KGÜ 9

Mittwoch, 22. Dezember 2010
15:58

Zur letzten KGÜ:

$$\Delta \underline{H} := \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) H$$

$$\underbrace{\Delta}\ \underline{\vec H} = \frac{\partial^2}{\partial x^2}\vec H + \frac{\partial^2}{\partial y^2}\vec H + \frac{\partial^2}{\partial z^2}\vec H$$

$$\boxed{(\vec\nabla\vec\nabla)\underline{\vec H} \neq \vec\nabla(\vec\nabla \vec H)}$$

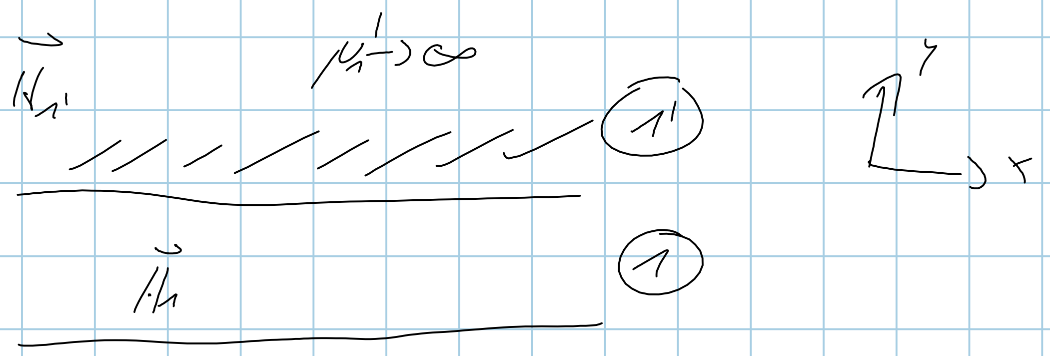
$$(\vec a \vec b)\vec c \neq \vec a(\vec b \vec c)$$

$$a(bc) = (ab)c$$

$$\Delta\vec H = \Delta H_x \vec e_x + \Delta H_y \vec e_y + \Delta H_z \vec e_z$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$$

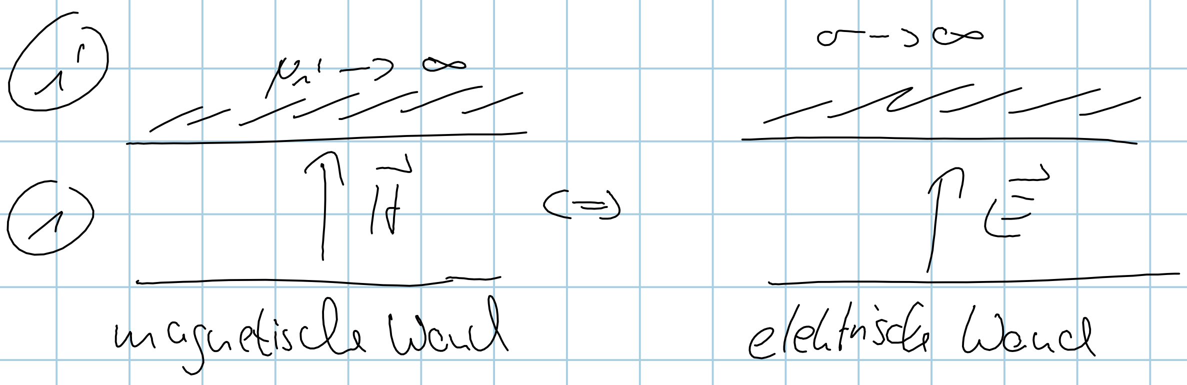
<u>H2000 A.4</u> Begründung für $\underline{\vec H} = \vec e_y \cdot \underline{H}_y(x)$



$B_{1n} = B_{1'n}$ an Grenzfläche

$$\mu_1 H_{1y} = \mu_1' H_{1'y}$$

$$\Rightarrow H_{1y} = \underbrace{\frac{\mu_1'}{\mu_1}}_{\to\infty} H_{1'y}$$



$H_t = 0$

a) $\Delta \underline{\vec{H}} - \underline{k}^2 \underline{\vec{H}} = 0 \qquad \underline{\vec{H}} = \vec{e}_y \underline{H}_y(x)$

$\mathrm{rot}\, \vec{H} = \vec{J} + \dot{\vec{D}} \qquad | \mathrm{rot}(\,) \qquad \mathrm{rot}\, \vec{E} = -\dot{\vec{B}}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = -j\omega\mu \vec{H}$

$\Rightarrow \Delta \vec{H} = \mathrm{rot}(\vec{J}) + j\omega\, \mathrm{rot}(\varepsilon \vec{E})$

$= \mathrm{rot}\, \vec{J} + j\omega\varepsilon\, \mathrm{rot}\, \vec{E}$

$= \mathrm{rot}\, \vec{J} + j\omega\varepsilon(-j\omega\mu \vec{H})$

$= \mathrm{rot}\, \underbrace{\vec{J}}_{\sigma \vec{E}} + \omega^2 \varepsilon\mu \vec{H}$

e) $\mathrm{rot}\, \underline{\vec{H}} = (\sigma_0 + j\omega\varepsilon)\underline{\vec{E}} = (1 + j\omega \underbrace{\frac{\varepsilon_0}{\tau_0}}_{\approx 0})\sigma_0 \underline{\vec{E}}$

$\Rightarrow \mathrm{rot}\,\mathrm{rot}\, \underline{\vec{H}} = \sigma_0 \cdot \mathrm{rot}\, \underline{\vec{E}} = -j\omega\mu_0\sigma_0 \underline{\vec{H}} = \mathrm{grad}\, \underbrace{\mathrm{div}\, \underline{\vec{H}}}_{=0} - \Delta \underline{\vec{H}}$

$\Rightarrow \Delta \underline{\vec{H}} - \underline{k}^2 \underline{\vec{H}} = 0 \qquad \underline{k}^2 = j\omega\mu_0\sigma_0$

$\underline{\vec{H}} = \underline{H}_y(x)\, \vec{e}_y$

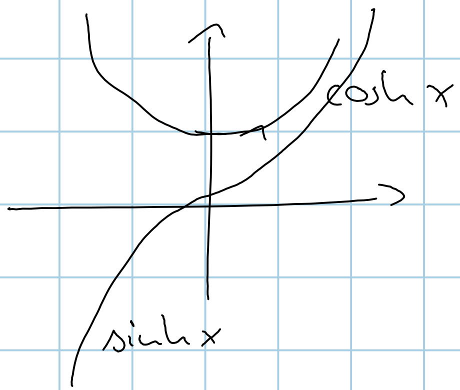
$\Rightarrow \frac{d^2}{dx^2} \underline{H}_y(x) - \underline{k}^2 \underline{H}_y(x) = 0$

$+\underline{k}x \qquad -\underline{k}x$

Allg. Lösung: $\underline{H}_y(x) = \underline{\alpha}' e^{+\underline{k}x} + \underline{\beta}' e^{-\underline{k}x}$

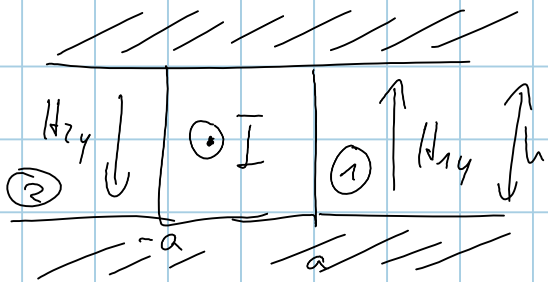
$$\underline{H}_y(x) = \underline{\alpha} \cosh(\underline{k}x) + \underline{\beta} \sinh(\underline{k}x)$$

$$\left[\cosh x = \frac{e^{x}+e^{-x}}{2}\right]$$

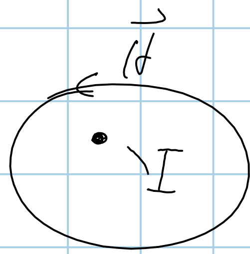


$x \rightarrow x + a$

b)



Analogie



$$\oint \underline{\vec{H}}\, d\vec{s} = \int \underline{\vec{J}}\, d\vec{F}$$

$$\int_0^h H_{1y}(x_1)\, d\vec{s} + \int_h^0 H_{2y}(x_2)\, d\vec{s} = \underline{I}$$

$$h \cdot H_{1y}(x_1) + (-H_{2y}(x_2)) = \underline{I}$$

$$2h H_{1y}(x_1) = \underline{I} \Rightarrow H_{1y} = \frac{\underline{I}}{2h}$$

$$H_{2y} = -\frac{\underline{I}}{2h}$$

c) $\underline{H}_y(x) = \alpha \cosh(\underline{k}x) + \underline{\beta} \sinh(\underline{k}x)$

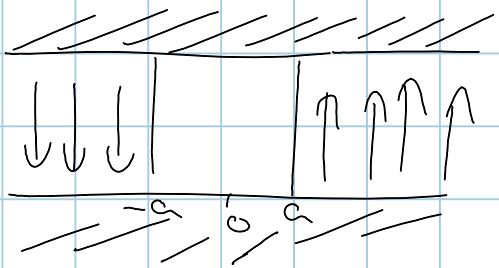
(x im Leiter)

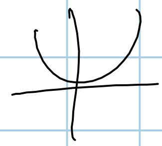
$\underline{H}_y(x) = \dfrac{\underline{I}}{2h}$ (für $x > a$)

$$\underline{H}_y(x) = -\frac{I}{2h} \quad (\text{für } x < -a)$$

$$\Rightarrow \text{für } x = a: \quad \underline{H}_y(a) = \underline{\alpha} \cosh(\underline{k}a) + \underline{\beta} \sinh(\underline{k}a) = \frac{I}{2h}$$

$$\text{"} \quad x = -a: \quad \underline{H}_y(-a) = \underline{\alpha} \cosh(-\underline{k}a) + \underline{\beta} \sinh(-\underline{k}a) = -\frac{I}{2h}$$



$$\Rightarrow \alpha = 0 \quad \text{wg. } \cosh(x) =$$ 

$$\Rightarrow \quad \underline{\beta} = \frac{I}{2h} \frac{1}{\sinh(\underline{k}a)}$$

$$\Rightarrow \underline{H}_y(x) = \frac{I}{2h} \frac{\sinh(\underline{k}x)}{\sinh(\underline{k}a)}$$

$$\vec{\underline{J}} = \vec{e}_z \frac{\partial \underline{H}(x)}{\partial x} = \frac{I}{2h} \underline{k} \frac{\cosh(\underline{k}x)}{\cosh(\underline{k}a)}$$

d)