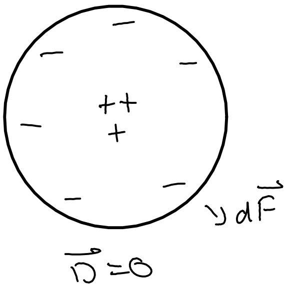
KGÜ 3

Dienstag, 9. November 2010
15:35

1.1 HÜ 7

negative Ladungen sind ortsfest

a) Außenraum <u>feldfrei</u>: $\vec{D} = 0$

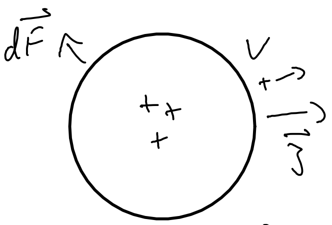


$\operatorname{div} \vec{D} = \rho$

$$\Rightarrow \int_V \operatorname{div} \vec{D}\, dV \overset{\text{Gauss}}{=} \int_F \vec{D}\, d\vec{F}$$

$$= Q_{ein} = Q_+ - Q_- \quad \text{mit } Q_{ein} = \int_V \rho\, dV$$

b)
$$\left.\begin{aligned} \operatorname{rot} \vec{H} &= \vec{J} + \dot{\vec{D}} \\ \operatorname{div} \vec{D} &= \rho \\ \operatorname{div}\operatorname{rot} &\equiv 0 \end{aligned}\right\} \quad \text{diff. Form} \quad \boxed{\operatorname{div} \vec{J} + \dot{\rho} = 0} \quad \text{Kontinuitätsgleichung}$$



$\rho > 0$ und $\dot{\rho} < 0$ $\qquad \operatorname{div} \vec{J} > 0$ Quelle

e)
$$\int_V \operatorname{div} \vec{J}\, dV \overset{\text{Gauss}}{=} \int_F \vec{J}\, d\vec{F} = \boxed{I = -\frac{d}{dt} Q_{ein}}$$

(int. Form)

c)
$$\left.\begin{aligned} \operatorname{div} \vec{J} + \dot{\rho} &= 0 \\ \vec{J} &= \sigma_{741} \vec{E} \\ \operatorname{div} \vec{D} &= \rho \end{aligned}\right\} \text{DGL}$$

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a) $Q_+ = -Q_-$

$Q_+ = \frac{4}{3} \pi r_i^3 \cdot \rho_+$

$$Q_- = \left(\frac{4}{3}\pi r_a^3 - \frac{4}{3}\pi (r_a - d)^3\right)\rho_-$$

$$+ r_i^3 \cancel{\rho_+} = \left(r_a^3 - (r_a - d)^3\right)\cancel{\rho_-} \qquad \rho_- = -\rho_+$$

$$\Rightarrow \quad r_i^3 - r_a^3 = (r_a - d)^3$$

Es ist leider ein Zylinder, keine Kugel

$$\Leftrightarrow \quad d = r_a + \sqrt[3]{r_i^{\cancel{3}2} \cancel{+} r_a^{\cancel{3}2}}$$

b)

$$\operatorname{div}(\operatorname{rot}\vec{H}) = \operatorname{div}(\vec{J} + \dot{\vec{D}})$$

$$0 = \operatorname{div}\vec{J} + \dot{\rho}$$

c)

$$\vec{J} = \sigma_{Zyl}\vec{E} = \frac{\sigma_{Zyl}}{\varepsilon}\vec{D}$$

$$\operatorname{div}\vec{J} + \dot{\rho} = 0$$

$$\Rightarrow \operatorname{div}\left(\frac{\sigma_{Zyl}}{\varepsilon}\vec{D}\right) + \dot{\rho} = 0$$

$$\frac{\sigma_{Zyl}}{\varepsilon}\rho + \dot{\rho} = 0$$

$$\int \frac{\dot{\rho}}{\rho}dt = \int -\frac{\sigma_{Zyl}}{\varepsilon}dt$$

$$\ln(\rho) = -\frac{\sigma}{\varepsilon}t$$

$$\underline{\rho = Ae^{-\frac{\sigma}{\varepsilon}t}}$$

$0 < r \leq r_i$: $\quad -\frac{\sigma}{\varepsilon}t$

$\rho = \rho_+ e$

$r_i < r < r_a - d$:

a) Gesamtladung des Zylinders $= 0$

$Q_{ein} = 0 = Q_+ + Q_-$

$\Rightarrow Q_+ = -Q_-$

$Q_+ = q_+ (2l)(\pi r_i^2)$

$Q_- = q_- (2l)\pi\left(r_a^2 - (r_a - d)^2\right)$

$\Rightarrow d = r_a \underset{\uparrow \; d<r_a}{\overset{(+)}{-}} \sqrt{r_a^2 - r_i^2}$

c) $\operatorname{div} \vec{J} + \dot{\rho} = 0 \qquad \vec{D} = \varepsilon \vec{E}$

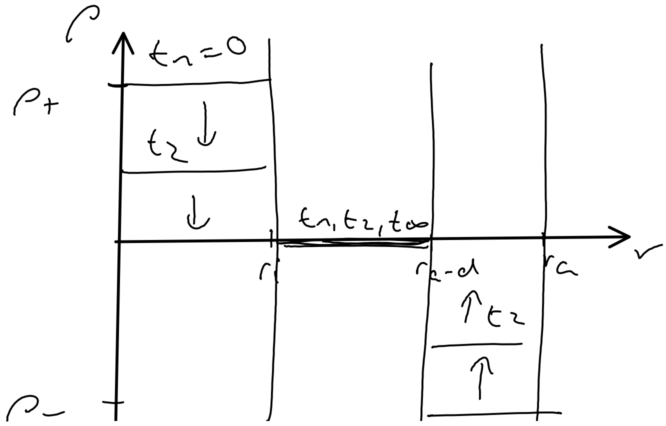
$\operatorname{div} \vec{J} = \sigma \operatorname{div} \vec{E} = \frac{\sigma}{\varepsilon} \operatorname{div} \vec{D} = \frac{\sigma}{\varepsilon} \rho$

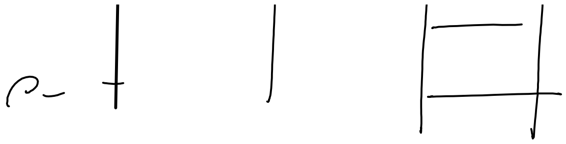
$\dot{\rho} = -\frac{1}{\tau} \rho \qquad \tau = \frac{\varepsilon}{\sigma}$

Lösung: $\rho(t) = \rho_0 e^{-\frac{t}{\tau}}$

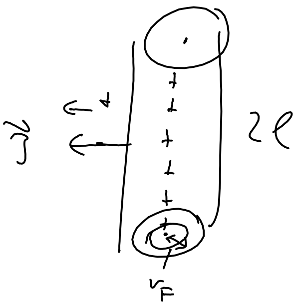
1. $0 \le r \le r_i$: $\rho_0 = \rho_+$
2. $r_i < r < r_a - d$: $\rho_0 = 0$
3. $r_a - d \le r \le r_a$: $\rho_0 = \rho_-$

d)





e) radialer Strom pro Längeneinheit



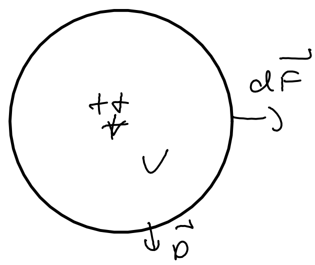
$$J' = \frac{J}{2l}$$

mit $J = \int_F \vec{J}\, d\vec{F}$ $\quad F = 2\pi r_F \cdot 2l$

und $J = -\frac{d}{dt} Q_{ein}$

$$J' = -\dot{\rho}(t)\, \frac{\pi r_i^2\, 2l}{2l} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}\, \rho_+\, e^{-\frac{t}{\tau}} \cdot \pi r_i^2$$

f) $\int_F \vec{D}\, d\vec{F} = Q_{ein}$



$$D_r(r_i,t) = \frac{2l\, \pi\, r_i^2\, \rho(t)}{2l\; 2\pi r_i}$$

$$= \rho_+\, e^{-t/\tau}\, \frac{r_i}{2}$$

allg. $D_r(r,t) = \frac{\rho_+\, e^{-t/\tau}\, r_i^2}{2r}$ für $r_i < r < r_a - d$

$$p = \vec{J}\vec{E} \qquad \vec{J} = \sigma \cdot \vec{E}$$

$$p = \frac{1}{(\varepsilon_0 \varepsilon_r)^2}\, \sigma_{zyl} \cdot \rho_+^2\, e^{-\frac{2t}{\tau}}\, \frac{\cdot r_i^4}{4 r^2}$$