Übung 10

Dienstag, 11. Januar 2011
11:54

Aufg. 5 (Klausur F2009)

a) ges.: $\underline{A}_1$ $\underline{B}_1$ $\underline{C}_1$ von Σ_1

bekannt $\underline{A}_{tot} = \underline{A}_2$ $\quad C_{tot} = C_2$ des Gesamtsys.

$\underline{B}_{tot} = \underline{B}_2$

$$\underline{A}_{tot} = \left[\begin{array}{c|c} \overbrace{\underline{A}_1}^{n\times n} & \overbrace{\underline{A}_4\, \underline{k}_2}^{n\times n} \\ \hline \underline{k}_2 \underline{C}_1 & \underline{A}_1 - \underline{k}_2 \underline{C}_1 + \underline{B}_1 \underline{k}_2 \end{array}\right] \stackrel{!}{=} \underline{A}_2 = \left[\begin{array}{cc|cc} 1 & 0 & a_1 & a_2 \\ 1 & 1 & a_1 & a_2 \\ \hline -2 & 2 & a_3 & a_4 \\ 1 & -1 & a_5 & a_6 \end{array}\right]$$

$$\underline{A}_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\underline{B}_{tot} = \begin{bmatrix} \tilde{\underline{b}}_1 \\ \underline{b}_2 \end{bmatrix} \stackrel{!}{=} \underline{B}_2 = \begin{pmatrix} 1 \\ 1 \\ b_1 \\ b_2 \end{pmatrix} \Rightarrow \tilde{\underline{b}}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$C_{tot} = \left(\underline{c} \quad \underline{0}^T \right) \stackrel{!}{=} \underline{C}_2 = \left(1 \; -1 \; 0 \; 0\right)$$

$$\Rightarrow \underline{C}_1 = \underline{\underline{\left(1 \; -1\right)}}$$

b)

$$A_{tot} = \begin{bmatrix} A_1 & B_1 k_2 \\ k_2 C_1 & A_1 - k_2 C_1 + B_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a_1 & a_2 \\ 1 & 1 & a_1 & a_2 \\ -2 & 2 & a_3 & a_4 \\ 1 & -1 & a_5 & a_6 \end{bmatrix} = A_2$$

ges: $\underline{k}_2$

$$\underline{k}_2 \underline{C}_1 \stackrel{!}{=} \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} (1 \ \ -1) = \begin{pmatrix} l_1 & -l_1 \\ l_2 & -l_2 \end{pmatrix} \overset{!}{=} \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix}$$

$$\Rightarrow l_1 = -2 \qquad \underline{L_2} = (-2, 1)^T$$
$$l_2 = 1$$

c) $$A_{GOL} = \left(\begin{array}{c|c} A_1 & B_1 K_2 \\ \hline L_2 C_1 & A_1 - L_2 C_1 + B_1 \end{array}\right) \overset{!}{=} \left(\begin{array}{cc|cc} 1 & 6 & a_1 & a_2 \\ 1 & 1 & a_3 & a_4 \\ \hline \end{array}\right) = \underline{A_2}$$

$$\underline{B_1} \cdot \underline{K_2} = \begin{bmatrix} a_1 & a_2 \\ a_1 & a_2 \end{bmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} (k_1 \ \ k_2) = \begin{pmatrix} k_1 & k_2 \\ k_1 & k_2 \end{pmatrix} \overset{!}{=} \begin{pmatrix} a_1 & a_2 \\ a_1 & a_2 \end{pmatrix} \Rightarrow k_2 = (a_1 \ \ k_2)$$

d) ges: k_2, s.d. Polstelle bei $z = \pm 1$

$$k_2 = (a_2, \ a_1)$$

$$\chi(z) = \det(\underline{I} z - \underline{A_1} - \underline{B_1} \cdot \underline{K_2}) \overset{!}{=} (z-1)(z+1)$$

$$= \det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} a_1 & a_2 \\ a_1 & a_2 \end{bmatrix}\right) \overset{!}{=} z^2 - 1$$

$$= \det\begin{pmatrix} z - 1 - a_1 & -a_2 \\ -1 - a_1 & z - 1 - a_2 \end{pmatrix} = z^2 - 1$$

$$\Leftrightarrow (z - 1 - a_1)(z - 1 - a_2) + a_2(-1 - a_1)$$

$$= (z - (1 + a_1))(z - (1 + a_2)) - a_2(1 + a_1)$$

$$= z^2 - \underbrace{(2 + a_2 + a_1)}_{\overset{!}{=} 0} z + a_1 + 1$$

$$\overset{!}{=} z^2 - 1$$

$$2 + a_1 + a_2 \overset{!}{=} 0 \quad \Rightarrow a_2 = 0$$

$$a_1 + 1 \overset{!}{=} -1 \quad \Rightarrow a_1 = -2$$

$$\Rightarrow K = \begin{pmatrix} -2 & 0 \end{pmatrix}$$

e) $$A_{tot} = \left(\begin{array}{c|c} A_1 & B_1 K_2 \\ \hline L_2 C_1 & A_1 - L_2 C_1 + B_2 K_2 \end{array}\right) = \left(\begin{array}{c|cc} & & \\ \hline & a_3 & a_4 \\ & a_5 & a_6 \end{array}\right)$$

$$A_1 - L_2 C_1 + B_2 K_2 \overset{!}{=} \begin{pmatrix} a_3 & a_4 \\ a_5 & a_6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} a_3 & a_4 \\ a_5 & a_6 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 4 & -1 \\ 1 & 3 \end{pmatrix} \overset{!}{=} \begin{pmatrix} a_3 & a_4 \\ a_5 & a_6 \end{pmatrix}$$