GGÜ 2

Freitag, 19. November 2010
10:00

Aufgabe 3

- Dicke $\delta \ll a, b$
- Widerstand R sehr groß
- $\mu = \mu_0$

a) $i_2(t) = I_0 \cos(\omega t) \quad \wedge v = 0$

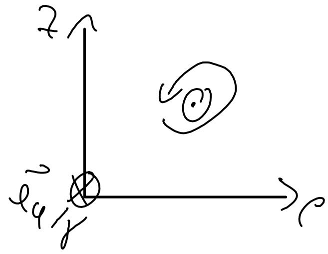
$$\vec{H} = \frac{I}{2\pi\rho}\vec{e}_\varphi \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi\rho}\vec{e}_\varphi$$

$$\phi = \iint \vec{B}\, d\vec{A} \qquad d\vec{A} = d\vec{x} \times d\vec{z}$$

$$= d\vec{\rho} \times d\vec{z} = -d\rho\, dz\, \vec{e}_\varphi$$

$b = vt + 2a$

$$= \int_0^a \int_a^{2a} \frac{\mu_0 I}{2\pi\rho}\, d\rho\, dz\, \underbrace{(-\vec{e}_\varphi)\vec{e}_\varphi}_{-1}$$



$$\phi = -a\frac{\mu_0 I}{2\pi}\left[\ln(\rho)\right]_a^{2a} = -a\frac{\mu_0 I}{2\pi}\ln(2)$$

$$I = I_0 \cos(\omega t)$$

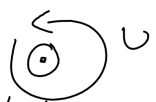
$$v(t) = -\frac{\partial \phi}{\partial t} = -a\frac{\omega\mu_0}{2\pi}\ln(2)\, I_0 \sin(\omega t)$$

$$i_1 = -\frac{v(t)}{R} = +\frac{a\omega\mu_0}{2\pi R}\ln(2)\, I_0 \sin(\omega t)$$

b) $i_2 = I_0 = \text{const} \qquad v > 0$

$$\phi = \iint \vec{B}\, d\vec{A} = -a\int_{vt+a}^{vt+2a} \frac{\mu_0 I}{2\pi\rho}\, d\rho = -a\frac{\mu_0 I}{2\pi}\left[\ln(\rho)\right]_{vt+a}^{vt+2a}$$

$$= -a\frac{\mu_0 I}{2\pi}\left[\ln(vt+2a) - \ln(vt+1a)\right]$$



$$U = -\frac{\partial\phi}{\partial t}$$

$$= \frac{a\cdot\mu_0 I}{2\pi}\cdot\left[\underbrace{\frac{\partial(vt+2a)}{\partial t}}_{v}\cdot\frac{\partial\ln(vt+2a)}{\partial(vt+2a)} - \underbrace{\frac{\partial(vt+1a)}{\partial t}}_{v}\cdot\frac{\partial\ln(vt+1a)}{\partial(vt+1a)}\right]$$

$$= \frac{a\mu_0 I_0 v}{2\pi}\left[\frac{1}{vt+2a} - \frac{1}{vt+1a}\right]$$

$$i_1 = \frac{-U}{R}$$

$$= \frac{a\mu_0 I_0 v}{2\pi R}\left[\frac{1}{vt+1a} - \frac{1}{vt+2a}\right]$$

$$\operatorname{rot} \vec{E} = -j\omega\mu_0 \vec{H}$$

nur für die „interessierten" Studenten

nicht Klausurrelevant

$$\vec{E}_{ind} = \vec{v} \times \vec{B}$$

$$= (v\vec{e}_\rho) \times (B\vec{e}_\varphi) = v \frac{\mu_0 I}{2\pi\rho} \vec{e}_z$$

$$U = \int_0^a E_z \, dz \Big|_{\rho = vt+a} + \cancel{\int_{vt+a}^{vt+2a} E_\rho \, d\rho} + \int_a^0 E_z \, dz \Big|_{\rho = vt+2a} + \cancel{\int E_\rho \, d\rho}$$

$$= a \frac{v\mu_0 I}{2\pi(vt+a)} - a \frac{v\mu_0 I}{2\pi(vt+2a)}$$

$$i_1 = \frac{U}{R} = \frac{a v \mu_0 I}{2\pi R} \left[\frac{1}{vt+a} - \frac{1}{vt+2a} \right]$$

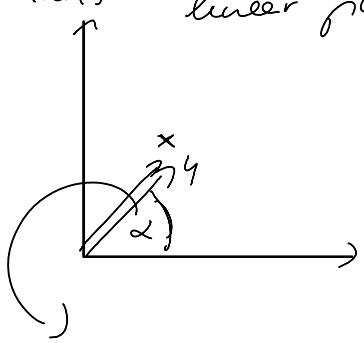
A.4)

$$\vec{E}(\vec{r},t) = \operatorname{Re}\{ \underset{\substack{\uparrow \\ \text{complex}}}{\underline{\vec{E}}_0} \exp(j\omega t - j\vec{k}\vec{r}) \}$$

$$\underline{\vec{E}}_0 = \vec{E}_{0,re} + j\,\vec{E}_{0,im}$$

- $\vec{E}_{0,im} = 0 \Rightarrow \underline{\vec{E}}_0$ ist reell $\Rightarrow$ linear polarisierte Welle
- $\underline{\vec{E}}_0 = \vec{E}'_{0,re} \cdot e^{j\varphi}$ ⊛ $\Rightarrow$ auch linear polarisiert

Im{}



Re{}

linear polarisiert!

x, y haben gleichen Winkel α

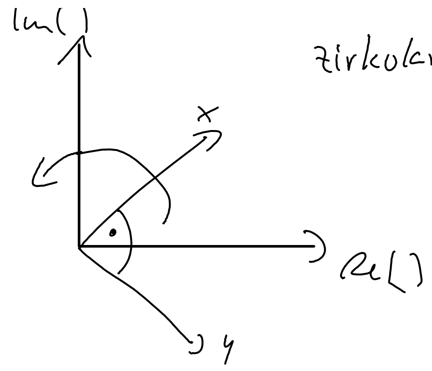
$\{\underline{E} \cdot e^{j\omega t}\}$

- allgemeiner Fall

$\underline{\vec{E}}_0$ lässt sich nicht wie ⊛ darstellen

Im()

zirkular polarisiert



zirkular polarisiert

a) $\vec{E} = (\vec{r}, t) \perp \vec{n} \quad (\forall \vec{r}, t)$

$$|\vec{E}(\vec{r}, t)| = |\vec{E}(\vec{r}, t=0)| = \text{const}$$

$$\vec{E}(\vec{r}=0, t=0) = \vec{E}_1 = A\,\vec{e}_z$$

$$\vec{E}\left(\vec{r}=0, t=\frac{T}{4}\right) = \vec{E}_2 = A\,\vec{e}_y$$

$$\vec{n} = \pm \frac{\vec{E}_1 \times \vec{E}_2}{|\vec{E}_1 \times \vec{E}_2|} = \pm \frac{\vec{E}_1 \times \vec{E}_2}{|E_1|^2} = \pm\, \vec{e}_z \times \vec{e}_y = \mp\, \vec{e}_x$$

Wegen $n_x > 0 \Rightarrow \vec{n} = \vec{e}_x$

$$\vec{k} = |\vec{k}|\,\vec{n} = \omega \sqrt{\varepsilon_0 \mu_0}\, \vec{e}_x$$

b) $\underline{\vec{E}}(\vec{r}) = \underline{\vec{E}}_0\, e^{-j\vec{k}\vec{r}} \quad \Rightarrow \quad \underline{\vec{E}}(\vec{0}) = \underline{\vec{E}}_0$

$$\vec{E}(\vec{0}, t) = Re\{\underline{\vec{E}}(\vec{0})\, e^{j\omega t}\} = Re\{\underline{\vec{E}}_0\, e^{j\omega t}\}$$

1.) $t = 0 \quad \vec{E}(\vec{0}, 0) = \vec{E}_1 = A\,\vec{e}_z = Re\{\underline{\vec{E}}_0\}$

2.) $t = \frac{T}{4} = \frac{\pi}{2\omega} \Rightarrow \vec{E}\left(\vec{0}, \frac{T}{4}\right) = \vec{E}_2 = A\,\vec{e}_y$

$$= Re\{\underline{\vec{E}}_0\, e^{j\frac{\pi}{2}}\} = Re\{j\,\underline{\vec{E}}_0\}$$

$$\Rightarrow Re\{j\,\underline{\vec{E}}_0\} = -Im(\underline{\vec{E}}_0) \Rightarrow Im\{\underline{\vec{E}}_0\} = -A\,\vec{e}_y$$

$$\underline{\vec{E}}(\vec{r}) = \underline{\vec{E}}_0\, e^{-j\vec{k}\vec{r}} = A(\vec{e}_z + j\,\vec{e}_y)\, e^{-j\vec{k}\vec{r}}$$

$$\vec{k}\vec{r} = |\vec{k}|\,\vec{n}\vec{r} = \omega\sqrt{\varepsilon_0\mu_0}\, x$$

Nur TEM-Welle: $\underline{\vec{H}}(\vec{r}) = \frac{1}{Z_0}\, \vec{n} \times \underline{\vec{E}} \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

(oder: rot $\underline{\vec{E}}(\vec{r}) = -j\omega\mu_0\, \underline{\vec{H}}(\vec{r})$)

$$\underline{\vec H}(\vec r) = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, A\, e^{-j\omega\sqrt{\varepsilon_0\mu_0}\cdot x} \left(\underbrace{\vec e_x \times \vec e_z}_{-\vec e_y} - j\,\underbrace{\vec e_x \times \vec e_y}_{\vec e_z}\right)$$

$$\Rightarrow \underline{\vec H}(\vec r) = -\sqrt{\frac{\varepsilon_0}{\mu_0}}\, A\, (\vec e_y + j\vec e_z)\, e^{-j\omega\sqrt{\varepsilon_0\mu_0}\, x}$$

$$= -\sqrt{\frac{\varepsilon_0}{\mu_0}}\, A\, (\vec e_y + j\vec e_z)\, e^{-j\vec k\vec r}$$

c) $\underline{\vec E}_{mod}(\vec r) = -(\vec e_z + j\vec e_y)\, A\, e^{-j\vec k\vec r}$

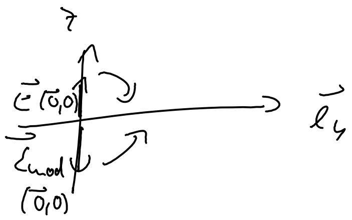
$$\vec E_{mod}(\vec 0, t) = Re\{\underline{\vec E}_{mod}(\vec 0)\, e^{j\omega t}\} = -A\, Re\{(\vec e_z + j\vec e_y)\, e^{j\omega t}\}$$

1.) $t = 0: \quad \vec E_{mod}(\vec 0, 0) = -\vec E_1$

$t = \frac{T}{4}: \quad ..... \quad A\vec e_y = \vec E_2$

$$\underline{\vec H}_{mod}(\vec r) = \frac{1}{Z_0}\, \vec n \times \underline{\vec E}_{mod}(\vec r) = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, (-A)\, [\vec e_x \times (\vec e_z + j\vec e_y)]\, e^{-j\vec k\vec r}$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}}\, A\, (\vec e_y - j\vec e_z)\, e^{-j\omega\sqrt{\varepsilon_0\mu_0}\, x}$$



$$\underline{\vec E}_{ges}(\vec r) = \underline{\vec E}(\vec r) + \underline{\vec E}_{mod}(\vec r) = -2j\, A\, \vec e_y\, e^{-j\vec k\vec r}$$

$$\underline{\vec H}_{ges}(\vec r) = \underline{\vec H}(\vec r) + \underline{\vec H}_{mod}(\vec r) = -\sqrt{\frac{\varepsilon_0}{\mu_0}}\, A\, 2j\, \vec e_z\, e^{-j\vec k\vec r}$$