KGÜ 10

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a) $\underline{\vec{H}} = \frac{1}{\underline{Z}} \vec{n} \times \underline{\vec{E}}$ TEM-Wellen mit $\underline{Z} = \sqrt{\frac{\mu}{\underline{\varepsilon}}}$

(komplexer: H aus E-Feld über Maxwell)

b) Grenzbedingungen bei $z=0$: $\underline{\vec{E}}_{tan}$ stetig mit $\underline{E}_2 = 0$ da $\sigma_2 \to \infty$

$\underline{\vec{E}}_{1tan} = \vec{0}$ mit $\underline{\vec{E}}_1 = \underline{\vec{E}}_e + \underline{\vec{E}}_r$

c) $\vec{n}_{12} \times (\underbrace{\underline{\vec{H}}_2}_{\substack{=0 \\ (\sigma_2 \to \infty)}} - \underline{\vec{H}}_1) = \vec{J}_F$

d) ✓

e) $\vec{E}_1(z,t) = \mathrm{Re}\{\underline{E}_1(z) e^{j\omega t}\}$

f) Poynting-Vektor

$\underline{\vec{S}} = \underline{\vec{E}} \times \underline{\vec{H}}^*$

$\overline{\vec{S}} = \frac{1}{2} \mathrm{Re}\{\underline{\vec{S}}\}$

$W_{el} = \frac{1}{2} \vec{E} \vec{D} \longrightarrow \overline{W_{el}} = \frac{1}{4} \mathrm{Re}\{\underline{\vec{E}}\, \underline{\vec{D}}^*\}$

a) $\underline{\vec{H}}_e = \frac{1}{\underline{Z}} \vec{n} \times \underline{\vec{E}} = \sqrt{\frac{\varepsilon_1}{\mu_1}} \cdot (-\vec{e}_x \sin\varphi_1 + \vec{e}_z \cos\varphi_1) \times (\vec{e}_y \cdot E_{e0}\, e^{-j\vec{k}_e \vec{r}})$

$= \sqrt{\frac{\varepsilon_1}{\mu_1}} \cdot (-\vec{e}_z \sin\varphi_1 E_{e0}\, e^{-j\vec{k}_e \vec{r}} - \vec{e}_x \cos\varphi_1 \cdot E_{e0}\, e^{-j\vec{k}_e \vec{r}})$

$$\vec{H}_r = \sqrt{\frac{\varepsilon_1}{\mu_1}} E_{r0}\, e^{-jk_r\vec{r}} \left(-\vec{e}_z \sin\varphi_1 + e_x \cos(\varphi_1)\right) \checkmark$$

b) $\varepsilon_{1tan} = E_{2tan} = 0$

$$-\vec{e}_z \times \underline{\vec{H}_1} = \vec{J}_F$$

c) $E_{e0} = -E_{r0}$ ✓

d) $\varphi_1 = 0$

$$\vec{k}_e = \vec{e}_z\, \omega \sqrt{\varepsilon_1 \mu_0} = -\vec{k}_r = -\vec{e}_z\, \omega \sqrt{\varepsilon_1 \mu_0} \checkmark$$

e) $\vec{E}(z,t) = E_e(z,t) + E_r(z,t)$

$$= \mathrm{Re}\left\{ \left(\underline{\vec{E}}_e(z) \cdot \underline{\vec{E}}_r(z)\right) e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ \left(\vec{e}_y\, \varepsilon_{e0}\, e^{-j\vec{k}\vec{r}} + \vec{e}_y\, E_{r0}\, e^{jk\vec{r}}\right) e^{j\omega t} \right\}$$

$$= \left(\varepsilon_{e0} \cos(-\vec{k}\vec{r}) + \varepsilon_{r0} \cos(\vec{k}\vec{r})\right) \cos(\omega t)$$

$$= \left(\varepsilon_{e0} - \varepsilon_{e0}\right) \cos(\vec{k}\vec{r}) \cos(\omega t) = 0$$

$$\vec{H}(z,t) = \mathrm{Re}\left\{ H_e(z,t) + H_r(z,t) \right\}$$

$$= \mathrm{Re}\left\{ \sqrt{\frac{\varepsilon_1}{\mu_1}} \left(e^{-jk_e\vec{r}} (-\vec{e}_x)\, \varepsilon_{e0} + \varepsilon_{r0}\, e^{-jk_r\vec{r}} (e_x)\right) e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ \sqrt{\frac{\varepsilon_1}{\mu_1}}\, \vec{e}_x\, \varepsilon_{e0}\, 2 \left(\frac{e^{-jk_e\vec{r}} + e^{jk_r\vec{r}}}{2} \right) e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ \sqrt{\frac{\varepsilon_1}{\mu_1}}\, \vec{e}_x\, \varepsilon_{e0}\, 2 \cdot \cos(k_r \vec{r})\, e^{j\omega t} \right\}$$

$$= \sqrt{\frac{\varepsilon_1}{\mu_1}}\, \vec{e}_x\, E_{e0}\, 2 \cdot \cancel{\cos}\, \sin(k_r \vec{r}) \cos(\omega t) \quad (v)$$

a) $$\underline{\vec{H}}_e = -\frac{E_{0e}}{Z_1} e^{-j\vec{k}_e \vec{r}} (\vec{e}_x \cos\varphi_1 + \vec{e}_z \sin\varphi_1)$$

$$\underline{\vec{H}}_r = -\frac{E_{0r}}{Z_1} e^{-j\vec{k}_r \vec{r}} (\vec{e}_x \cos\varphi_1 - \vec{e}_z \sin\varphi_1)$$

b) ✓

c) $\underset{(z=0)}{\underline{\text{Grenzbed}}}$: $\vec{E}_{etan} + \underline{\vec{E}}_{rtan} = \vec{0}$

$$e^{-j\vec{k}_e\vec{r}} = e^{-j\vec{k}_r\vec{r}} \quad \text{wenn } \vec{r} = x\vec{e}_x + y\vec{e}_y$$

$$\Rightarrow E_{e0} = -E_{r0}$$

d) $\varphi_1 = 0 \Rightarrow \vec{k}_e = -\vec{k}_r$

e) $$\underline{\vec{E}}_{1ges} = -\vec{e}_y\, 2j\, E_{0e} \sin(k_e z)$$

$$\underline{\vec{H}}_{1ges} = -\vec{e}_x\, 2 \cdot \frac{E_{e0}}{Z_1} \cos(k_e z)$$

$$\vec{E}_{1ges}(t) = \vec{e}_y\, 2 \cdot E_{0e} \sin(k_e z) \sin(\omega t)$$

$$\vec{H}_{1ges}(t) = -\vec{e}_x\, 2\, \frac{E_{0e}}{Z_1} \cos(k_e z) \cos(\omega t)$$

f) $$\underline{\vec{S}} = \vec{e}_y \times \vec{e}_x \cdot 2j\, E_{e0} \sin(k_e z) \cdot 2 \frac{E_{e0}}{Z_1} \cos(k_e z)$$

$$\Rightarrow \overline{\vec{S}} = \frac{1}{2} \mathrm{Re}\{\underline{\vec{S}}\} = 0$$

[$\underline{\text{hier:}}$ $$-\frac{1}{2} \operatorname{div} \underline{\vec{S}} = 2j\omega \underbrace{(\overline{w}_m - \overline{w}_e)}_{\text{Blindleistungsdichten}} + \ldots$$

$$\nearrow \quad \overline{w(t)} = \varepsilon_1 E_{0e}^2$$

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$$\begin{cases} \overline{w_{el}(t)} = \varepsilon_1 E_{0e}^2 \sin^2(k_e z) \\ \overline{w_{mag}(t)} = \quad " \quad \cos^2(k_e z) \end{cases}$$

g) $\vec{\underline{J}}_F = 2\vec{e}_y \cdot \frac{E_{0e}}{Z_1}$

$J_F = 2\vec{e}_y \frac{E_{0e}}{Z_1} \cos(\omega t)$