KGÜ7

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15:48

$\underline{\vec{J}} = \sigma \underline{\vec{E}}$ im Zwischenraum $\underline{\vec{E}} \sim \vec{e}_\rho$

$\text{Rot}\, \underline{\vec{H}} = \underline{\vec{J}}_F$ Innen- bzw. Außenleiter $\underline{\vec{J}}_F \sim \vec{e}_z$

a) $\underline{\vec{H}} = \underline{H}_\varphi \cdot \vec{e}_\varphi$ und $\underline{H}_\varphi = \underline{h}(\rho)\, e^{-\gamma z}$

$$\text{rot}\, \underline{\vec{H}} = -\frac{\partial \underline{H}_\varphi}{\partial z} \vec{e}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \underline{H}_\varphi) \vec{e}_z$$

b) $\Delta \underline{\vec{H}} + \underline{k}^2 \underline{\vec{H}} = \vec{0}$ Helmholtz-Gleichung

$$\Delta \underline{\vec{H}} = \frac{\partial}{\partial \rho}\left(\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \underline{H}_\varphi)\right) \vec{e}_\varphi + \frac{\partial^2}{\partial z^2} \underline{H}_\varphi \vec{e}_\varphi$$

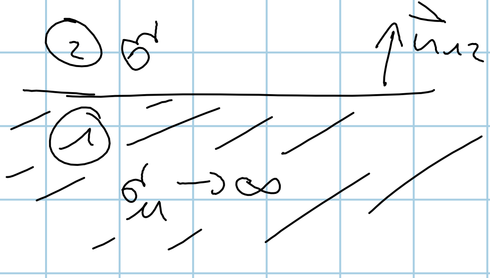
$\sim \vec{e}_\varphi$

Vektorfeld

c) (Volumen-)stromdichte : $J(u) = \frac{J}{F}$

Flächenstromdichte : $J_F = \frac{J}{L}$

Grenzbeleg für $\underline{\vec{H}}$:



$\vec{n}_{12} \times (\underline{\vec{H}}_2 - \underline{\vec{H}}_1) = \underline{\vec{J}}_F$
$= 0$

$(\vec{e}_\rho \times \vec{e}_\varphi = \vec{e}_z)$

d) $\underline{\vec{H}} = \frac{1}{\underline{Z}_F} (\vec{n} \times \underline{\vec{E}})$

$\Rightarrow \underline{\vec{S}} = \underline{\vec{E}} \times \underline{\vec{H}}^* = \underline{Z}_F \cdot |\underline{\vec{H}}|^2 \vec{n}$

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\, (\vec{a}\vec{c}) - \vec{c}\, (\vec{a}\vec{b})$

$$\langle \underline{a} | \underline{b} \rangle = \underline{a}_x \underline{b}_x^* + \underline{a}_y \underline{b}_y^* + \underline{a}_z \underline{b}_z^*$$

a) $\text{rot}\, \vec{\underline{H}} = \sigma \vec{\underline{E}}$

$$\text{rot}\, \vec{\underline{H}} = \text{rot}\big(\underline{h}(\rho) \exp(-\gamma z) \vec{e}_\varphi\big)$$
$$= -h(\rho)(-\gamma)\exp(-\gamma z)\vec{e}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \rho}\, \rho \underline{h}(\rho)\exp(-\gamma z)\vec{e}_\varphi$$

$$\vec{E} = \frac{h(\rho)}{\sigma}\gamma e^{-\gamma z}\vec{e}_\rho + \frac{1}{\sigma\rho}\frac{\partial}{\partial\rho}\rho h(\rho) e^{-\gamma z}\vec{e}_z$$

$$= h(\rho)\frac{\gamma}{\sigma}e^{-\gamma z}\vec{e}_\rho + \frac{1}{\sigma\rho}h(\rho)e^{-\gamma z}\vec{e}_z$$
$$+ \frac{1}{\sigma\cancel{\rho}}\cdot\cancel{\rho}\cdot h'(\rho)e^{-\gamma z}\vec{e}_z$$

$$= h(\rho)\frac{1}{\sigma}\left(\gamma e^{-\gamma z}\vec{e}_\rho + \frac{1}{\rho}e^{-\gamma z}\vec{e}_z\right) + \frac{1}{\sigma}h'(\rho)e^{-\gamma z}\vec{e}_z$$

inhomog. 1. Ordn

$$h(\rho) = A e^{-\lambda\rho} + B e^{\lambda\rho}$$

c) $\text{rot}\, \vec{\underline{H}} = \vec{\underline{J}} + \dot{\vec{\underline{D}}}$ mit $\vec{\underline{J}} = \sigma \vec{\underline{E}}$

$$\vec{\underline{E}} = \frac{1}{j\omega\varepsilon + \sigma}\,\text{rot}\, \vec{\underline{H}}$$

$$\Rightarrow \vec{\underline{E}} = \frac{\gamma}{j\omega\varepsilon + \sigma}\underline{h}(\rho)e^{-\gamma z}\vec{e}_\rho$$

$$j\omega\varepsilon+\sigma \quad \frac{1}{\rho}\frac{d}{d\rho}(\rho \underline{h}(\rho))\, e \quad e_z$$

TEM: $\underline{E}_z = 0 \Rightarrow \frac{d}{d\rho}(\underbrace{\rho\, \underline{h}(\rho)}_{\text{konst.}}) = 0$

$$\Rightarrow \underline{h}(\rho) = \frac{1}{\rho}$$

b) $\Delta \vec{\underline{H}} + \underline{k}^2 \vec{\underline{H}} = 0$ mit $\underline{k}^2 = -j\omega\mu(j\omega\varepsilon + \sigma)$

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}(\rho\, \underline{h}(\rho))\right] + \gamma^2 \underline{h}(\rho) + \underline{k}^2 \underline{h}(\rho) = 0$$

$\Rightarrow \gamma^2 = -\underline{k}^2$ allgemein für TEM-Wellen! $(\rho^2 = 0)$

c) elektr. Wand: $(\rho = a)$ $\vec{\underline{E}}_{tan} = \vec{0}$ ✓ nur $\underline{E}_\rho \neq 0$ u. $\underline{E}_z = 0$

$\vec{\underline{J}}_{Fa} = \frac{\underline{I}_0}{2\pi a} e^{-\gamma z} \vec{e}_z$ Flächenstromdichte bei $\rho = a$

Grenzbedingung für $\vec{\underline{H}}$ bei $\rho = a$:

$$-\underline{H}_{tan} = \underbrace{\vec{n}_{12}}_{e_\rho} \times \vec{\underline{J}}_{Fa} = -\frac{\underline{I}_0}{2\pi a} e^{-\gamma z} \vec{e}_\varphi$$

$$\Rightarrow \underline{A} = \frac{I_0}{2\pi}$$

Grenzbed. für $\vec{\underline{H}}$ bei $\rho = b$: $\vec{n}_{12} = -\vec{e}_\rho$

$$\vec{\underline{J}}_{Fb} = -\frac{\underline{I}_0}{2\pi b} e^{-\gamma z} \vec{e}_z$$

d) $\underline{Z}_F = \frac{\underline{E}_\rho}{\underline{H}_\varphi} = \sqrt{\frac{j\omega\mu}{\underbrace{j\omega\varepsilon + \sigma}}}$

$$\frac{\mu\omega}{\underbrace{\sqrt{j\omega\varepsilon+\sigma}}}$$

$$\underline{\vec{S}} = \sqrt{\varepsilon} \cdot \frac{|\underline{J}_0|^2}{4\pi^2\rho^2}\, e^{-2\alpha z}\, \vec{e}_z$$

$\underline{\underline{\omega\varepsilon \gg \sigma}}$

$$\gamma \approx \underbrace{j\omega\sqrt{\varepsilon\mu}}_{\beta} + \underbrace{\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}_{\alpha}$$

$$v_{ph} = \frac{\omega}{\beta} = \frac{\lambda}{T} = \lambda f = \frac{1}{\sqrt{\varepsilon\mu}} \neq v_{ph}(\omega)$$

<u>keine</u> Dispersion!