1}^4] - \sigma_x^4 \sigma_h^4$$

$$= \sigma_x^4 \left[3 \cdot \left(\frac{\sigma_h^2}{2}\right)^2 + \frac{\sigma_h^2}{2} \frac{\sigma_h^2}{2} + \frac{\sigma_h^2}{2} \frac{\sigma_h^2}{2} + 3\left(\frac{\sigma_h^2}{2}\right)^2\right] - \sigma_x^4 \sigma_h^4$$

$$= \sigma_x^4 \cdot \sigma_h^4 \neq 0$$

=> $|y_1|^2$ und $|y_2|^2$ im Allg. nicht unabh.

$\Rightarrow$ $\underline{y}$ kann nicht gemeinsam normalverteilt sein.