Übung 8

Dienstag, 9. November 2010
10:01

V-Transformation

Rechenregeln

- $\{f(n)\} + \{g(n)\} \circ\!\!-\!\!\bullet \; F(V) + G(V)$
- $\{f(n)\} * \{g(n)\} \circ\!\!-\!\!\bullet \; F(V) \cdot G(V)$
- $\{f(n-k)\} \circ\!\!-\!\!\bullet \; F(V) \cdot V^{k}$ (Rechtsverschiebung)
- $\{f(n+k)\} \circ\!\!-\!\!\bullet \; F(V) \cdot V^{-k} - \sum_{i=0}^{k-1} f(i)\, V^{-k+i}$

Aufg. 1

a) $(V^3+1) : (V^2+V+1) = V + 1$

$$-(V^3+V^2+V)$$
$$V^2+V+1$$
$$-(V^2+V+1)$$
$$0$$

$\Rightarrow F(V) = V+1$

b) $\{f(n)\} = \{1, 1, 0, \dots\}$

Aufg. 2

a) $F(V) = 1 + \cancel{V^3} + V^4 + \cancel{V^6} + \cancel{V^7} + \cancel{V^9} + V^{10} \; \dots$

$-V^3 \cdot F(V) = \cancel{V^3} + \cancel{V^6} + \cancel{V^7} + \cancel{V^9}$

$$(1-v^3)\cdot F(v) = 1 + v^4$$

$$\Leftrightarrow F(v) = \frac{1+v^4}{1+v^3}$$

b) $F(v) = 1 + v^2$

c) rein Periodisch, da Nennergrad > Zählergrad

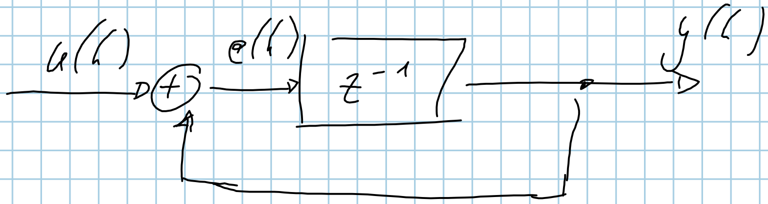
$$F(v) = \frac{v+1}{v^2+1} = \frac{1}{v+1}$$

$$\Rightarrow \{f(n)\} = \{1, \bar{1}, \dots\}$$

Aufg. 3

a) A $x(k+1)$ B $x(k-1)$

b)



1 $E(v) = U(v) + Y(v)$

$$\Rightarrow Y(v) = E(v)\cdot v$$

$$\Leftrightarrow Y(v) = U(v)\cdot v + Y(v)\cdot v$$

$$\Leftrightarrow \frac{Y(v)}{U(v)} = \frac{v}{1-v}$$

(2) $E(z) = U(z) + Y(z)$

$Y(z) = E(z) \cdot z^{-1}$

$$\Rightarrow \frac{Y(z)}{U(z)} = \frac{z^{-1}}{1 - z^{-1}} = \frac{1}{z - 1}$$

Aufg. 4

A) $x_2(k) = x_2(k+1) \cdot V$

$x_2(k+1) = u(k) + x_1(k)$

$x_1(k+1) = x_2(k) + x_1(k)$

$x_1(k) = x_1(k+1) \cdot V$

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \overset{A}{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \overset{B}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \cdot u(k)$$

$y(k) = x_1(k)$

$\Rightarrow C = (1 \quad 0) \qquad D = (0)$

B) $x_1(k+1) = x_2(k)$

$x_2(k+1) = u(k)$

$$\Rightarrow \underline{x}(k+1) = \overset{A}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} \underline{x}(k) + \overset{B}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} u(k)$$

$y(k) = x_1(k)$

$\Rightarrow C = (1 \, 0) \qquad D = (0)$

(c)

$$x_1(k+1) = x_1(k) + u(k)$$
$$x_2(k+1) = x_1(k) + x_2(k)$$

$$\Rightarrow \underline{x}(k+1) = \overset{A}{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}} \underline{x}(k) + \overset{B}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} u(k)$$

$$y(k) = x_2(k) + u(k)$$

$$\Rightarrow C = (0 \ 1) \qquad D = (1)$$

b) (c)

$$X_1(v) \cdot v^{-1} = X_1(v) + U(v)$$
$$\Leftrightarrow X_1(v) = U(v) \cdot \frac{v}{v+1}$$
$$X_2(v) = X_1(v) \cdot \frac{v}{v+1}$$
$$Y(v) = X_2(v) + U(v)$$
$$\Leftrightarrow Y(v) = U(v) \cdot \frac{v^2}{(v+1)^2} + U(v)$$
$$\Leftrightarrow \frac{Y(v)}{U(v)} = \frac{1}{v^2+1}$$

(A)

$$Y(v) = X_1(v)$$
$$X_1(v) \cdot v^{-1} = X_2(v) + Y(v)$$
$$X_2(v) \cdot v^{-1} = U(v) + Y(v)$$

$$\Rightarrow Y(z) = Y(z)\cdot z + U(z)\cdot z^2 + Y(z)\cdot z^2$$

$$\Leftrightarrow \frac{Y(z)}{U(z)} = \frac{z^2}{z^2+z+1}$$

c) $\{x_2(k+1)\} = \{1, 0, 0, 0, 0, \dots\}$

$\{x_2(k)\} = \{0, 1, 0, 0, 0, \dots\} = \{x_1(k+1)\}$

$\{x_1(k)\} = \{0, 0, 1, 0, 0, \dots\} = \{y(k)\}$

d) $\{x_1(k)\} = \{1, 1, 1, 1, \bar{1} \dots\}$

$\{x_1(k+1)\} = \{1, 1, 1, 1, \bar{1}, \dots\}$

$\{x_2(k)\} = \{0, 1, 0, 1, \overline{0, 1}, \dots\}$

Aufg. 5

a) $x_1(k+1) = x_1(k) + x_3(k) + u(k)$

$x_2(k+1) = x_1(k)$

$x_3(k+1) = x_2(k)$

$y(k) = x_1(k) + x_2(k) + u(k)$