Übung 12

Dienstag, 25. Januar 2011
12:17

Klausur H2008 – Aufg. 6

a) ges: Kalman-Gleichung

MOD $\hat{x}_{k,k} = \hat{x}_{k,k-1} + \frac{\Sigma_{k,k-1} \cdot C(y_k - x_{k,k-1})}{c^2 \Sigma_{k,k-1} + \underbrace{R_k}_{\sigma_n^2}}$

$R_k = E\{n_k^2\}$

$Var\{n_k\} = E\{n_k^2\} - (\underbrace{E\{n_k\}}_{=0})^2 = \sigma_n^2$

$$\Sigma_{k,k} = \Sigma_{k,k-1} - \frac{\Sigma_{k,k-1}^2 \cdot C^2}{c^2 \cdot \Sigma_{k,k-1} + \sigma_n^2}$$

TUD $\hat{x}_{k+1,k} = a \cdot \hat{x}_{k,k} + \delta \cdot a_k$

$\Sigma_{k+1,k} = a^2 \Sigma_{k,k} + \underbrace{Q}_{\sigma_u^2}$

$Q = E\{u_k^2\} \quad Var\{u_k\} = \sigma_u^2$

Kalman Gain $K_k = \frac{a \cdot c \cdot \Sigma_{k,k-1}}{c^2 \Sigma_{k,k-1} + \sigma_n^2}$

zu zeigen $\Sigma_{k+1,k} = \frac{a^2 \cdot \sigma_n^2 \cdot \Sigma_{k,k-1}}{\sigma_n^2 \cdot c^2 \Sigma_{k,k-1}} + \sigma_u^2$

TUD $\Sigma_{k+1,k} = a^2 \Sigma_{k,k} + \sigma_u^2$

MOD $\Sigma_{k,k} = \Sigma_{k,k-1} - \frac{\Sigma_{k,k-1}^2 \cdot C^2}{c^2 \Sigma_{k,k-1} + \sigma_n^2}$

$$c^2\,\Sigma_{k,k-1} + \sigma_m^2 \qquad \acute{c}$$

$$\Sigma_{k+1,k} = a^2\left(\frac{\Sigma_{k,k-1}\left(c^2\,\Sigma_{k,k-1} + \sigma_m^2\right) + \Sigma_{k,k-1}\cdot c^2}{c^2\cdot\Sigma_{k,k-1} + \sigma_m^2}\right) + \sigma_n^2$$

d) $a = 1 \quad \delta = 1/2 \quad c = 1$

$\sigma_m^2 = 2 \quad \sigma_n^2 = 1$

$$\lim_{k\to\infty}\Sigma_{k,k-1} = \lim_{k\to\infty}\Sigma_{k,k} = \Sigma$$

$$\Sigma_{k+1,k} = \frac{2\cdot\Sigma_{k,k-1}}{\Sigma_{k,k-1} + 2} + 1$$

$$\bar{\Sigma} = \frac{2\,\bar{\Sigma}}{2+\Sigma} + 1$$

$\to$ quadr. Gl.

$\to$ positive Variante $\bar{\Sigma} > 0$

[...]

$\bar{\Sigma} = 2$

$$\lim_{k\to\infty} K_k = 1/2$$