Übung 7

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11:53

Klausur F2006 A5

f) ges duales System

$$\underline{A}_{2,Dual} = A_2^T = \begin{pmatrix} 2 & -1 & 0 \\ -3 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\underline{B}_{2,Dual} = C_2^T = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$C_{2,Dual} = B_2^T = (1\ 1\ 0)$$

g) ges S $\Sigma_2 \to \Sigma_2^*$ dass in RNF vorliegt

$$\underline{S} = \underline{Q}_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_{n-1} & 1 & & \\ \vdots & & \ddots & \\ a_1 & \cdots & a_{n-1} & 1 \end{pmatrix}$$

$$\chi = \det(\underline{I}s - A) \Rightarrow a_0 \ldots a_{n-1}$$

$$Q_s = (A_2^2 B_2 \quad A_2 B_2 \quad B_2)$$

$$= \begin{pmatrix} -5 & -1 & 1 \\ 2 & 1 & 1 \\ -3 & -1 & 0 \end{pmatrix}$$

$$\chi = \det(Is - A_2) = \left| \begin{pmatrix} s & 0 & 0 \\ \end{pmatrix} \quad \begin{pmatrix} 2 & -3 & 0 \\ \end{pmatrix} \right|$$

$$\chi = \det(\underline{I}z - \underline{A}_2) = \left| \begin{pmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{pmatrix} - \begin{pmatrix} 2 & -3 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 2 \end{pmatrix} \right|$$

$$= \begin{vmatrix} z-2 & 3 & 0 \\ 1 & z-2 & -1 \\ 0 & 1 & z-2 \end{vmatrix} = z^2 - 6z + 10z - 4$$

$$\overset{!}{=} z^3 + a_2 z^2 + a_1 z + a_0$$

$$\Rightarrow \quad a_2 = -6$$

$$a_1 = 10$$

$$a_0 = -9$$

$$\underline{S} = \underbrace{\begin{pmatrix} -5 & -1 & 1 \\ 2 & 1 & 1 \\ -3 & -1 & 0 \end{pmatrix}}_{Q_S} \begin{pmatrix} 1 & 0 & 0 \\ \underbrace{-6}_{a_2} & 1 & 0 \\ \underbrace{10}_{a_1} & -6 & 1 \end{pmatrix} = \begin{pmatrix} 11 & -7 & 1 \\ 6 & -5 & 1 \\ 3 & -1 & 0 \end{pmatrix}$$

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d) ges: Systemmatrizen

$$\underline{x}(k+1) = \underbrace{\begin{pmatrix} 1 & 0 & 4 \\ 1/2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}}_{\underline{A}:[3\times 3]} \underline{x}(k) + \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\underline{B}:[3\times 1]} u(k)$$

$$y(k) = \underbrace{\begin{pmatrix} 2 & 2 & 1 \end{pmatrix}}_{\underline{C}:[1\times 3]} \cdot x(k) + \begin{pmatrix} 0 \end{pmatrix} u(k)$$

c) ges: ex. ein min. äquiv. System?

$$\underline{Q}_B = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 \\ 4 & 6 & 7 \\ 14 & 18 & 17 \end{pmatrix}$$

$$2z_2 + 3z_1 \Rightarrow \begin{pmatrix} 2 & 2 & 1 \\ 8+6 & 12+6 & 14+3 \\ 14 & 18 & 17 \end{pmatrix}$$

$\Rightarrow$ rang $(Q_B) = 2$

$\hookrightarrow$ Das min. äquiv. System hat nur noch 2 Zustände

$$\longrightarrow \quad R = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 \\ 4 & 6 & 7 \end{pmatrix}$$

e) ges. min äquiv. System Σ_2^*

$\rightarrow$ ges A_2^* B_2^* C_2^* D_2^*

$$A^* R = R \cdot A \Rightarrow RA = \begin{pmatrix} 2 & 2 & 1 \\ 4 & 6 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 & 4 \\ 12 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$RA = \begin{pmatrix} C \\ CA \end{pmatrix} A \qquad = \underline{\underline{\begin{pmatrix} 4 & 6 & 7 \\ 14 & 18 & 17 \end{pmatrix}}}$$

$$= \begin{pmatrix} CA \\ CA^2 \end{pmatrix}$$

$m = 2$

$n = 3$

$$\begin{pmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{pmatrix} \begin{pmatrix} 4 & 6 & 7 \\ 14 & 18 & 17 \end{pmatrix}$$

$$\left.\begin{aligned} 2a_{11}^* + 4a_{12}^* &= 4 \\ 2a_{11}^* + 6a_{12}^* &= 6 \end{aligned}\right\} \Rightarrow a_{11}^* = 0 \; ; \; a_{12}^* = 1$$

$$\left.\begin{aligned} 2a_{21}^* + 4a_{22}^* &= 14 \\ 2a_{11}^* + 6a_{12}^* &= 18 \end{aligned}\right\} a_{22}^* = 2 \; ; \; a_{21}^* = 3$$

$$A^* = \underline{\underline{\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}}}$$

$$B^* = R \cdot B = \begin{pmatrix} 2 & 2 & 1 \\ 4 & 6 & 7 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 \\ 17 \end{pmatrix}}}$$

$$\underline{C}^* \cdot \underline{R} = \underline{C}$$

$$\underbrace{\begin{pmatrix} c_1^* & c_2^* \end{pmatrix}}_{\underline{C}^*} \cdot \underbrace{\begin{pmatrix} 2 & 2 & 1 \\ 4 & 6 & 7 \end{pmatrix}}_{R} \overset{!}{=} \underbrace{\begin{pmatrix} 2 & 2 & 1 \end{pmatrix}}_{C}$$

$$\left.\begin{aligned} 2c_1^* + 4c_2^* &= 2 \\ 2c_1^* + 6c_2^* &= 2 \end{aligned}\right\} c_1^* = 1 \; ; \; c_2^* = 0$$

$$\underline{C}^* = \begin{pmatrix} 1 & 0 \end{pmatrix}$$