KGÜ 5

Dienstag, 23. November 2010
15:52

A.1) HOG

a) $\phi(\rho, t=0) = \int_F \vec{B}\, d\vec{F}$ magn. Fluss

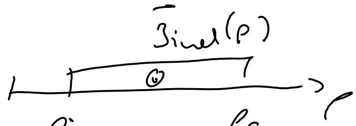
$$d\vec{F} = \rho\, d\varphi\, d\rho\, \vec{e}_z$$

b) siehe KGÜ A3

c) $\vec{J}_{ind} = J_{ind\varphi}(\rho)\, \vec{e}_\varphi$

div $J_{ind} = 0$

[Quellenfrei



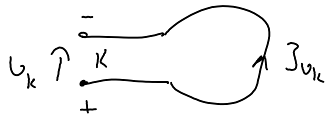
$I_K(t) = \int_F J_{ind\varphi}(\rho)\, dF$

$\vec{J}_{ind} = \sigma_s \vec{E}_{ind}$

$\oint_C \vec{E}_{ind}\, d\vec{s} = -\dot{\Phi}$ Faradaysche Induktionsgesetz

$d\vec{s} = \rho\, d\varphi\, \vec{e}_\varphi$

d)

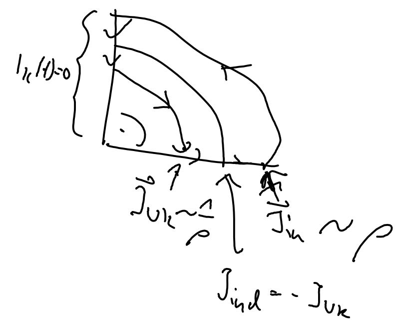


$J_K(t) = \int_F (J_{ind\varphi}(\rho) + J_{U_K\varphi}(\rho))\, d\vec{F} \overset{!}{=} 0$

$J_{U_K} = \sigma_s \vec{E}_K$

$\uparrow$

$U_K = \int_{C_S} \vec{E}_K\, d\vec{s}$



$J_{U_K\varphi} \sim \frac{1}{\rho}$ und $J_{ind\varphi} \sim \rho$

$\Rightarrow$ Wirbelströme in den Scheiben

a) $\Phi(t=0) = \frac{1}{2}\pi \rho_a^2 \cdot B_0$ ✓

b) $\Phi(t) = \Phi(t=0) \cdot \cos(\omega t)$ ✓

c) $\vec{J}_{ind} = \sigma_s \vec{E}_{ind}$

$\text{div}\, \vec{J}_{ind} = \text{div}\, J_{ind\varphi}(\rho) = \frac{d}{d\varphi} J_{ind\varphi}(\rho) = 0$ ✓

$\Rightarrow$ Quellenfrei

$\oint E_{ind} \ldots = -\dot{\Phi} = \ldots$

$$\oint_C E_{ind} = -\dot{\Phi} = \pi \rho E_{ind} \qquad \rho_i < \rho < \rho_a$$

$$E_{ind} = \frac{1}{\cancel{\pi}\rho} \cdot \cancel{(-1)} \cdot \frac{1}{2}\pi \rho^{\cancel{2}} B_0 \cancel{(-} \sin(\omega t)\,\omega)$$

$$= \frac{1}{2} \cdot \frac{\rho^{\cancel{2}}}{\cancel{\rho}} B_0 \omega \sin(\omega t)$$

$$\vec{J}_{ind}(\rho, t) = \sigma_S \cdot E_{ind}(\rho, t)$$

$$= \frac{\sigma_S}{2} B_0 \omega \sin(\omega t)\rho$$

$$I_K(t) = \int_{\rho_i}^{\rho_a} J_{ind}(\rho, t) \cdot d \cdot d\rho$$

$$= \frac{\sigma_S}{2} \rho_a^2 B_0 \omega \sin(\omega t)\, d \int_{\rho_i}^{\rho_a} \rho \, d\rho$$

$$= \frac{\sigma_S}{2} \rho_a^2 B_0 \omega \sin(\omega t)\, d \frac{1}{2}(\rho_a^2 - \rho_i^2) \quad \checkmark$$

d) $$I_K(t) = \int_F J_{ind\,\varphi}(\rho) + J_{UK\,\varphi}(\rho)\, d\vec{F} \overset{!}{=} 0$$

$$J_{ind} = \frac{\sigma_S}{2} \quad B_0 \omega \sin(\omega t)\rho$$

$$J_{UK\,\varphi}(\rho) = \sigma_S E_{UK}(\rho) = \sigma_S \frac{1}{\pi} \frac{1}{\rho} U_K(t) \quad \checkmark$$

$$U_K(t) = \int E\, d\vec{s} = E_{UK} \cdot \pi \rho$$

$$\Leftrightarrow \quad E_{UK}(\rho, t) = \frac{1}{\pi} \frac{1}{\rho} U_K(t)$$

$$I_K(t) = \int_{\rho_i}^{\rho_a} \frac{\sigma_S}{2} B_0 \omega \sin(\omega t)\rho + \sigma_S \frac{1}{\pi} \frac{1}{\rho} U_K(t)\; d\vec{F} \overset{!}{=} 0$$

$$\frac{\cancel{\sigma_s}}{4} B_0 \omega \sin(\omega t)(\rho_a^2-\rho_i^2) = -\cancel{\sigma_s} \frac{1}{\pi} U_k(t) \cdot \ln \frac{\rho_a}{\rho_i}$$

$$U_k(t) = -\frac{\pi}{4} B_0 \omega \sin(\omega t) \ln \frac{\rho_i}{\rho_a} (\rho_a^2-\rho_i^2) \checkmark$$

a) $\phi(\rho, t=0) = \frac{\pi}{2} B \rho^2$

b) $\phi(\rho) = \phi(\rho, t=0) \cdot \cos(\omega t)$

c) $\vec{J} = J_\varphi(\rho) \vec{e}_\varphi$

$$\operatorname{div} \vec{J} = \frac{1}{\rho} \underbrace{\frac{\partial J_\varphi}{\partial \varphi}}_{=0} = 0 \qquad (\checkmark)$$

$$I_k(t) = d \cdot \int_{\rho_i}^{\rho_a} J_\varphi(\rho) d\rho$$

$$\oint_{1-2-3-8} \vec{E} d\vec{s} = -\frac{1}{2} \dot{\phi}$$

$$\frac{\pi}{2} \rho \frac{J_\varphi}{\sigma_s} = \frac{1}{2} \frac{\pi}{2} B_0 \rho^2 \sin(\omega t) \omega$$

$$J_\varphi = \frac{\sigma_s}{2} B_0 \rho \omega \sin(\omega t)$$

$$I_k(t) = \frac{\sigma_s \cdot d B_0 \omega}{4} \sin(\omega t) (\rho_a^2 - \rho_i^2)$$

d) $U_k = \int_{C_S} \vec{E}_k d\vec{s} = E_k \cdot 2 \cdot \frac{\pi}{2} \rho$

$$E_k = \frac{U_k}{\pi \rho}$$

$$I_k(t) = 0$$

$$\Rightarrow U_k(t) = -\frac{\pi}{4} B_0 \omega \sin(\omega t) \frac{(\rho_a^2-\rho_i^2)}{\ln\left(\frac{\rho_a}{\rho_i}\right)}$$