V 4

Freitag, 26. November 2010
10:02

Grenzfälle der Maxwell-Gl.

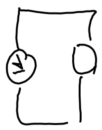
a)
$$\operatorname{rot}\vec{H} = \vec{J} + \dot{\vec{D}}$$
$$\operatorname{rot}\vec{E} = -\dot{\vec{B}}$$
$$\operatorname{div}\vec{D} = \rho$$
$$\operatorname{div}\vec{B} = 0$$

b) Elektrostatisches Feld
$$\vec{J} = \dot{\vec{D}} = 0 \Rightarrow \vec{H} = \vec{0}$$
$$\operatorname{rot}\vec{E} = \vec{0}$$
$$\operatorname{div}\vec{D} = \rho$$

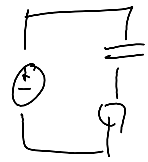
c) stationäres Strömungsfeld
$$\operatorname{rot}\vec{H} = \vec{J} \qquad \dot{\vec{D}} = 0$$
$$\operatorname{rot}\vec{E} = \vec{0} \qquad \dot{\vec{B}} = 0$$
$$\operatorname{div}\vec{D} = \rho$$
$$\operatorname{div}\vec{B} = 0$$

d) Quasistatischer Grenzfall
$$\operatorname{rot}\vec{E} = \vec{0}$$
$$\operatorname{div}\vec{D} = \rho$$
$$\vec{H} = \vec{0}\ ;\ \vec{B} = \vec{0}\ ,\ \dot{\vec{B}} = \vec{0}$$

e) Quasistationärer Grenzfall
$$\operatorname{rot}\vec{H} = \vec{J} \qquad |\dot{\vec{D}}| \ll |\vec{J}|$$
$$\operatorname{rot}\vec{E} = \vec{0} \qquad \dot{\vec{B}} = \vec{0}$$
$$\operatorname{div}\vec{D} = \rho$$
$$\operatorname{div}\vec{B} = \vec{0}$$

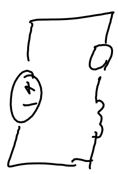


f) Wirbelstromfreier Grenzfall
$$\operatorname{rot}\vec{H} = \vec{J} + \dot{\vec{D}}$$
$$\operatorname{rot}\vec{E} = \vec{0} \qquad |\dot{\vec{B}}| \approx 0$$
$$\operatorname{div}\vec{D} = \rho$$
$$\operatorname{div}\vec{B} = 0$$



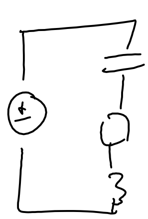
$$\dot{\vec{D}} \approx \vec{0}$$

g) Verschiebungsstromfreier Grenzfall
$$\operatorname{rot}\vec{H} = \vec{J} \qquad |\dot{\vec{D}}| \approx \vec{0}$$
$$\operatorname{rot}\vec{E} = -\dot{\vec{B}}$$
$$\operatorname{div}\vec{D} = \rho$$
$$\operatorname{div}\vec{B} = 0$$

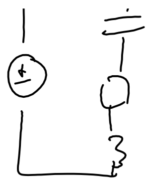


$$\dot{\vec{B}} \neq 0$$

h)



$$\dot{\vec{D}} \neq 0,\ \vec{B} = \vec{0}$$
$$\dot{\vec{B}} \neq 0\ \dot{\vec{D}} \neq \vec{0}$$



$D\neq 0, B=0$

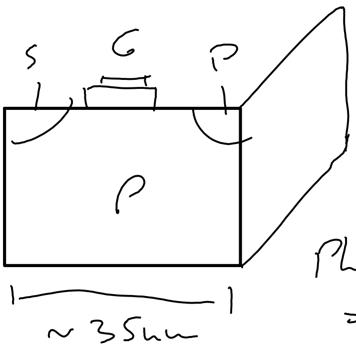
$\dot{\vec{B}}\neq 0 \quad \dot{\vec{D}}\neq\vec{0}$

HEZ: $c=\lambda f \qquad c=\frac{1}{\sqrt{\varepsilon\mu}}\sim\frac{1}{\sqrt{\varepsilon_r}}$

$\lambda >> d$

1GHz : $\lambda = 30\,cm$

Bsp: Kondensator << 30cm bzw 3cm (mit Materialabhängigem λ)



Phänomene Vernachlässigbar in MOSFETS
=> keine vollständige Maxwell-Gl. nötig

12 Felder in leitenden Medien

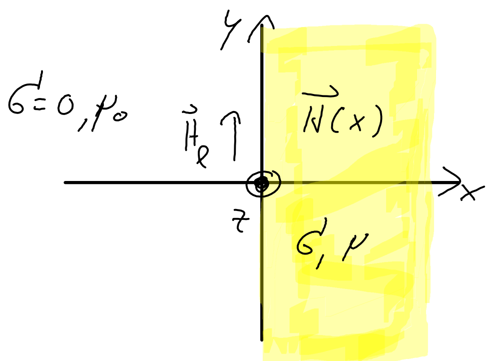
12.1 Skineffekt

$$\vec{G}=\vec{J}+\dot{\vec{D}}$$

HEZ $\quad \underline{\vec{G}}=\underline{\vec{J}}+j\omega\underline{\vec{D}}=(\sigma+j\omega\varepsilon)\underline{\vec{E}} \qquad \sigma\mu \hat{=} \text{const.}$

$\sigma >> \omega\varepsilon \rightarrow |\vec{J}| >> |\dot{\vec{D}}|$

$\mathrm{rot}\,\vec{H}=\vec{J}$



$$\vec{H}_L = H_0(t)\cdot\vec{e}_y$$

$x\geq 0$

$$\mathrm{rot}\,\vec{H}=\mathrm{rot}(H_y(x)\vec{e}_y)$$

$$=\frac{\partial H_y}{\partial x}\vec{e}_z=\vec{J}=J_z(x)\vec{e}_z$$

$$\vec{H}(x)=H_y(x)\vec{e}_y$$

$$\underbrace{\mathrm{rot}\,\mathrm{rot}}_{\mathrm{grad}\,\mathrm{div}\vec{H}-\Delta\vec{H}}\vec{H}=\mathrm{rot}\,\vec{J}=\sigma\,\mathrm{rot}(\vec{E})=-\sigma\dot{\vec{B}}=-\sigma\mu\dot{\vec{H}}$$

$$\Rightarrow \Delta\vec{H}-\sigma\mu\dot{\vec{H}}=\vec{0}$$

$$\underbrace{\text{grad div}\,\vec{H}}_{=0} - \Delta\vec{H} \qquad \Rightarrow \Delta\vec{H} - \sigma\mu\dot{\vec{H}} = \vec{0}$$

$$\Delta\vec{E} = -\sigma\mu\dot{\vec{E}} = \vec{0}$$

$$\Delta\vec{J} - \sigma\mu\dot{\vec{J}} = \vec{0}$$

HEZ: $\Delta\underline{\vec{J}} - j\omega\sigma\mu\underline{\vec{J}} = \vec{0}$

$$\underline{\vec{J}} = J_z(x)\,\vec{e}_z \qquad \frac{\partial^2}{\partial x^2}J_z - j\omega\sigma\mu J_z = 0$$

$$J_z(x) = \underline{A}\,e^{+\lambda x} + \underline{B}\,e^{-\lambda x}$$

$$\underline{\lambda}^2 - j\omega\sigma\mu = 0$$

$$\underline{\lambda} = \pm\sqrt{j\omega\sigma\mu} \qquad \sqrt{j} = \frac{1+j}{\sqrt{2}}$$

$$\lambda = \pm\frac{1+j}{\cancel{\sqrt{2}}}\sqrt{\cancel{2}\pi f\sigma\mu} = \pm\frac{(1+j)}{a}$$

$$a = \frac{1}{\sqrt{\pi f\sigma\mu}}$$

$$\underline{\vec{J}}(x) = \underline{J_0}\,e^{-\frac{1+j}{a}}\,\vec{e}_z$$

$$\text{rot}\,\underline{\vec{J}} = \text{rot}\,\sigma\underline{\vec{E}} = -j\omega\mu\sigma\underline{\vec{H}}$$

$$\underline{\vec{H}} = -\frac{1}{j\omega\mu\sigma}\text{rot}\,\underline{\vec{J}} = -\frac{1}{j\omega\sigma\mu}\,\frac{dJ_z}{dx}\,\vec{e}_y$$

$$= -\frac{1}{j\omega\mu\sigma}\,\frac{1+j}{a}\,\underline{J_0}\exp\left(-\frac{1+j}{a}x\right)\vec{e}_y$$

$$\text{Rot}\,\vec{H}\Big|_{x=0} = \vec{0} \qquad H_{y,e}(x=0) = H_0 = H_y(0) = -\frac{1}{j\omega\sigma\mu}\,\frac{1+j}{a}\,\underline{J_0}$$

$$= +\frac{a^2}{2j}\,\frac{(\cancel{1}-j)}{a}\,\underline{J_0}$$

$$\underline{J_0} = -\frac{1+j}{a}H_0$$

$$\underline{I}' = \int_0^\infty \underline{J_z}(x)\,dx = \underline{J_0}\int_0^\infty e^{-\frac{1+j}{a}x}\,dx$$

$$= \quad a \quad - \quad (1$$

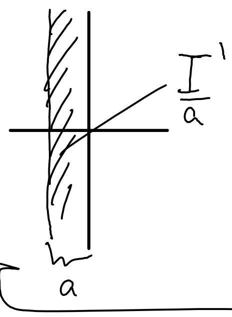
mit $\underline{J}_0 = - \nu_0$

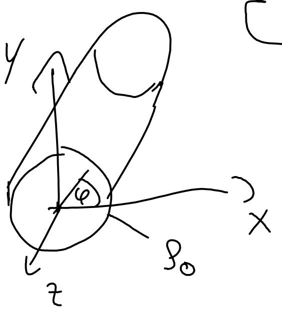
$$p_{Joule} = \frac{1}{2} \mathrm{Re}\{\underline{\vec{E}}\, \underline{\vec{J}}^*\} = \frac{1}{2\sigma} |\underline{\vec{J}}|^2 = \frac{1}{2\sigma} |\underline{J}_z|^2$$

$$P'_{Joule} = \int_0^\infty p_{Joule}\, dx = \frac{1}{2\sigma} \int_0^\infty |\underline{J}_0|^2\, e^{-\frac{1+j}{a}x}\, e^{-\frac{1-j}{a}x}\, dx$$

$$= \frac{1}{2\sigma} \int_0^\infty |\underline{J}_0|^2\, e^{-\frac{2}{a}x}\, dx = \frac{a}{4\sigma} |\underline{J}_0|^2 = \frac{1}{2} |\underline{I}'|^2 R_{eq}$$

$$R_{eq} = \frac{1}{\sigma a}$$





$$\vec{H}(\vec{r}) = H_\varphi(\rho, t)\, \vec{e}_\varphi$$

$$\mathrm{rot}\, \vec{E} = -\mu \dot{\vec{H}}$$

$$(\mathrm{rot}\, \vec{E})\, \vec{e}_\varphi = -\mu \dot{H}_\varphi$$

$$(\mathrm{rot}\, \vec{E})\, \vec{e}_\varphi = \Big(\underbrace{\frac{\partial E_\rho}{\partial z}}_{\substack{=0 \\ E_\varphi = 0}} - \frac{\partial E_z}{\partial \rho}\Big) = -\frac{\partial E_z}{\partial \rho}$$

$$\frac{\partial E_z}{\partial \rho} = \mu \dot{H}_\varphi$$

$$\vec{J} = \sigma \vec{E} = \sigma E_z \vec{e}_z$$

$$\Delta \vec{J} - \mu\sigma \dot{\vec{J}} = \vec{0}$$

$$\Delta J_z - \mu\sigma \dot{J}_z = 0$$

HEZ:

$$\frac{\partial^2 \underline{J}_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \underline{J}_z}{\partial \rho} - j\omega\sigma\mu\, \underline{J}_z = 0$$

Besselsche DGL f. $n = 0$

$$\underline{\eta} = \sqrt{-j\omega\sigma\mu}\,\varrho$$

$$\frac{\partial^2 \underline{J}_z}{\partial \underline{\eta}^2} - \frac{1}{\underline{\eta}}\frac{\partial \underline{J}_z}{\partial \underline{\eta}} + \underline{J}_z = 0$$

$$\underline{J}_z = \underline{c}_1 \underline{J}_0(\underline{\eta}) + \cancel{\underline{c}_2 \underline{N}(\underline{\eta})}$$

$c_2 = 0$ da $\underline{N}(0) \to \infty$
und $\underline{J}_z(0)$ endlich
$\Rightarrow \underline{c}_1 = 0$

$$\underline{J}_z = \underline{c}_1 \underline{J}_0(\underline{\eta}) = \underline{c}_1 \underline{J}_0\left((1-j)\frac{\varrho}{a}\right)$$

$$\underline{\eta} = \sqrt{-j\omega\sigma\mu}\,\varrho = (1-j)\frac{\varrho}{a}$$

$$\underline{I} = \int_{F_\varphi} \underline{J}_z\, dF = \int_0^{2\pi}\int_0^{\varrho_0} \underline{J}_z\, \varrho\, d\varrho\, d\varphi$$

$$= \underline{c}_1 2\pi \int_0^{\varrho_0} \underline{J}_z\left(\frac{1-j}{a}\varrho\right)\varrho\, d\varrho = \underline{c}_1 2\pi \frac{a^2}{(1-j)^2}\underbrace{\int_0^{\eta} \underline{J}_0(\underline{\eta})\underline{\eta}\, d\underline{\eta}}_{\underline{\eta}_0 \underline{J}_1(\underline{\eta}_0)}$$

$$= (1+j)\pi \underline{c}_1\, a\, \varrho_0\, \underline{J}_1\left(\frac{1-j}{a}\varrho_0\right)$$

$$\underline{c}_1 = \frac{\underline{I}}{(1+j)\pi a \varrho_0 \underline{J}_1\left(\frac{1-j}{a}\varrho_0\right)}$$

$$\underline{J}_z(\varrho) = \frac{\underline{I}\, \underline{J}_0\left(\frac{1-j}{a}\varrho\right)}{(1+j)\pi a \varrho_0 J_1\left(\frac{1-j}{a}\varrho_0\right)}$$

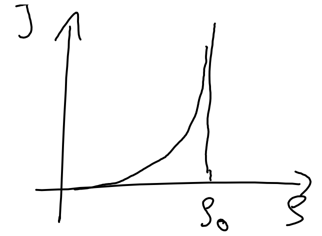
sehr hohe Frequenz.

$$\varrho_0 \gg a$$

$$\underline{J}_z(\varrho) \sim \frac{\underline{J}_0\left(\frac{1-j}{a}\varrho\right)}{\underline{J}_1\left(\frac{1-j}{a}\varrho_0\right)}$$

$$\approx \frac{\sqrt{\frac{2}{\pi\left(\frac{1-j}{a}\right)\varrho}}}{\sqrt{\frac{2}{\pi\left(\frac{1-j}{a}\right)\varrho_0}}}\;\frac{\cos\left(\frac{1-j}{a}\varrho - \frac{\pi}{4}\right)}{\cos\left(\frac{1-j}{a}\varrho_0 - \frac{3}{4}\pi\right)} = \sqrt{\frac{\varrho_0}{\varrho}}\;\frac{e^{j\frac{1-j}{a}\varrho - j\frac{\pi}{4}}}{e^{j\frac{1-j}{a}\varrho_0 + j\frac{3\pi}{4}}}$$

$$= j\sqrt{\frac{\varrho_0}{\varrho}}\; e^{\frac{1+j}{a}(\varrho - \varrho_0)} \approx j \cdot e^{\frac{1+j}{a}(\varrho - \varrho_0)}$$

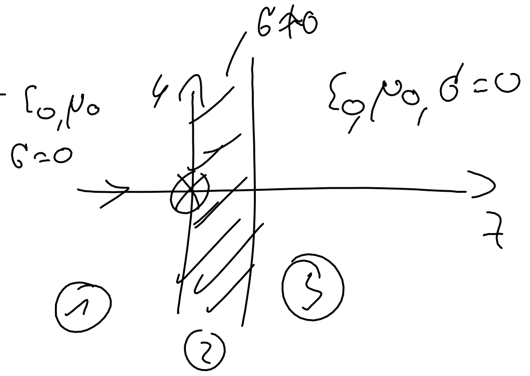


$$\underline{J}_z(\rho) = \frac{\underline{I}}{(1+j)\pi a \rho_0}\, j e^{-\frac{1+j}{a}(\rho_0-\rho)} = \frac{(1+j)\,\underline{I}}{2\pi a\,\rho_0}\, e^{-\frac{1+j}{a}(\rho_0-\rho_0)}$$

16kHz : a = 1μm

50Hz: a = 4,5mm

12.2



HEZ: $\Delta \underline{\vec{E}} + \underline{k}^2 \underline{\vec{E}} = \vec{0}$

1 3 $\quad k_0^2 = \omega^2 \varepsilon_0 \mu_0$

$k_0 = \omega\sqrt{\varepsilon_0\mu_0}$

2 $\quad \underline{k}^2 = \omega^2 \varepsilon\mu - j\omega\sigma\mu$

$= k_0 \varepsilon_r \mu_r \left(1 - \frac{j\sigma}{\omega\varepsilon}\right)$

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linear polarisierte Welle

$\underline{\vec{E}} = \underline{E}_x \vec{e}_x$

1 $\quad \underline{E}_{1x} = E_0\, e^{-jk_0 z} - \underline{r}\, E_0\, e^{+jk_0 z}$

2 $\quad \underline{E}_{2x} = E_0\left(\underline{A}\, e^{-j\underline{k} z} + \underline{B}\, e^{j\underline{k} z}\right)$

3 $\quad \underline{E}_{3x} = \underline{t}\, E_0\, e^{-jk_0 z}$

① $\underline{H}_{1y} = \frac{E_0}{Z_0}\left(e^{-jk_0 z} - \underline{r}\, e^{jk_0 z}\right)$

② $\underline{H}_{2y} = \frac{E_0}{\underline{Z}}\left(\underline{A}\, e^{-j\underline{k}z} - \underline{B} e^{+j\underline{k}z}\right)$

③ $H_{3y} = \frac{E_0}{Z_0}\, \underline{t}\; e^{-jk_0 z}$

$z=0$ und $z=d$

$\text{rot}\, \vec{E} = \vec{0}$

$E_{1x}(z=0) = E_{2x}(z=0)$

$\cancel{E_0}(1+\underline{r}) = \cancel{E_0}(\underline{A}+\underline{B})$

$\text{Rot}\, \vec{H} = \vec{0}$

$H_{1y}(0) = H_{2y}(0)$

$\frac{1}{Z_0}(1-\underline{r}) = \frac{1}{\underline{Z}}(\underline{A}+\underline{B})$

$z = d$

$E_{2x}(d) = E_{3x}(d)$

$\cancel{E_0}\left(\underline{A} e^{-j\underline{k} d} + \underline{B} e^{j\underline{k} d}\right) = \cancel{E_0}\, \underline{t}\, e^{-jk_0 d}$

$\frac{1}{\underline{Z}}\left(\underline{A}\, e^{-j\underline{k}d} - \underline{B}\, e^{j\underline{k}d}\right) = \frac{1}{Z_0}\underline{t}\, e^{-jk_0 d}$

$\underline{r} = \dfrac{-j\left(1 - \frac{Z^2}{Z_0^2}\right)\tan(\underline{k} d)}{2\frac{Z}{Z_0} + j\left(1+\frac{Z^2}{Z_0^2}\right)\tan(\underline{k} d)}$

$\underline{t} = \left((1+\underline{r})\cos(\underline{k} d) - j\frac{Z}{Z_0}(1-\underline{r})\sin(\underline{k}d)\right)e^{jk_0 d}$

Silber $\sigma \gg \omega\varepsilon$ $\qquad |d'| = \frac{d}{\delta} < 1$

Silber $\sigma >> \omega\varepsilon$

$$\underline{k}^2 = -j\omega\sigma\mu$$

$$\underline{k} = \frac{1-j}{a} \; ; \; \underline{Z} = \frac{1+j}{\sigma a}$$

$$d' = \frac{d}{a} < 1$$

$$Z' = \frac{Re\{Z\}}{Z_0} = \frac{1}{\sigma a Z_0} << 1$$

$$\underline{r} = -j \frac{\tan(\underline{k}d)}{2\frac{Z}{Z_0} + j\tan(\underline{k}d)} = -j \frac{\sin(\underline{k}d)}{2\frac{Z}{Z_0}\cos(\underline{k}d) + j\sin(\underline{k}d)}$$

$$\underline{k}d = \frac{1-j}{a} d = (1-j)d' \quad = -j \frac{(1-j)d'}{2\frac{Z}{Z_0}1 + j(1-j)d'}$$

$$= -\frac{1}{1 + 2\frac{Z}{Z_0}\frac{1}{j(1-j)d'}} = -\frac{1}{1+2\frac{Z'(1+j)}{(1+j)d'}}$$

$$= -1 + 2\frac{Z'}{d'} \quad (\text{u. } 10\,\text{GHz} - Z' = 5 \cdot 10^{-5}$$