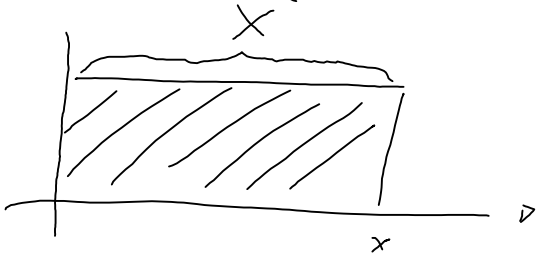
Übung 7

Dienstag, 26. Oktober 2010
09:03

Zusammenfassung

Verteilungsfunktion

$$P(x) = P(X \leq x)$$



Eigenschaften

a) $P(\infty) = 1 \qquad P(-\infty) = 0$

b) $P(X > x) = 1 - P(x)$

c) $P(x_1 < X \leq x_2) = P(x_2) - P(x_1)$

d) $x_1 > x_2 \Rightarrow P(x_1) > P(x_2)$

Verteilungsdichtefunktion

$$p(x) = \frac{d}{dx} P(x)$$

Eigenschaften:

a) $p(x) \geq 0$

b) $\int_{-\infty}^{\infty} p(x)\, dx = 1$

c) $P(x) = \int_{-\infty}^{x} p(\xi)\, d\xi$

d) $P(x_1 < X < x_2) = P(x_2) - P(x_1) = \int_{x_1}^{x_2} p(x)\, dx$

Erwartungswert einer Zufallsvariable

$$E\{x\} = \int_{-\infty}^{\infty} x \cdot p(x)\, dx = m$$

Varianz

$$E\{(x-m)^2\} = \sigma^2 = E\{x^2\} - E^2\{x\}$$

$$= \int_{-\infty}^{\infty} (x-m)^2 p(x)\, dx$$

Bedingte W'keit

$$P(A|M) = \frac{P(A, M)}{P(M)}$$

totale W'keit

$$P(B) = \sum_{i=1}^{N} P(B|A_i) \cdot P(A_i)$$

wobei $\sum_{i=1}^{N} P(A_i) = 1$

Aufg. 1

a) Ω: Alle Möglichkeiten

$|\Omega| = 36$

E: Alle mit Augensumme 7

$|E| = |\{(4,6),(6,1),(1,5),(5,2),(3,4),(4,3)\}|$

$= 6$

$\Rightarrow P(E) = \frac{|E|}{|S_4|} = \frac{1}{6}$

5) R: Karte Rot

$\Rightarrow P(R) = \frac{1}{2}$

A: Karte Ass

$\Rightarrow P(A) = \frac{1}{13}$

$P(A,R) = \frac{2}{52} = \frac{1}{26}$

$\Rightarrow P(A|R) = \frac{\frac{1}{26}}{\frac{1}{2}} = \frac{2}{26} = \frac{1}{13}$

c) G: Ball weiß

$P(G) = \frac{1}{2}$

$P(G,1) = \frac{999}{2000}$

$P(1|G) = \frac{\frac{999}{2000}}{\frac{1}{2}} = \frac{999}{1000} = 99,9\%$

Aufg. 2

$P(Z_1) = 0,2 \quad P(Z_2) = 0,3 \quad P(Z_3) = 0,5$

$P(K|Z_1) = 0,15 \quad P(K|Z_2) = 0,18 \quad P(K|Z_3) = 0,09$

a) $P(K, Z_1) = 0,15 \cdot 0,2 = 0,03$

$P(K, Z_2) = 0{,}18 \cdot 0{,}3 = 0{,}054$

$P(K, Z_3) = 0{,}09 \cdot 0{,}5 = 0{,}045$

b) $P(K) = P(K, Z_1) + P(K, Z_2) + P(K, Z_3)$

$= 0{,}129$

c) $P(Z_1 | K) = \frac{P(Z_1, K)}{P(K)} = \frac{0{,}03}{0{,}129} = 0{,}2326$

Aufg. 4

a) $\int = 1$

$$\frac{d}{dx} F(x) = \begin{cases} 0 & ; x < 0 \\ a & \\ 0 & \end{cases}$$

$$\int_0^3 a \, dx = 1 \iff ax \Big|_0^3 = 1$$

$$\iff a \cdot 3 = 1$$

$$\iff a = 1/3$$

b)

$$\Rightarrow F(x) = \begin{cases} 0 & ; x < 0 \\ 1/3 \, x & ; 0 \leq x < 3 \\ 1 & ; x \geq 3 \end{cases} \qquad f(x) = \begin{cases} 0 & ; x < 0 \\ 1/3 & ; 0 \leq x < 3 \\ 0 & ; x \geq 3 \end{cases}$$

∞

c) $E\{x\} = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$= \int_{0}^{3} \frac{1}{3} x \, dx = \frac{1}{6} x^2 \Big|_0^3 = \frac{9}{6} = \frac{3}{2}$

$Var(x) = E(x^2) - E^2(x)$

$= \int_{-\infty}^{\infty} x^2 f(x^2)\, dx - \frac{9}{4}$

$= \int_{0}^{3} \frac{1}{3} x^2 - \frac{9}{4} = \frac{1}{9} x^3 \Big|_0^3 - \frac{9}{4} = 3 - \frac{9}{4}$

$= \frac{3}{4}$ //

Aufg. 5

a) $p(x) = \frac{0 - 2/3}{3 - 0} x + 2/3 = -\frac{2}{9} x + \frac{6}{9}$

$P(x) = \int_{0}^{x} p(x)\, dx = \int_{0}^{x} -\frac{2}{9} x + \frac{6}{9}$

$= -\frac{2}{18} x^2 + \frac{6}{9} x \Big|_0^x$

$= -\frac{2}{18} x^2 + \frac{6}{9} x = -\frac{1}{9} x^2 + \frac{2}{3} x$

b) $P(X>1) = 1 - P(X \leq 1) = \frac{4}{9}$

c) $P(X>2 \mid X>1) = \frac{P(2,1)}{P(1)} = \frac{P(2)}{P(1)} = \frac{1/9}{4/9} = \frac{1}{4}$

Aufg. 6

a) $\int_0^{x_0} \frac{x}{2}\, dx \overset{!}{=} 1$

$\Leftrightarrow \frac{1}{4} x^2 \Big|_0^{x_0} = 1$

$\Leftrightarrow x_0^2 = 4 \quad \Leftrightarrow x_0 = 2$

b)