Übung 11

Dienstag, 18. Januar 2011
12:01

Aufg. 21

tatsächliche Position = Zustand $\vec{S}_k = [x(k), y(k)]^T$

Eingang: $\vec{a}_K = [\Delta x(k), \Delta y(k)]^T$

System rauschen: $\vec{n}_K = [\omega_x(k), \omega_y(k)]^T$

Messung: $\vec{S}_{mess,k} = [x_{mess}(k), y_{mess}(k)]^T$

gemessene Position

Messrauschen $m_k = [m_x(k), m_y(k)]^T$

a) ges: S_{K+1}

$$\underbrace{\begin{pmatrix} x(k+1) \\ y(k+1) \end{pmatrix}}_{\vec{S}_{K+1}} = \underbrace{\begin{pmatrix} x(k) \\ y(k) \end{pmatrix}}_{\vec{S}_K} + \underbrace{\begin{pmatrix} \Delta x(k) \\ \Delta y(k) \end{pmatrix}}_{\vec{a}_a} + \underbrace{\begin{pmatrix} \omega_x(k) \\ \omega_y(k) \end{pmatrix}}_{\vec{n}_K}$$

b)

$$\begin{pmatrix} x(k+1) \\ y(k+1) \end{pmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\underline{A}} \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\underline{B}} \vec{a}_K + \vec{n}_K$$

a)

$$\underbrace{\begin{pmatrix} x_{mess,K} \\ y_{mess,K} \end{pmatrix}} = \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} + \begin{pmatrix} m_x(k) \\ m_y(k) \end{pmatrix}$$

zu d) $\vec{S}_{mess,K} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \vec{S}_K + \vec{m}_K$

ρ

b) $\underline{Q} = E\{\vec{n}_K \cdot \vec{n}_K^{\,T}\} = E\left\{\begin{pmatrix} \omega_x(k) \\ \omega_y(k) \end{pmatrix} (\omega_x(k), \omega_y(k))\right\}$

$$= E\left\{\begin{pmatrix} \omega_x^2(k) & \omega_x(k)\,\omega_y(k) \\ \omega_x(k)\,\omega_y(k) & \omega_y^2(k) \end{pmatrix}\right\}$$

$$\rho = \frac{E\{[\omega_x(k) - E\{\omega_x(k)\}][\omega_y(k) - E\{\omega_y(k)\}]\}}{\sqrt{\sigma^2 \cdot \sigma^2}}$$

$$\rho = \frac{E\{[\omega_x(k) - E\{\omega_x(k)\}][\omega_y(k) - E\{\omega_y(k)\}]\}}{\sqrt{\sigma_{\omega_x}^2 \cdot \sigma_{\omega_y}^2}}$$

$$Q = \begin{bmatrix} E\{\omega_x^2(k)\} & E\{\omega_x(k)\omega_y(k)\} \\ E\{\omega_x(k)\omega_y(k)\} & E\{\omega_y^2(k)\} \end{bmatrix}$$

$$E\{\omega_x(k)\} = 0 \qquad E\{\omega_y(k)\} = 0$$

$$\rho = \frac{E\{\omega_x(k)\omega_y(k)\}}{\sigma_{\omega_x}\sigma_{\omega_y}} \iff E\{\omega_x(k)\omega_y(k)\} = \sigma_{\omega_x} \cdot \sigma_{\omega_y} \cdot \rho$$

$$\Rightarrow Q = \begin{bmatrix} \sigma_{\omega_x}^2 & \rho\sigma_{\omega_x}\sigma_{\omega_y} \\ \rho\sigma_{\omega_x}\sigma_{\omega_y} & \sigma_{\omega_y}^2 \end{bmatrix}$$

c) ges.: $\hat{\vec{s}}_{0,0}$ $\underline{\Sigma}_{0,0}$ $\underline{K}_0$

Hinweis: $\hat{\vec{s}}_{0,-1} = \hat{\vec{s}}_0$

$\Sigma_{0,-1} = \Sigma_0$

gegeben: $\underline{\Sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\qquad \vec{s}_{mess,0} = \begin{pmatrix} 0{,}9 \\ 0{,}6 \end{pmatrix}$

$R_0 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

<u>MaD</u>

$$\hat{\vec{s}}_{k,k} = \hat{\vec{s}}_{k,k-1} + \underline{\Sigma}_{k,k-1} \cdot \underline{C}^T(\underline{C}\,\Sigma_{k,k-1}\underline{C}^T + R_k)^{-1} \cdot (\vec{s}_{mess,k} - C \cdot \hat{\vec{s}}_{k,k-1})$$

$k = 0$

$$\hat{s}_{0,0} = \underbrace{\hat{s}_{0,-1}}_{\hat{s}_0} + \underbrace{\Sigma_{0,-1}}_{\Sigma_0} \cdot \underline{C}^T(\underline{C}\underbrace{\Sigma_{0,-1}}_{\Sigma_0} \cdot \underline{C}^T + R_0)^{-1}(\hat{s}_{mess,0} - \underline{C} \cdot \hat{s}_0)$$

$$\hat{s}_{0,0} = s_0 + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right]^{-1}\left(\begin{pmatrix} 0{,}9 \\ 0{,}6 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\hat{s}_0\right)$$

$$s_{0,0} = s_0 + \begin{pmatrix} \cdot & \cdot \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cdot & \cdot \\ 0 & 1 \end{pmatrix} \left[\begin{pmatrix} \cdot & \cdot \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \cdot & \cdot \\ 0 & 2 \end{pmatrix} \right] \left(\begin{pmatrix} \cdot \\ 0{,}6 \end{pmatrix} - \begin{pmatrix} \cdot & \cdot \\ 0 & 1 \end{pmatrix} s_0 \right)$$

$$\vec{s}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ Erwartungswert des Startzustandes mittelwertfrei

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} 0{,}9 \\ 0{,}6 \end{pmatrix} = \begin{pmatrix} 0{,}3 \\ 0{,}2 \end{pmatrix}$$

$$\Sigma_{k,k} = \underbrace{\Sigma_{k,k-1}}_{\Sigma_0} - \Sigma_{k,k-1} \cdot C^T \left(C \cdot \Sigma_{k,k-1} C^T + R_k \right)^{-1} \cdot C \cdot \Sigma_{k,k-1}$$

$k = 0$

$$\Sigma_{0,0} = \Sigma_0 - \Sigma_0 C^T \left(C \cdot \Sigma_0 \cdot C^T + R_0 \right)^{-1} \cdot C \cdot \Sigma_0$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \underbrace{\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \right]^{-1}}_{\begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix}} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix} = \begin{pmatrix} 2/3 & 0 \\ 0 & 2/3 \end{pmatrix}$$

$$K_0 = A \cdot \Sigma_0 \cdot C \underbrace{\left(C \cdot \Sigma_0 \cdot C^T + R_0 \right)^{-1}}_{= \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix}}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix}$$