Übung 6

Donnerstag, 25. November 2010
10:01

Aufg. 6.3

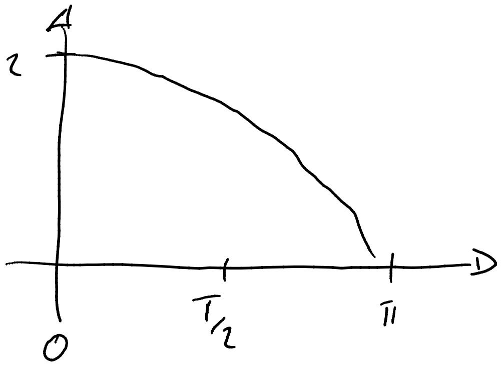
b) $|H(\Omega)| = \sqrt{1 - 2a \cdot \cos(\Omega) + a^2}$

$\overset{a=1}{=} \sqrt{2 + 2 \cdot \cos(\Omega)}$

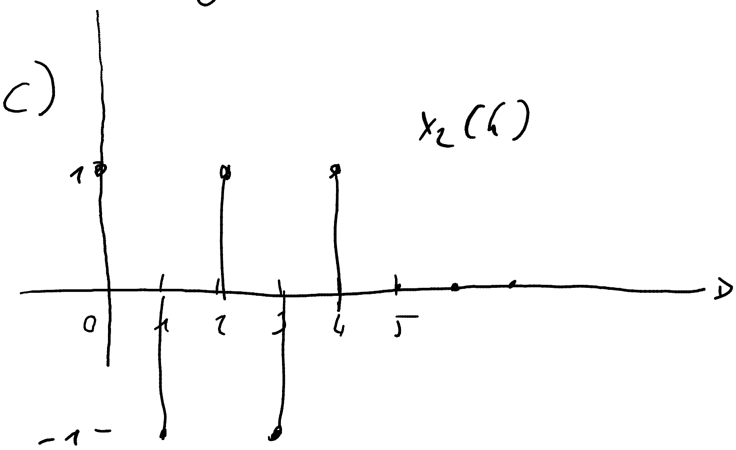
$= \sqrt{4 \cdot \cos^2\left(\frac{\Omega}{2}\right)}$

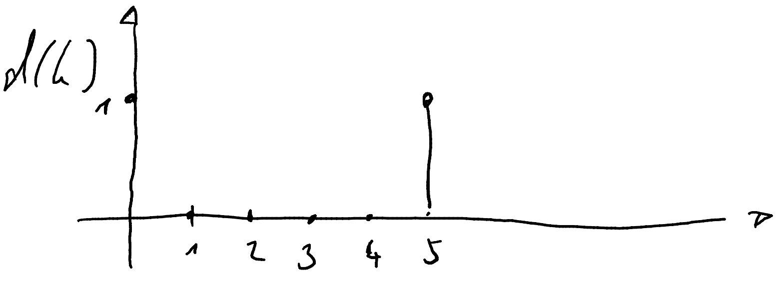
$= \left|2 \cdot \cos\left(\frac{\Omega}{2}\right)\right|$

[$1 + \cos(2\alpha) = 2 \cdot \cos^2(\alpha)$]



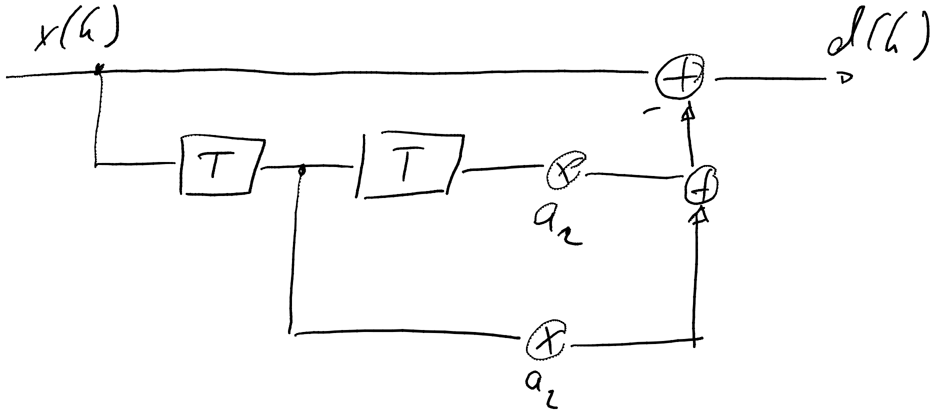
c)





d)

d)



Normalgleichungen

$$\begin{pmatrix} \varphi_{x_1x_1}(1) \\ \varphi_{x_1x_1}(2) \end{pmatrix} = \begin{pmatrix} \varphi_{xx}(0) & \varphi_{xx}(1) \\ \varphi_{xx}(1) & \varphi_{xx}(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{bmatrix} a_1 - a_2 = -1 \\ -a_1 + a_2 = 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_2 = a_1 + 1 \\ a_2 = a_1 + 1 \end{bmatrix} \Rightarrow a_1 \text{ bel.}$$

z.B. $a_1 = -1 \Rightarrow a_2 = 0$

$$H(\Omega) = 1 - a_1 \cdot e^{-j\Omega} - a_2 \cdot e^{-j2\Omega}$$

$$= 1 - a_1 e^{-j\Omega} - e^{-j2\Omega} - a_1 e^{-2j\Omega}$$

$$\Rightarrow |H(\Omega = 0)| = |1 - a_1 - 1 - a_1| = 2|a_1|$$

$$|H(\Omega=\pi)| = |1 + a_1 - 1 - a_1| = 0$$

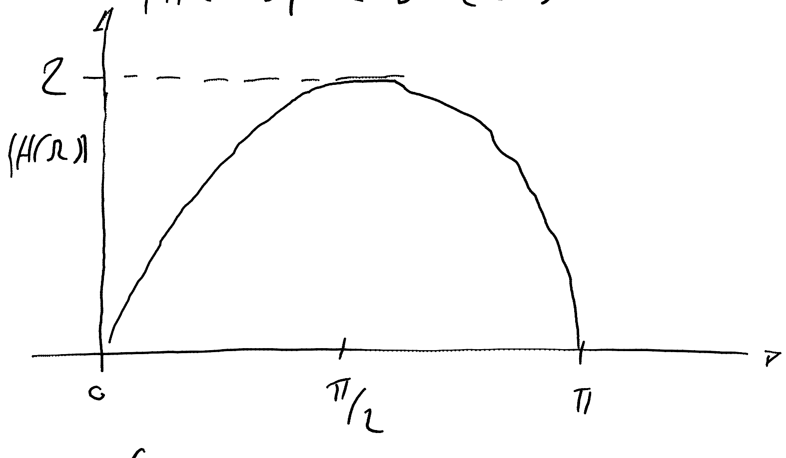
e) $$\begin{pmatrix}\varphi_{x_2x_2}(1)\\ \varphi_{x_2x_2}(2)\end{pmatrix} = \begin{pmatrix}\varphi_{x_2x_2}(0) & \varphi_{x_2x_2}(-1)\\ \varphi_{x_2x_2}(1) & \varphi_{x_2x_2}(0)\end{pmatrix}\begin{pmatrix}a_1\\ a_2\end{pmatrix}$$

$$\begin{pmatrix}1\\ 1\end{pmatrix} = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}a_1\\ a_2\end{pmatrix}$$

$$\left.\begin{array}{l} \Rightarrow a_1 + a_2 = 1 \\ (\rightarrow d)\quad a_2 = a_1 + 1 \end{array}\right\} \Rightarrow \begin{array}{l} a_1 = 0 \\ a_2 = 1 \end{array}$$

f) $$H(\Omega) = 1 - e^{-2j\Omega}$$
$$= e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$$
$$= e^{-j\Omega} \cdot 2 \cdot \sin(\Omega)$$

$$\Rightarrow |H(\Omega)| = 2 \cdot \sin(\Omega)$$



Aufg. 4.4

a) ges.: a_{opt}

$$E\{d^2(k)\} \longrightarrow \min$$

$$E\{d^2\} = E\left\{\left(x(k) - a \cdot x(k-1)\right)^2\right\}$$

$$= \varphi_{xx}(0) - 2a\,\varphi_{xx}(1) + a^2 \varphi_{xx}(0)$$

$$\frac{dE\{d^2\}}{da} \overset{!}{=} 0 - 2\,\varphi_{xx}(1) + 2a\,\varphi_{xx}(0) \overset{!}{=} 0$$

$$\Rightarrow a_{opt} = \frac{\varphi_{xx}(1)}{\varphi_{xx}(0)}$$

$$\frac{d^2 E(d^2)}{da} = 2\,\varphi_{xx}(0) > 0 \Rightarrow \text{Tiefpunkt}$$

$$\Rightarrow a_{opt} = \frac{2\,si\left(\frac{\pi}{2} \cdot 1\right)}{2 \cdot si\left(\frac{\pi}{2} \cdot 0\right)} = \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \frac{2}{\pi} \cdot 0{,}637$$

b) ges: Prädiktionsgewinn

$$G_p = \frac{\varphi_{xx}(0)}{\varphi_{xx}(0) - 2a\,\varphi_{xx}(1) + a^2\,\varphi_{xx}(0)}$$

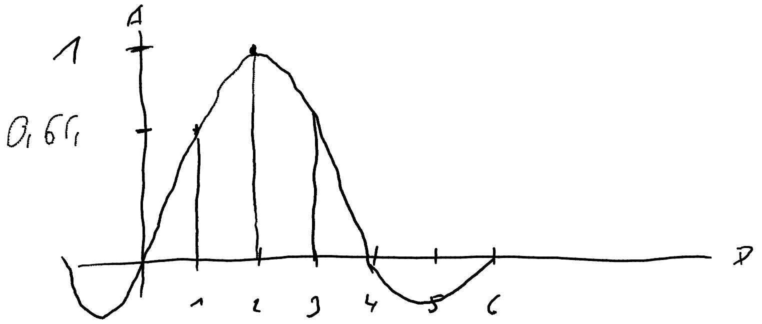
hier $a = a_{opt}$

$$\Rightarrow G_p = \frac{1}{1 - 2\,\frac{\varphi_{xx}(1)}{\varphi_{xx}(0)} \cdot \frac{\varphi_{xx}(1)}{\varphi_{xx}(0)} + \frac{\varphi_{xx}^2(1)}{\varphi_{xx}^2(0)} \cdot 1}$$

$$= \frac{1}{1 - a_{opt}^2}$$

$$\frac{G_p}{dB} = 10 \cdot \log\left(\frac{1}{1-a_{opt}^2}\right) = 2{,}26\ dB$$

c) ges: d_{max}, N_q



$$d(1) = x(1) - a_{opt}\, x(0) = 0{,}64$$

$$d(2) = x(2) - a_{opt}\, x(1) = 1 - 0{,}64 \cdot 0{,}64) = 0{,}59$$

$$d(3) = x(3) - a_{opt}\, x(2) = 0{,}64 - 0{,}64 \cdot 1 = 0$$

$$d_{max} = 0{,}64$$

$$w = 5\,bit \quad \Rightarrow \Delta d = \frac{2 d_{max}}{2^w} = \frac{0{,}64 \cdot 2}{2^5} = 0{,}04$$

$$\Rightarrow N_q = \frac{\Delta d^2}{12} = 1{,}33 \cdot 10^{-4}$$

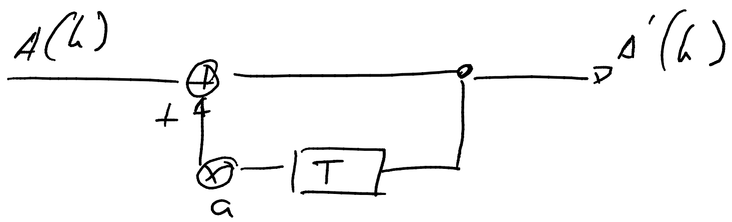
d) ges: $N_{out} = \varphi_{zz}(0)$

Wiener-Lee-Beziehung

$$\varphi_{zz}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\Delta\Delta}(\Omega) \cdot |G(\Omega)|^2\, d\Omega$$

$\Delta(k)$: gleichverteilt, weiß

$\Rightarrow S_{\Delta\Delta}(\Omega) = N$ für alle Ω



$$a'(k) = a(k) + a \cdot a'(k-1)$$

$$A'(z) = A(z) + a \cdot z^{-1} \cdot A(z)$$

$$\Rightarrow G(z) = \frac{A'(z)}{A(z)} = \frac{1}{1 - a \cdot z^{-1}}$$

$$\Rightarrow |G(\Omega)| = \frac{1}{\sqrt{(1 - a \cdot \cos(\Omega))^2 + a^2 \sin^2(\Omega)}}$$

$$= \frac{1}{\sqrt{1 - 2a\cos\Omega + a^2}}$$

$$\Rightarrow \varphi_{zz}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} N \cdot \frac{1}{1 + a^2 - 2a\cos\Omega}$$

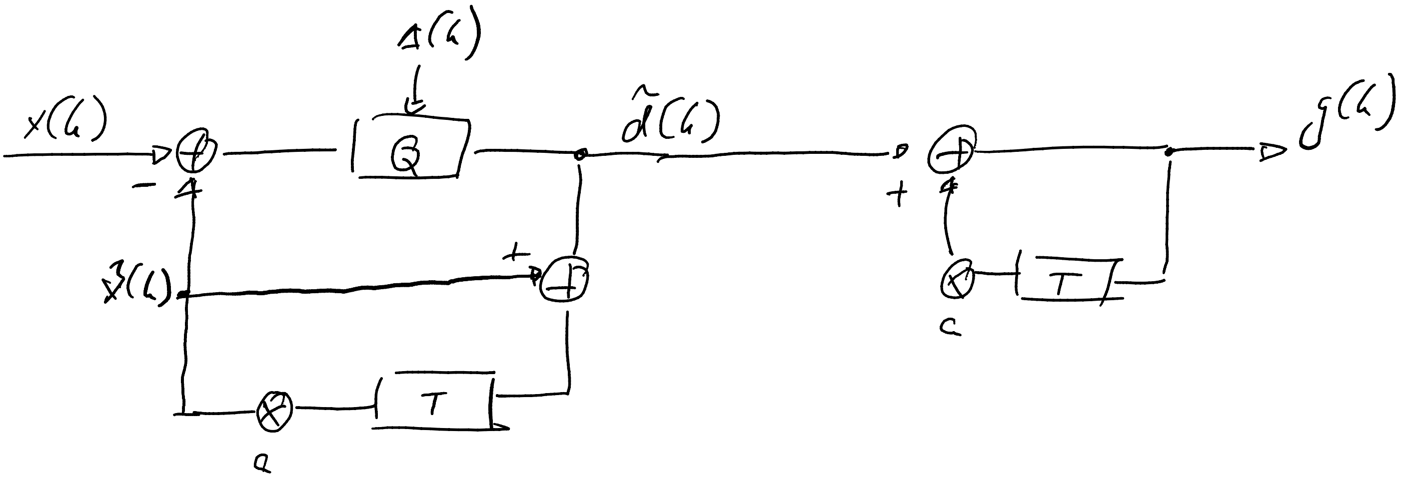
mit $b = (1+a^2)$ und $c = 2a$

$$\Rightarrow \varphi_{zz}(0) = \frac{N}{\sqrt{(1+a^2)^2 - 4a^2}} = \frac{N}{\sqrt{1 - 2a^2 + a^4}}$$

$$= \frac{N}{1 - a^2} = 2{,}22 \cdot 10^{-4}$$

e) ges: $\varphi_{tz}(0) = N_z$

geg: Rückwärtsprädiktion



Betrachtung von $\Delta(k)$: $x(k) = 0$ $\qquad y(k) = \Delta'(k)$

$$\tilde{D}(z) = \Delta(z) - \hat{X}(z)$$

$$\hat{X}(z) = \tilde{D}(z) + \hat{X}(z)\cdot a \cdot z^{-1} \Leftrightarrow \hat{X}(z) = \frac{a \cdot z^{-1}}{1 - a\cdot z^{-1}}\,\tilde{D}(z)$$

$$\Rightarrow \tilde{D}(z) = \Delta(z) - \frac{a\cdot z^{-1}}{1-a\cdot z^{-1}}\cdot \tilde{D}(z)$$

$$\Leftrightarrow \tilde{D}(z) = \Delta(z)\,(1 - a\cdot z^{-1})$$

Empfänger

$$\Delta'(z) = \tilde{D}(z) + \Delta'(z)\cdot a \cdot z^{-1}$$

$$\Rightarrow \Delta'(z) = \tilde{D}(z)\cdot\frac{1}{1-a\cdot z^{-1}} = \Delta(z)\,\frac{(1 \cancel{- a\cdot z^{-1}})}{(1 \cancel{- a\cdot z^{-1}})}$$

$1 - a \cdot z^{-1}$ $\quad (1 - \cancel{a \cdot z^{-1}})$

$\Rightarrow \boxed{A'(z) = A(1)}$

$\Rightarrow$ Quantisierungsrauschen am Empfänger ist identisch mit Rauschen im Prädiktor

$\Rightarrow \varphi_{zz}(0) = N_{Q_c)} = 1{,}33 \cdot 10^{-4}$