Übung 9

Dienstag, 23. November 2010
10:03

Übertragungsfunktion berechnen

1) allgem. Formel:

$$\text{diskret: } \underline{Y}(z) = \overbrace{\left[\underline{C}(z)\cdot(z\underline{I}-\underline{A}(z))^{-1}\cdot\underline{B}(z)+\underline{D}(z)\right]}^{\ddot{U}\text{-Fkt}}\cdot\underline{U}(z)$$

$$\text{kontinuierlich: } \underline{Y}(s) = \left[\underline{H}(s)\,(s\cdot\underline{I}-\underline{F}(s))^{-1}\cdot\underline{G}(s)+\underline{D}(s)\right]\cdot\underline{U}(s)$$

2)
$$\begin{pmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1)\end{pmatrix} = A\begin{pmatrix} x_1(k) \\ \vdots \\ x_n(k)\end{pmatrix} + B\cdot\underline{u}(k) \quad \circ\!\!\xrightarrow{z}\!\!\bullet \quad \text{n-Gl. im z-Bereich}$$

$$y(k) = \underline{c}\cdot\underline{x}(k) + D\cdot\underline{u}(k) \quad \circ\!\!\xrightarrow{z}\!\!\bullet$$

3) (Kap. 17)

Hinweis Aufg. 4: $i_C = C\cdot\frac{du_C(t)}{dt}$ $\quad u_L = L\cdot\frac{di_L(t)}{dt}$

Aufg. 1

$$y(k) = e(k) + b_1\cdot x_1(k) + b_2\cdot x_3(k)$$

$$y(k) = x_1(k+1)$$

$$e(k) = a_0\cdot u(k) + a_1\cdot x_2(k)$$

$$x_1(k+1) = a_0\cdot u(k) + a_1\cdot x_2(k) + b_1\, x_1(k) + b_2\cdot x_3(k)$$

$$x_2(k+1) = u(k)$$

$$x_3(k+1) = x_1(k)$$

$$\Rightarrow \begin{pmatrix} x_1(k+1)\\ x_2(k+1)\\ x_3(k+1)\end{pmatrix} = \overbrace{\begin{pmatrix} b_1 & a_1 & b_2\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}}^{A}\begin{pmatrix} x_1(k)\\ x_2(k)\\ x_3(k)\end{pmatrix} + \overbrace{\begin{pmatrix} a_0\\ 1\\ 0\end{pmatrix}}^{B} u(k)$$

$$\begin{pmatrix} \vdots \\ x_3(k+1) \end{pmatrix} = \begin{pmatrix} \vdots & \vdots & \vdots \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vdots \\ x_3(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(k)$$

$$y(k) = (b_1 \quad a_1 \quad b_2)\, \underline{x}(k) + a_0 \cdot u(k)$$

b) $\dot{x}_1 = u(t)$

$$\dot{x}_2 = -x_2(t) + u(t)$$

$$\dot{x}_3 = -2 \cdot x_3(t) + u(t)$$

$$\Rightarrow \underline{\dot{x}}(t) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \underline{x}(t) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = (4 \quad -6 \quad 2)\, \underline{x}(t) + 0 \cdot u(t)$$

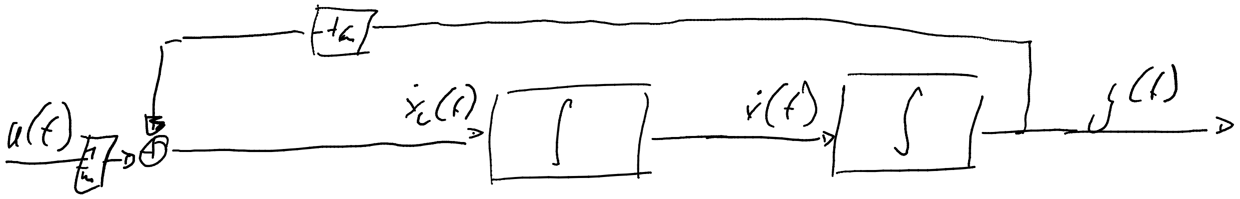
Aufg. 2

a) $m \cdot \dot{x}_2(t) = -f \cdot x_1(t) + u(t)$

$$\dot{x}_1(t) = x_2(t)$$

$$\Rightarrow \underline{\dot{x}}(t) = \overbrace{\begin{pmatrix} 0 & 1 \\ -f/m & 0 \end{pmatrix}}^{F} \underline{x}(t) + \overbrace{\begin{pmatrix} 0 \\ 1/m \end{pmatrix}}^{G} u(t)$$

$$y(t) = \underbrace{(1 \quad 0)}_{H} \underline{x}(t) + \underbrace{0}_{D} \cdot u(t)$$



$$s\cdot\underline{I} - F = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -f/m & 0 \end{pmatrix} = \begin{pmatrix} s & -1 \\ f/m & s \end{pmatrix}$$

$$\left(s\underline{I} - F\right)^{-1} = \frac{1}{s^2 + f/m} \begin{pmatrix} s & 1 \\ -f/m & s \end{pmatrix}$$

$$\left(s\underline{I} - F\right)^{-1} \cdot G(s) = \frac{1}{s^2 + f/m} \begin{pmatrix} 1/m \\ s/m \end{pmatrix}$$

$$H(s)\left(s\cdot\underline{I} - F\right)^{-1} G(s) = \frac{1}{s^2 + f/m} \cdot \frac{1}{m}$$

$$\Rightarrow g(s) = \frac{1}{m\cdot s^2 + f}$$

Aufg. 3

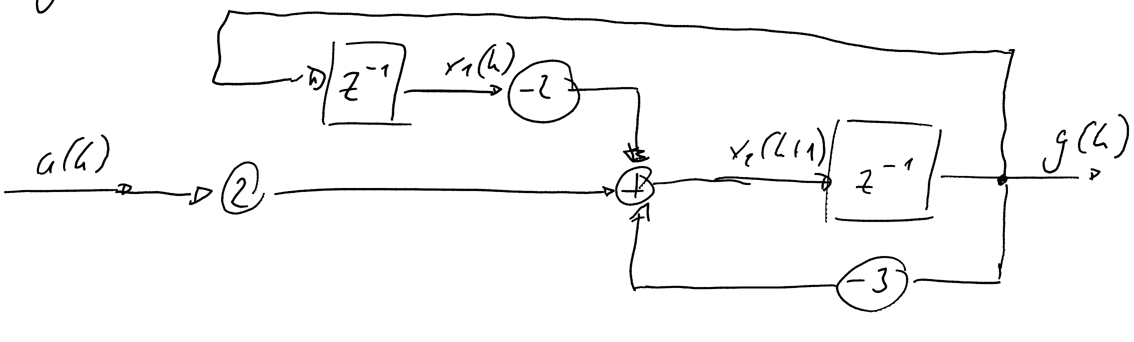
a)

$$x_2(k+1) + 3x_2(k) + 2x_1(k) = 2u(k)$$
$$x_1(k+1) = x_2(k)$$

$$\underline{x}(k+1) = \overbrace{\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}}^{A} \underline{x}(k) + \overbrace{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}^{B} u(k)$$

$$y(k) = \overbrace{\begin{pmatrix} 0 & 1 \end{pmatrix}}^{C} \underline{x}(k) + \overbrace{0}^{D} \cdot u(k)$$

b)



c)

$$z\underline{I} - A(z) = \begin{pmatrix} z & -1 \\ 2 & z+3 \end{pmatrix}$$

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$$(z\underline{I} - A)^{-1} = \frac{1}{z(z-3)+2} \begin{pmatrix} z+3 & +1 \\ -2 & z \end{pmatrix}$$

$$((z\underline{I} - A)^{-1} \cdot B) = \frac{1}{z(z-3)+2} \begin{pmatrix} 2 \\ 2z \end{pmatrix}$$

$$C\,(z\underline{I} - A)^{-1} \cdot B = \frac{1}{z(z+3)+2} \cdot 2z$$

$$\Rightarrow g(z) = \frac{2z}{z(z+3)+2}\, u(z)$$

<u>Aufg. 4</u>

$u_c(t) = x_1(t)$

$i(t) = x_2(t)$

$$i_c = C \cdot \frac{du_c(t)}{dt} \qquad u_L = L \cdot \frac{di_L(t)}{dt}$$

$$x_2(t) = C \cdot \dot{x}_1(t)$$

$$\dot{x}_2(t) \cdot L = u_1(t) - x_1(t) - R \cdot x_2(t)$$

$$\Rightarrow \underline{\dot{x}}(t) = \begin{pmatrix} 0 & 1/C \\ -1/L & -R/L \end{pmatrix} \underline{x}(t) + \begin{pmatrix} 0 \\ 1/L \end{pmatrix} u_1(t)$$

$$u_2(t) = (1 \;\; 0)\, \underline{x}(t) + 0 \cdot u_1(t)$$