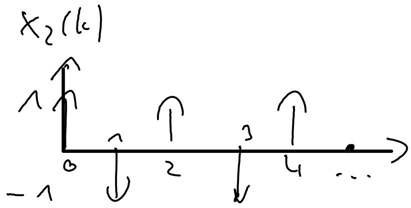
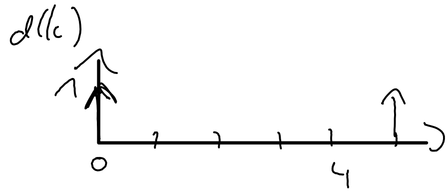
GGÜ 6

Donnerstag, 25. November 2010
09:47

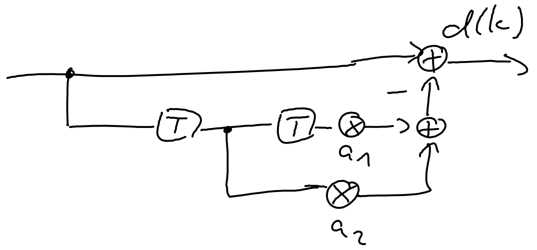
A.4.3)

c)





d) $x(k)$



Normalengleichungen:

$$\begin{pmatrix} \varphi_{x_1x_1}(1) \\ \varphi_{x_1x_1}(2) \end{pmatrix} = \begin{pmatrix} \varphi_{xx}(0) & \varphi_{xx}(1) \\ \varphi_{xx}(1) & \varphi_{xx}(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Rightarrow \begin{matrix} a_1 - a_2 = -1 \\ -a_1 + a_2 = 1 \end{matrix} \quad \Rightarrow \quad a_2 = a_1 + 1$$

a_1 beliebig

z. B. $a_1 = -1$, $a_2 = 0$

$a_1 = 0$, $a_2 = 1$

$$H(\Omega) = 1 - a_1 e^{-j\Omega} - a_2 e^{-j2\Omega}$$
$$= 1 - a_1 e^{-j\Omega} - e^{-j2\Omega} - a_1 e^{-j2\Omega}$$

$$\Rightarrow |H(\Omega = 0)| = |1 - a_1 - 1 - a_1| = 2|a_1|$$

$$|H(\Omega = \pi)| = |1 + a_1 - 1 - a_1| = 0$$

e)
$$\begin{pmatrix} \varphi_{x_2x_2}(1) \\ \varphi_{x_2x_2}(2) \end{pmatrix} = \begin{pmatrix} \varphi_{xx}(0) & \varphi_{xx}(-1) \\ \varphi_{xx}(1) & \varphi_{xx}(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

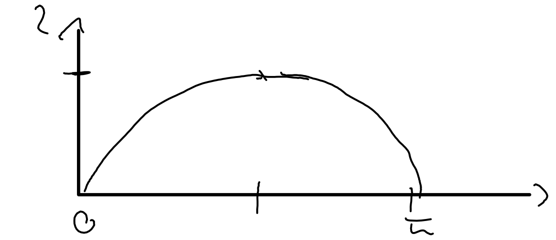
$$\begin{pmatrix} \varphi_{x_2x_2}(1) \\ \varphi_{x_2x_2}(2) \end{pmatrix} = \begin{pmatrix} \varphi_{xx}(0) & \varphi_{xx}(1) \\ \varphi_{xx}(1) & \varphi_{xx}(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \Rightarrow a_1 + a_2 = 1$$

aus d) $a_2 = 1 + a_1 \Rightarrow \begin{matrix} a_1 = 0 \\ a_2 = 1 \end{matrix}$

f) $H(\Omega) = 1 - e^{-2j\Omega} = e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$

$|H(\Omega)| \Rightarrow |H(\Omega)| = |2 \sin \Omega|$



A. 4.4

a) Gesucht a_{opt}

$$E\{d^2(\Omega)\} \to \min$$

$$E\{d^2(\Omega)\} = E\{(x(\Omega) - a\,x(k-1))^2\}$$
$$= \varphi_{xx}(0) - 2a\,\varphi_{xx}(1) + a^2 \varphi_{xx}(0)$$

$$\frac{d\,E\{d^2(\Omega)\}}{da} = 0 - 2\varphi_{xx}(1) + 2a\,\varphi_{xx}(0) \overset{!}{=} 0$$

$$\Rightarrow a_{opt} = \frac{\varphi_{xx}(1)}{\varphi_{xx}(0)}$$

$$\frac{d^2 E\{d^2(\Omega)\}}{da^2} = 2\varphi_{xx}(0) > 0 \Rightarrow TP$$

$$\Rightarrow a_{opt} = \frac{2\,si(\pi/2)}{2\,si(\pi/2 \cdot 0)} = \frac{\sin(\pi/2)}{\pi/2} = \frac{2}{\pi} = 0{,}637$$

b) Gesucht: Prädiktionsgewinn

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$$G_p = \frac{E\{x^2\}}{E\{d^2\}} = \frac{\varphi_{xx}(0)}{\varphi_{xx}(0) - 2a\varphi_{xx}(1) + a^2 \varphi_{xx}(0)}$$

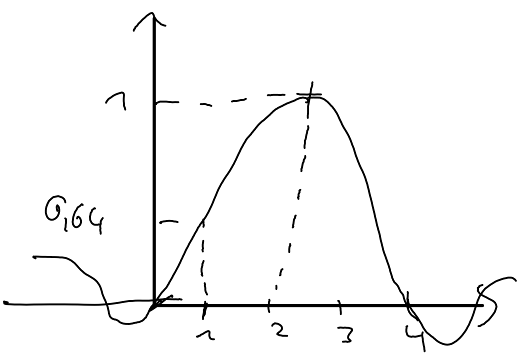
für $a = a_{opt}$

$$\Rightarrow G_p = \frac{1}{1 - 2\frac{\varphi_{xx}(1)}{\varphi_{xx}(0)} \cdot \frac{\varphi_{xx}(1)}{\varphi_{xx}(0)} + \frac{\varphi_{xx}^2(1)}{\varphi_{xx}^2(0)} \cdot 1}$$

$$= \frac{1}{1 - a_{opt}^2}$$

$$G_{p\,dB} = 10 \log_{10}\left(\frac{1}{1 - a_{opt}^2}\right) = 2{,}26\,dB$$

c) Gesucht: d_{max}, N_q



$d(1) = x(1) - a_{opt}x(0) = 0{,}64$

$d(2) = x(2) - a_{opt}x(1) = 1 - 0{,}64 \cdot 0{,}64 = 0{,}59$

$d(3) = x(3) - a_{opt}x(2) = 0{,}64 - 0{,}64 \cdot 1 = 0$

$d_{max} = 0{,}64$

$5\,bit \Rightarrow w = 5 \qquad \Rightarrow \Delta d = \frac{2 d_{max}}{2^w} = \frac{0{,}64 \cdot 2}{2^5} = 0{,}04$

$$\Rightarrow N_q = \frac{\Delta d^2}{12} = 1{,}33 \cdot 10^{-4}$$

d) Gesucht: $N_{out} = \varphi_{zz}(0)$

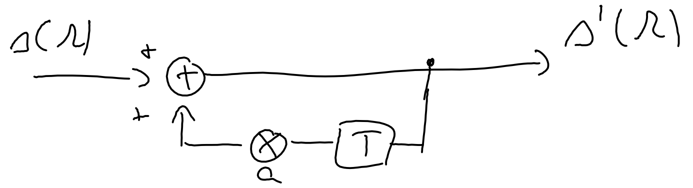
Wiener-Lee-Beziehung

$$\varphi_{zz}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\Delta\Delta}(\Omega)\, |G(\Omega)|^2\, d\Omega$$

$\Delta(\Omega)$: gleichverteilt, weiß

$\Rightarrow S_{\Delta\Delta}(\Omega) = N$ f. alle Ω

$\Delta(\Omega)$ $\longrightarrow$ $\Delta'(\Omega)$



$$s'(n) = s(n) + a\, s'(n-1)$$

$$s'(z) = s(z) + a\, s(z)\, z^{-1}$$

$$\Rightarrow G(z) = \frac{s'(z)}{s(z)} = \frac{1}{1-a z^{-1}} \Rightarrow |G(\Omega)| = \frac{1}{\sqrt{(1-a\cos(\omega))^2 + a^2 \sin^2(\Omega)}}$$

$$= \frac{1}{\sqrt{1-2a\cos\Omega + a^2}}$$

$$\Rightarrow \varphi_{zz}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} N \frac{1}{1+a^2-2a\cos\Omega}$$

mit $b = (1+a^2)$ und $c = 2a$

$$\Rightarrow \varphi_{zz}(0) = \frac{N}{\sqrt{(1+a^2)-4a^2}} = \frac{N}{\sqrt{1-2a^2+a^4}}$$

$$= \frac{N}{1-a^2} = 2{,}22 \cdot 10^{-4}$$

e) Rückwärtsprädiktion

$\Rightarrow$ Quant-Rauschen am Empfänger identisch mit Q-Rauschen am Sender

$$\Rightarrow \varphi_{zz}(0) = N_q = 1{,}33 \cdot 10^{-4}$$