GGÜ 4

Freitag, 17. Dezember 2010
10:07

a.) $\vec{\underline{E}}_1 = \vec{\underline{E}}_e + \vec{\underline{E}}_r$ $\qquad \vec{\underline{E}}_2 = \vec{\underline{E}}_d$

$\vec{\underline{H}}_1 = \vec{\underline{H}}_e + \vec{\underline{H}}_r$ $\qquad \vec{\underline{H}}_2 = \vec{\underline{H}}_d$

AS: $\vec{\underline{E}}_e = \vec{e}_x \, \underline{E}_{e0} \, e^{-j\underline{k}_1 z}$

$\vec{\underline{E}}_r = \vec{e}_x \, \underline{E}_{r0} \, e^{-j\underline{k}_1 z}$

$\vec{\underline{E}}_d = \vec{e}_x \, \underline{E}_{d0} \, e^{-j\underline{k}_2 z}$

$\underline{E}_{e0}$ gegeben, $\underline{E}_{r0}$ $\underline{E}_{d0}$ müssen berechnet werden

$$\underline{k}_1^2 = \omega^2 \mu_0 \varepsilon_1 - j\omega\mu_0 \sigma_1 = \omega^2 \mu_0 \left(\varepsilon_1 - j \frac{\sigma_1}{\omega}\right) = \omega^2 \mu_0 \underline{\varepsilon}_1$$

$$\underline{k}_2^2 = \omega^2 \mu_0 \varepsilon_2 - j\omega\mu_0 \sigma_2 = \omega^2 \mu_0 (\varepsilon_2 - j \cdots) = \omega^2 \mu_0 \underline{\varepsilon}_2$$

$$\vec{\underline{H}} = \frac{1}{\underline{Z}} (\vec{n} \times \vec{\underline{E}})$$

$$\Rightarrow \vec{\underline{H}}_e = \frac{1}{\underline{Z}_1} \underline{E}_{e0} \, e^{-j\underline{k}_1 z} \, \vec{e}_y$$

$$\vec{\underline{H}}_d = \frac{1}{\underline{Z}_2} \underline{E}_{d0} \, e^{-j\underline{k}_2 z} \, \vec{e}_y$$

$$\vec{\underline{H}}_r = \frac{1}{\underline{Z}_1} (\underbrace{-\vec{e}_z \times \vec{\underline{E}}_r}_{\sim \vec{e}_y}) = -\frac{1}{\underline{Z}_1} \underline{E}_{r0} \, e^{j\underline{k}_1 z} \, \vec{e}_y$$

$$\underline{Z}_1 = \sqrt{\frac{\mu_0}{\underline{\varepsilon}_1}} = \frac{\omega\mu_0}{\underline{k}_1} \qquad \underline{Z}_2 = \sqrt{\frac{\mu_0}{\underline{\varepsilon}_2}} = \frac{\omega\mu_0}{\underline{k}_2}$$

1) $\operatorname{Div} \vec{\underline{B}} = 0 \Rightarrow \vec{\underline{H}}_{\text{normal}}$ stetig

$= \operatorname{Div}(\mu_0 \vec{\underline{H}}) = \mu \operatorname{Div}(\vec{\underline{H}})$

2) $\operatorname{Div} \vec{\underline{D}} = \rho_F$ $\quad\quad$ ρ_F ist unbekannt

3.) $\operatorname{Rot} \vec{\underline{E}} = 0 \Rightarrow \underline{E}_{tan}$ stetig

$$\underline{E}_{1x}\big|_{z=0} = \vec{\underline{E}}_1 \vec{e}_x = \underline{E}_{2x}\big|_{z=0} = \vec{\underline{E}}_2 \vec{e}_x$$

4.) $\operatorname{Rot} \vec{\underline{H}} = \vec{0} \Rightarrow \vec{\underline{H}}_{tan}$ stetig

$$\vec{\underline{H}}_1 \vec{e}_y\big|_{z=0} = \vec{\underline{H}}_2 \vec{e}_y\big|_{z=0}$$

Betrachte GB bei $\vec{r} = 0$

$$\underline{E}_{e0} + \underline{E}_{r0} = \underline{E}_{d0}$$

$$\frac{1}{\underline{Z}_1} \underline{E}_{e0} - \frac{1}{\underline{Z}_1} \underline{E}_{r0} = \frac{1}{\underline{Z}_2} \underline{E}_{d0} \quad\quad \| \, \underline{Z}_1$$

$$2 \underline{E}_{e0} = \underline{E}_{d0} \left(1 + \frac{\underline{Z}_1}{\underline{Z}_2}\right) = \underline{E}_{d0} \frac{\underline{Z}_1 + \underline{Z}_2}{\underline{Z}_2}$$

$$\Rightarrow \underline{E}_{d0} = \frac{2 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \cdot \underline{E}_{e0}$$

$$\underline{E}_{r0} = \underline{E}_{d0} - \underline{E}_{e0} = \underline{E}_{e0} \left(\frac{2 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} - 1 \right)$$

$$\Rightarrow \underline{E}_{r0} = \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1} \underline{E}_0$$

$$\vec{\underline{E}}_r = \vec{e}_x \, \underline{E}_{e0} \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1} e^{j \underline{k}_1 z}$$

$$\vec{\underline{H}}_r = -\vec{e}_y \frac{\underline{E}_{e0}}{\underline{Z}_1} \cdot \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1} e^{j \underline{k}_1 z}$$

$$\vec{\underline{E}}_d = \vec{e}_x \, \underline{E}_{e0} \cdot \frac{2 \underline{Z}_2}{\underline{Z}_2 + \underline{Z}_1} e^{-j \underline{k}_2 z}$$

$$-j k_2 z$$

$$\underline{\vec{H}}_d = \vec{e}_y \, \underline{E}_{e0} \, \frac{2}{\underline{Z}_2 + \underline{Z}_1} \, e^{-jk_2 z}$$

$$\underline{r} = \frac{\underline{\vec{E}}_r \vec{e}_x}{\underline{E}_e \vec{e}_x} = \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1}$$

$$\underline{t} = \frac{\underline{\vec{E}}_d \vec{e}_x}{\underline{\vec{E}}_e \vec{e}_x \,|_{z=0}} = \frac{2 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

$$\underline{t} = \frac{\underline{Z}_2 + \underline{Z}_1 + \underline{Z}_2 - \underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} = 1 + \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1} = 1 + \underline{r}$$

$$\underline{t} - \underline{r} = 1$$

c) Fall 1 : $\sigma_1 = \sigma_2 = 0$

$\underline{k}_1 = k_1 \qquad \underline{Z}_{1,2} = Z_{1,2}$

$\underline{k}_2 = k_2$

$$r = \frac{\frac{1}{\sqrt{\varepsilon_2}} - \frac{1}{\sqrt{\varepsilon_1}}}{\frac{1}{\sqrt{\varepsilon_2}} + \frac{1}{\sqrt{\varepsilon_1}}} \quad | \cdot \sqrt{\varepsilon_1 \varepsilon_2}$$

Optik: $n: \sqrt{\frac{\varepsilon}{\varepsilon_0}}$ Brechungsindex

$$= \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t = \frac{2 n_1}{n_1 + n_2}$$

Fall 2: $\sigma_1 = 0 \quad \underline{k}_1 = \underline{k}_1 \quad \underline{Z}_1 = \underline{Z}_1$

$$\sigma_2 >> \omega \underline{\varepsilon}_2 \Rightarrow \omega \frac{\varepsilon_2}{D_2} << 1$$

$$\uparrow$$
$$\tau_2$$

$$\underline{k}_2 = \frac{1}{a_2}(1-j) \quad \text{mit } a_2 = \sqrt{\frac{2}{\mu_0 \omega \sigma_2}} \quad \text{aus A.5}$$

$$\underline{Z}_2 = \frac{\omega \mu_0}{\underline{k}_2} = \omega \mu_0 \sqrt{\frac{2}{\mu_0 \sigma_2}} \frac{1+j}{(1-j)(1+j)} = \sqrt{\frac{\omega \mu_0}{\sigma_2}} \frac{1+j}{\sqrt{2}}$$

$$= \sqrt{\frac{\omega \mu_0}{\sigma_2}} \cdot e^{j\frac{\pi}{4}}$$

$$\varepsilon_1 = \varepsilon_2$$

$$\frac{\underline{Z}_2}{\underline{Z}_1} = \sqrt{\frac{\omega \varepsilon_1}{\sigma_2}}\, e^{j\frac{\pi}{4}} = \sqrt{\omega \tau_2}\, e^{j\frac{\pi}{4}}$$

$$\underline{t} = \frac{2\,\underline{Z}_2/\underline{Z}_1}{1 + \underline{Z}_2/\underline{Z}_1} = \frac{2\sqrt{\omega\tau_2}\, e^{j\frac{\pi}{4}}}{1 + \underbrace{\sqrt{\omega\tau_2}}_{\omega\tau_2 \ll 1}\, e^{j\pi/4}}$$

$$\underline{t} = 2\sqrt{\omega\tau_2}\, e^{j\frac{\pi}{4}} \qquad |\underline{t}| \ll 1$$

$$\underline{r} = \underline{t} - 1 \approx 2\sqrt{\omega\tau_2}\, e^{j\frac{\pi}{4}} - 1 = -1 + \delta$$

$$\delta = 2\sqrt{\omega\tau_2}\, e^{j\frac{\pi}{4}} \;;\; |\delta| \ll 1$$

$$\underline{r} \approx -1$$

$$\sigma_2 \to 0 \implies \underline{t} = 0$$
$$\underline{r} = -1$$

Kl. Aufg. 2 Herbst 2010

- Strom in positive z-Richtung

$$I = I_0 \sin(\omega t)$$

- $d << L$
- Leitfähigkeit σ_0
  => keine Rückwirkung
- ω klein => keine Welleneffekte
- $\vec{A} = |A| \vec{e}_y$

a) $\oint H ds = I$ umschlossen

$$H \cdot 2\pi r = I_0 \sin(\omega t)$$

$$\vec{H}(r,t) = H \cdot (-\vec{e}_x)$$

$$= -\frac{I_0}{2\pi r} \sin(\omega t) \vec{e}_x$$

b) Strecke 12341 entspricht
Umlauf nach rechter Hand bei Orientierung
des Normalen nach $\vec{A}$

$$\int \frac{1}{r} = [\ln r]$$

$$\phi = \iint \vec{B} d\vec{A} = \mu_s \int_0^L \int_{L}^{2L} H \, dy \, dz$$

$$= -\mu_0 \frac{I_0}{2\pi} \sin(\omega t) L$$

c) U ? $\oint \vec{E} d\vec{s}$

$$\text{rot } \vec{E} = \vec{H}$$

$$\frac{\partial}{\partial z} E_\rho - \frac{\partial}{\partial \rho} E_z = H_\varphi$$

$$\oint \vec{E} d\vec{s} = U_{\text{Umlauf}} = -\frac{\partial \phi}{\partial t}$$

$$U_{12341} = -\frac{\partial \Phi_{12341}}{\partial t}$$

$$= +\mu_0 \frac{I_0 \omega}{2\pi} \cos(\omega t) L \cdot \ln(2)$$

$$U_{12561} = \mu_0 \frac{I_0 \omega}{2\pi} \cos(\omega t)\; L \cdot \ln(3)$$

d) Spannung fällt an der Widerstandsplatte ab

$$\vec{J} = J_z(y)\, \vec{e}_z \qquad \vec{J} = \sigma_0 \vec{E}$$

$$\vec{E} = E_z(y)\, \vec{e}_z$$

$$E_z(y) = \frac{U(y)}{\Delta x} = \frac{U(y)}{L}$$

$$U(y) = \frac{\mu_0 I_0}{2\pi} \omega \cos(\omega t) L \cdot \ln\left(\frac{y}{1L}\right)$$

$$J_z(y) = \frac{\mu_0 I_0 \sigma_0}{2\pi} \omega \cos(\omega t) \ln\left(\frac{y}{L}\right)$$

$$I = \iint \vec{J} \cdot d\vec{A}_{Platte,\,xy}$$

$$= d \mu_0 \sigma_0 \frac{I_0 \omega}{2\pi} \cos(\omega t) \cdot \int_{2L}^{3L} \ln\left(\frac{y}{1L}\right) dy$$

$$L \int_{y=2L}^{y=3L} \ln\underbrace{\left(\frac{y}{L}\right)}_{t} \underbrace{\frac{dy}{L}}_{dt} = \left[1L\left(\frac{y}{L} \ln\frac{y}{L} - \frac{y}{L}\right)\right]_{2L}^{3L}$$

$$= 1L\,(3\ln 3 - 3 - (2\ln 2 - 2))$$

$$= 1L\,(3\ln 3 - 2\ln 2 - 1)$$

$$I = d \mu_0 \sigma_0 \frac{I_0\, \omega \cos(\omega t)}{2\pi} L$$

$$(3\ln 3 - 2\ln 2 - 1)$$

e) die Ströme bzw. Spannungen können überlagert werden

$\underline{I}_Q = \underline{I}_{ind}$ mit $\underline{I}_Q = \frac{U_Q}{R_{DC}}$

$U_Q = \underline{I}_{ind}\, R_{DC}$

$$R_{DC} = \frac{1}{\sigma_0}\, \frac{L}{A_{xy}\, \cancel{L \cdot d}} = \frac{1}{\sigma_0 d}$$

$$\underline{\underline{U_Q = \mu_0 \frac{\underline{I}_0 \omega}{2\pi} \cos(\omega t)\, L \cdot (3\ln 3 - 2\ln 2 - 1)}}$$

$$J_{z\,ges} = J_{z\,ind} + J_{zQ} = \frac{\mu_0 \sigma_0 \underline{I}_0}{2\pi}\, \omega \cos(\omega t) \ln \frac{y}{L}$$

$$- \frac{\mu_0 \sigma_0 \underline{I}_0}{2\pi}\, \omega \cos(\omega t) \cdot (3\ln 3 - 2\ln 2 - 1)$$

$$= \frac{\mu_0 \sigma_0 \underline{I}_0}{2\pi}\, \omega \cos(\omega t) \cdot \left(\ln \frac{y}{L} - 3\ln 3 + 2\ln 2 + 1\right)$$