Übung 12

Dienstag, 25. Januar 2011
08:17

$$w(k) \longrightarrow \boxed{h(k)} \longrightarrow g(k)$$

$$g(k) = h(k) * w(k)$$

$$g(z) = h(z) \cdot w(z)$$

$$h(k) = h_0\, \delta(k) + h_1\, \delta(k-1) + h_2 \cdot \delta(k-2)$$

$$\Rightarrow g(k) = h_0\, w(k) + h_1\, w(k-1) + h_2\, w(k-2)$$

$$h(k) \circ\!\!-\!\!\bullet\; H(z) = h_0 + h_1 \cdot z^{-1} + h_2 \cdot z^{-2}$$

$$g(z) = h_0 \cdot w(z) + h_1 \cdot w(z) \cdot z^{-1} + h_2 \cdot w(z) \cdot z^{-2}$$

$$g(k) = h_0 \cdot w(k) + h_1\, w(z-1) + h_2 \cdot w(z-2)$$

<u>Vektorielle Schreibweise</u>

$$g(k) = \begin{pmatrix} h_0 & h_1 & h_2 \end{pmatrix} \cdot \begin{pmatrix} w(k) \\ w(k-1) \\ w(k-2) \end{pmatrix}$$

$$= \underline{h}^T \cdot \underline{w}(k)$$

$$\underbrace{E\{ g(k) \cdot \underline{w}(k) \}}_{= r_{gw}}$$

$$= E\{ \underline{w}(k) \cdot g(k) \}$$

$$= E\{ \underline{w}(k) \cdot \underline{h}^T \cdot \underline{w}(k) \}$$

$$\left(\begin{array}{l} g(k) \rightarrow \text{skalar} \\ \underline{w}(k) \rightarrow \text{vektoriell} \end{array} \right)$$

$$g(k) = \underline{h}^T \cdot \underline{w}(k)$$

$$= g^T(k) = (\underline{h}^T \cdot \underline{w}(k))^T$$

$$= \underline{w}^T(k) \cdot \underline{h}$$

$$= E\{\underbrace{\underline{\omega}(k)\cdot\underline{\omega}^T(k)}_{R_{\omega\omega}}\}\cdot \underline{h}$$

$$= R_{\omega\omega}\cdot \underline{h}$$

Aufg. 1

a) $g(k) = \begin{pmatrix} 0{,}5 & 0{,}25 & 0{,}25 \end{pmatrix} \cdot \begin{pmatrix} \omega(k) \\ \omega(k-1) \\ \omega(k-2 \end{pmatrix} + \eta(k)$

b)

$$R_{\omega\omega} = \begin{pmatrix} 0{,}25 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}25 \end{pmatrix}$$

c) $R_{g\omega} = E\{g(k)\cdot\omega(k)\}$

$= E\{(h^T\cdot\omega(k) + \eta(k))\cdot\omega(k)\}$

$= E\{h^T\cdot\omega(k)\cdot\omega(k) + \eta(k)\cdot\omega(k)\}$

$= E\{\omega(k)\omega^T(k)\cdot h \quad + \eta(k)\cdot\omega(k)\}$

$= R_{\omega\omega} \cdot h \quad + \begin{pmatrix} 0 \\ 0 \\ 0{,}1 \end{pmatrix}$

$= \begin{pmatrix} 0{,}125 \\ 0{,}0625 \\ 0{,}0625 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0{,}1 \end{pmatrix} = \begin{pmatrix} 0{,}125 \\ 0{,}0625 \\ 0{,}1625 \end{pmatrix}$

d) 3, da $h(k)$ ebenfalls 3 Koeffizienten hat

e)

$$g_{opt} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0{,}125 \\ 0{,}0625 \\ \end{pmatrix}$$

$$g_{opt} = \begin{pmatrix} 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0{,}0625 \\ 0{,}1625 \end{pmatrix}$$

$$= \begin{pmatrix} 0{,}5 \\ 0{,}25 \\ 0{,}65 \end{pmatrix}$$

1) $E\{e^2(k)\} = E\{(y(k) - \hat{y}(k))^2\}$

$= E\{(h^T \underline{\omega}(k) + n(k) - g_{opt} \cdot \underline{\omega}(k))^2\}$

$= E\{((h^T - g_{opt}) \cdot \underline{\omega}(k) + \eta(k))^2\}$

$= E\{((0 \quad 0 \; -0{,}4) \cdot \underline{\omega}(k) + \eta(k))^2\}$

$= E\{(0 \quad 0 \; -0{,}4) \cdot \underline{\omega}(k) \cdot \underline{\omega}^T(k) \cdot \begin{pmatrix} 0 \\ 0 \\ -0{,}4 \end{pmatrix}\}$

$+ 2 \cdot E\{(0 \quad 0 \; -0{,}4) \cdot \underline{\omega}(k) \cdot \eta(k)\}$

$+ E\{\eta^2(k)\}$

$= (0 \quad 0 \; -0{,}4) \; E\{\underline{\omega}(k) \cdot \underline{\omega}^T(k)\} \begin{pmatrix} 0 \\ 0 \\ -0{,}4 \end{pmatrix}$

$+ 2 \cdot (-0{,}4) \cdot 0{,}1$

$+ 0{,}1$

$= (0 \quad 0 \; -0{,}4) \begin{pmatrix} 0{,}25 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}25 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -0{,}4 \end{pmatrix}$

$- 0{,}8 \cdot 0{,}1 \qquad + 0{,}1$

$= (0 \quad 0 \; -0{,}1) \begin{pmatrix} 0 \\ 0 \\ -0{,}4 \end{pmatrix} \quad - 0{,}08 + 0{,}1$

$= + 0{,}04 - 0{,}08 + 0{,}1 = 0{,}06$

II. 1 7

9.4

a) $Y(z) = W(z) \cdot H(z) \cdot C(z)$

$$= W(z) \cdot (0{,}5 + 1{,}5 \cdot z^{-1})(1 + 0{,}5 \cdot z^{-1})$$

$$= W(z) \, (0{,}5 + 1{,}75 \, z^{-1} + 0{,}75 \cdot z^{-2}$$

$$\circ\!\!-\!\!\bullet$$

$$y(k) = 0{,}5 \, w(k) + 1{,}75 \cdot w(k-1) + 0{,}75 \cdot w(k-2)$$

$$= \begin{pmatrix} 0{,}5 & 1{,}75 & 0{,}75 \end{pmatrix} \begin{pmatrix} w(k) \\ w(k-1) \\ w(k-2) \end{pmatrix}$$

b)

$$R_{ww} = \begin{pmatrix} 0{,}25 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}25 \end{pmatrix}$$

$$r_{yw} = R_{ww} \cdot h = \begin{pmatrix} 0{,}125 \\ 0{,}4375 \\ 0{,}1875 \end{pmatrix}$$

c) $v(k) = \begin{pmatrix} 0{,}5 & 1{,}5 \end{pmatrix} \begin{pmatrix} w(k) \\ w(k-1) \end{pmatrix}$

$$= h^T \cdot \underline{w}(k)$$

$$R_{vv} = E\{ \underline{v}(k) \cdot \underline{v}^T(k) \}$$

$$= E\left\{ \begin{pmatrix} v^2(k) & v(k) \cdot v(k-1) \\ v(k) \cdot v(k-1) & v^2(k-1) \end{pmatrix} \right.$$

$$E\{ v(k) \cdot v(k) \} = 0{,}625$$

$$E\{ v(k) \cdot v(k-1) \} = E\{ \underline{h}^T \cdot \underline{w}(k) \cdot w^T(k-1) \cdot h \}$$

$$= h^T \cdot E\{ \underline{w}(k) \cdot w^T(k-1) \} \cdot h$$

$$= h^T \cdot E\left\{ \begin{pmatrix} w(k)\cdot w(k-1) & w(k)\cdot w(k-2) \\ w^2(k-1)\cdot & w(k-1)\cdot w(k-2) \end{pmatrix} \right\} h$$

$$= h^T \cdot E\left\{ \begin{pmatrix} 0 & 0 \\ 0,25 & 0 \end{pmatrix} \right\} \cdot h$$

$$= \begin{pmatrix} 0,375 & 0 \end{pmatrix} \begin{pmatrix} 0,5 \\ 1,5 \end{pmatrix} = 0,1875$$

$$\Rightarrow R_{vv} = \begin{pmatrix} 0,625 & 0,1875 \\ 0,1875 & 0,625 \end{pmatrix}$$

$$r_{yv} = E\{y(k)\cdot v(k)\}$$
$$= E\{v(k)\cdot y(k)\}$$
$$= E\{v(k)\cdot v^T(k)\cdot c\}$$
$$= E\{v(k)\cdot v^T(k)\}\cdot c$$
$$= R_{vv}\cdot c$$
$$= \begin{pmatrix} 0,7187 \\ 0,5 \end{pmatrix}$$

Aufg. 3

a) $r_{yw} = E\{y(k)\, w(k)\}$

$$= E\{ w(k) \cdot h^T \cdot w(k) \}$$

$$= E\{ w(k) \cdot w^T(k) \} \cdot h$$

$$= R_{ww} \cdot h$$

$$= \begin{pmatrix} \underbrace{w^2(k)}_{x} & \underbrace{w(k) \cdot w(k-1)}_{y} \\ w(k) \cdot w(k-1) & w^2(k-1) \end{pmatrix} \cdot h$$

$$\Rightarrow \left. \begin{array}{l} 0{,}1 \overset{!}{=} x \cdot 0{,}5 + y \cdot 1{,}5 \\ 0{,}3 = y \cdot 0{,}5 + x \cdot 0{,}5 \end{array} \right\} \Rightarrow x = 0{,}2 \;,\; y = 0$$

$$\Rightarrow R_{ww} = \begin{pmatrix} 0{,}2 & 0 \\ 0 & 0{,}2 \end{pmatrix}$$

b) $$\int_{0{,}4 - \frac{\sqrt{2}}{2}}^{0{,}4 + \frac{\sqrt{2}}{2}} a \, d_y \overset{!}{=} 1$$

$$\Leftrightarrow a \left[\cancel{0{,}4} + \frac{\sqrt{2}}{2} - \cancel{0{,}4} + \frac{\sqrt{2}}{2} \right] = 1$$

$$\Leftrightarrow a = \frac{\sqrt{2}}{2}$$

$$E\{n(k)\} = \int_{-\infty}^{\infty} p_n(n) \cdot n \, d_n$$

$$= \int_{0{,}4 - \frac{\sqrt{2}}{2}}^{0{,}4 + \frac{\sqrt{2}}{2}} \frac{\sqrt{2}}{2} \cdot n \, d_n$$

$$= \frac{\sqrt{2}}{4} n^2 \Big|_{0{,}4 - \frac{\sqrt{2}}{2}}^{0{,}4 + \frac{\sqrt{2}}{2}} = 0{,}4$$

c) 2 Koeffizienten, so wie h

d) $g_{opt} = R_{ww}^{-1} \cdot R_{gw}$

$$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \cdot R_{gw}$$

$$R_{gw} = E\{g(k) \cdot w(k)\}$$

$$= E\{w(k) \cdot \tilde{h}^T \cdot w(k)\}$$

$$= E\{w(k) \cdot w^T(k) \cdot \tilde{h}\} \quad \text{mit } \tilde{h}^T = h^T + n(k)$$

$$= (h_0 + n(k) \quad h_1 + n(k))$$

$$= R_{ww} \cdot \begin{pmatrix} 0{,}9 \\ 1{,}9 \end{pmatrix}$$

$$\Rightarrow g_{opt} = R_{ww}^{-1} \cdot R_{ww} \cdot \begin{pmatrix} 0{,}9 \\ 1{,}9 \end{pmatrix} = \begin{pmatrix} 0{,}9 \\ 1{,}9 \end{pmatrix}$$