Übung 11

Dienstag, 11. Januar 2011
08:16

Kalman filter

Zeitabhängigkeit der Matrizen

$$x(k+1) = A_k x(k) + B_k u(k) + n(k)$$

$$y(k) = C_k x(k) + m(k)$$

Σ : Kovarianzmatrix

Aufg. 1

a) $E\{s(k)\} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

b)

$$\Sigma = E\left\{ \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} \begin{pmatrix} x(k) & y(k) \end{pmatrix} \right\}$$

$$= E\left\{ \begin{pmatrix} x^2(k) & x(k)\,y(k) \\ x(k)\,y(k) & y^2(k) \end{pmatrix} \right\}$$

(1) $$= \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$$

(2) $$= \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}$$

(2)

$$= \begin{pmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{pmatrix}$$

Aufg. 2

a) $A = \frac{1}{2} \qquad B = 0 \qquad C = 1$

$$R_k = E\{v_k \cdot v_k^T\} = E\{v_k^2\} = \left(\frac{1}{2}\right)^k$$

$$Q = E\{u_k u_k^T\} = E\{u_k^2\} = 2$$

b) $$\hat{x}_{k,k} = \hat{x}_{k,k-1} + \Sigma_{k,k-1} \cdot \left(\Sigma_{k,k-1} + \left(\tfrac{1}{2}\right)^k\right)^{-1} \left(y_k - \hat{x}_{k,k-1}\right)$$

$$\Sigma_{k,k} = \Sigma_{k,k-1} - \Sigma_{k,k-1}\left(\Sigma_{k,k-1} + (1/2)^k\right)^{-1} \cdot \Sigma_{k,k-1}$$

$$\hat{x}_{k+1,k} = \frac{1}{2}\hat{x}_{k,k}$$

$$\Sigma_{k+1,k} = \frac{1}{4}\Sigma_{k,k} + 2$$

$$K_k = \frac{1}{2}\Sigma_{k,k-1}\left(\Sigma_{k,k-1} + (1/2)^k\right)^{-1}$$

c) $$\hat{x}_{0,0} = 0 + 1 \cdot \left(1 + (1/2)^0\right)^{-1} (0{,}6 - 0)$$

$$= \frac{1}{2} \cdot 0{,}6 = 0{,}3$$

$$\Sigma_{0,0} = 1 - 1\left(1 + (1/2)^0\right)^{-1} = 1 - \frac{1}{2} = 0{,}5$$

$$K_0 = \frac{1}{2} \cdot 1 \left(1 + (1/2)^6 \right)^{-1} = \frac{1}{4}$$

d) $$K_\infty = \lim_{k \to \infty} \frac{1}{2} \cdot \Sigma_{k,k-1} \left(\Sigma_{k,k-1} + (1/2)^6 \right)^{-1}$$

$$= \lim_{k \to \infty} \frac{1}{2} \frac{\Sigma_{k,k-1}}{\Sigma_{k,k-1}} = 1/2$$

$$\Sigma_{\infty,\infty} = \lim_{k \to \infty} \Sigma_{k,k-1} - \Sigma_{k,k-1} \left(\Sigma_{k,k-1} + (1/2)^6 \right)^{-1} \cdot \Sigma_{k,k-1}$$

$$= 0$$

Aufg. 5

d) $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $C = \begin{pmatrix} h_2 & h_1 \end{pmatrix}$ $D = 0$

$h_k = \begin{pmatrix} 0 \\ e_k \end{pmatrix}$ $v_k = \omega_k$

$R = E\{v_k^2\} = \beta^2$

$Q = E\left\{ \begin{pmatrix} 0 & 0 \\ 0 & e_k^2 \end{pmatrix} \right\} = \begin{pmatrix} 0 & 0 \\ 0 & \gamma^2 \end{pmatrix}$