V 2

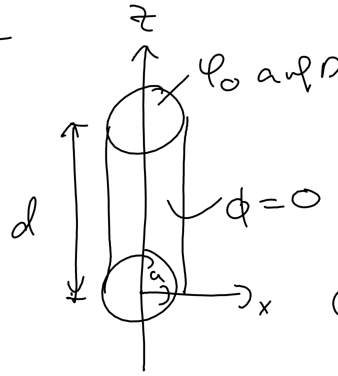
Freitag, 5. November 2010
10:00

nächste Woche Vorlesung
10.12. Tag d. Elektrotechnik -> frei

3D $(k_z \neq 0)$ $\quad 0 \leq z \leq \ell_z$

$$\phi_{mn}(\rho,\varphi,z) = \left[A_{mn} J_m(k_{zn}\rho) + B_{mn} N_m(k_{zn}\rho)\right] \cdot$$
$$\left[C_m \sin(m\varphi) + D_m \cos(m\varphi)\right] \cdot$$
$$\left[E_n \sinh(k_{zn} z) + F_n \cosh(k_{zn} z)\right]$$

8.2.3



$$\phi(\rho,\varphi,z) = \phi(\rho,z)$$
$$\Rightarrow m = 0$$

$$\phi_{mn}(\rho,\varphi,z) = \phi_{0n}(\rho,z)$$
$$= \left[A_{0n} J_0(k_{zn}\rho) + \cancel{B_{0n} N_0(k_{zn}\rho)}\right] \cdot$$

Kreuzprodukt oder Null.? → $\times \left[E_n \sinh(k_{zn} z) + \underbrace{F_n \cosh(k_{zn} z)}_{=0}\right]$ da $\phi(\rho, z=0) = 0$

$$\phi_{0n}(\rho,z) = \phi_n(\rho,z) = \underbrace{A_n}_{A_{0n}\cdot E_n} J_0(k_{zn}\rho) \sinh(k_{zn} z)$$

$\phi(\rho = a, z) = 0$ für $0 \leq z \leq d$

$$k_{zn} a = X_{0n} \Rightarrow k_{zn} = \frac{x_{0n}}{a}$$

$$\phi(\rho,z) = \sum_{n=1}^{\infty} \underbrace{A_n'}_{A_n \sinh(k_{zn} d)} J_0(k_{zn}\rho) \frac{\sinh(k_{zn} z)}{\sinh(k_{zn} d)}$$

$$\phi(\rho, z=d) = \Phi_0 = \sum_{n=1}^{\infty} A_n' J_0\left(\frac{x_{0n}}{a}\rho\right) \quad \text{(Fourier - Bessel - Reihe)}$$

$$\varphi_0 \cdot \int_0^a \Phi_0 \rho J_0\left(\frac{x_{0\nu}}{a}\rho\right) d\varphi = \sum_{n=1}^{\infty} A_n' \int^a \rho J_0\left(\frac{x_{0\nu}}{a}\rho\right) J_0\left(\frac{x_{0\nu}}{a}\rho\right) d\rho$$

$$\Phi_0 \cdot \underbrace{\int_0^a \Phi_0 \, \rho \, J_0\left(\frac{x_{0\nu}}{a}\rho\right) d\varphi}_{\frac{a^2}{x_{0\nu}} J_1(x_{0\nu})} = \sum_{n=1}^{\infty} A_n' \underbrace{\int_0^a \rho \, J_0\left(\frac{x_{0n}}{a}\rho\right) J_0\left(\frac{x_{0\nu}}{a}\rho\right) d\rho}_{\neq 0 \text{ für } n=\nu}$$

$$\frac{1}{2} a^2 J_1^2(x_0\nu)$$

$$A_n' = \frac{2\Phi_0}{x_{0n} J_1(x_{0n})} \sim \frac{1}{\sqrt{n}} \qquad n \gg 1$$

$$\Phi(\rho, z) = 2 \cdot \Phi_0 \sum_{n=1}^{\infty} \frac{J_0\left(\frac{x_{0n}}{a}\rho\right)}{x_{0n} J_1(x_{0n})} \; \frac{\sinh\left(\frac{x_{0n}}{a} z\right)}{\sinh\left(\frac{x_{0n}}{a} d\right)}$$

9.6 Wechselgleichungen

Zweite Maxwellgleichung.

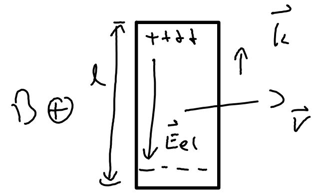
(A:) stationäres Magnetfeld

$\vec{K} = I \cdot (\vec{l} \times \vec{B})$

Kraft auf eine bewegl. Ladung

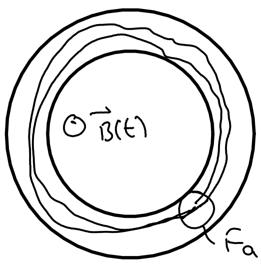
$\vec{K} = q(\vec{v} \times \vec{B})$

$\vec{E}_{mi} = \vec{v} \times \vec{B} = -\vec{E}_{el}$



$U_{ind} = vBl$

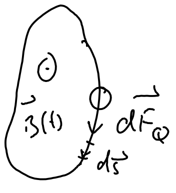
(B)



quasistationäres $\vec{B}$

$iR = -\frac{d\Psi}{dt} \qquad \Psi = \frac{1}{i} \int_{Fa} \Phi \, di$

Verkettung wird zu Stromröhre



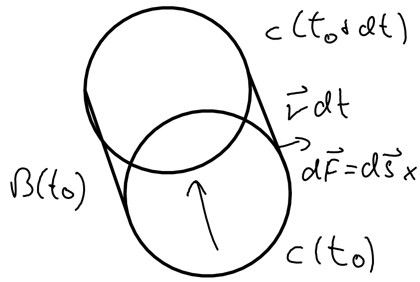
$d\vec{s} \parallel \vec{J} \parallel d\vec{F}_Q$

$di \, R_c = -\frac{d\Phi}{dt} \quad ; \quad di = \vec{J} \cdot d\vec{F}_Q = J \, dF_Q \, ; \quad \vec{J} = \sigma \vec{E}_{ind} \, ; \quad R_c = \oint_c \frac{1}{\sigma} \frac{ds}{dF_Q}$

$$di \, R_c = \oint_c \frac{1}{\sigma} \underbrace{\frac{di}{dF_Q}}_{J} ds \Rightarrow \oint \vec{E}_{ind} \, d\vec{s} = -\frac{d\Phi}{dt} = -\int_{F_c} \frac{\partial \vec{B}}{\partial t} d\vec{F}$$

(C) bewegter Leiter $\vec{B}(t)$

$$\oint \frac{\vec{J}}{\sigma}\, d\vec{s} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_{F_e(t)} \vec{B}(t)\, d\vec{F} = \underbrace{-\frac{d}{dt}\int_{F_e(t)} \vec{B}(t_0)\, d\vec{F}}_{\text{Bewegungsschwund}} \underbrace{- \frac{d}{dt}\int_{F(t_0)} \vec{B}(t)\, d\vec{F}}_{\text{Ruheschwund}}$$



$$d\vec{F} = d\vec{s} \times \vec{v}\, dt \qquad \int_F \vec{B}\, d\vec{F} = 0$$

$$\int_{F_\gamma} B(t_0)\, dF = \underbrace{\int_{F_e(t_0+dt)} \vec{B}(t_0)\, d\vec{F} - \int_{F_e(t_0)} \vec{B}(t_0)\, d\vec{F}}_{d\phi} + \int_{\text{Mantel}} \vec{B}(t_0)\, dF = 0$$

$$d\phi = -\int_{\text{Mantel}} \vec{B}(t_0)\, d\vec{F} = -\oint_{c(t_0)} \vec{B}(t_0)\,(d\vec{s} \times \vec{v}\, dt)$$

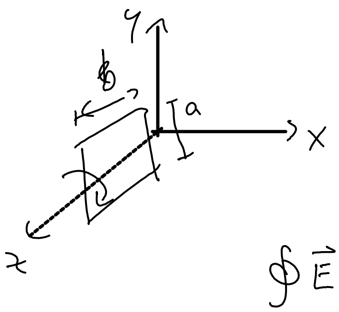
$$= -\oint (\vec{v} \times \vec{B})\, d\vec{s}\, dt$$

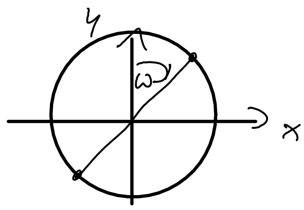
$$-\frac{d\phi}{dt} = \oint (\vec{v} \times \vec{B})\, d\vec{s} \qquad \oint_c \vec{E}\, d\vec{s} = -\frac{d}{dt}\int_{F_c(t)} \vec{B}\, d\vec{F}$$

C ruhend

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

9.6.1.



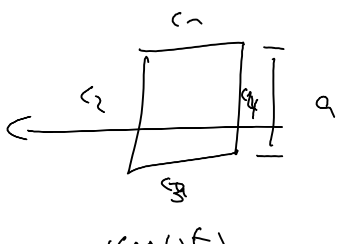


$$\vec{B} = B\,\vec{e}_x \qquad \dot{\vec{B}} = 0$$

$$\alpha = \text{const} \qquad \omega = \frac{2\pi}{T}$$

$$\vec{r}(t) = \frac{a}{2}\begin{pmatrix} \sin \omega t \\ \cos \omega t \\ 0 \end{pmatrix}$$

$$\oint_c \vec{E}\, d\vec{s} = U_{ind} = -\frac{d\phi}{dt}$$



$$\oint_c \vec{v} \times \vec{B}\, d\vec{s} = \int_{c_2} \underbrace{\vec{v} \times \vec{B}\, d\vec{s}}_{=0} + \int_{c_4} \underbrace{\vec{v} \times \vec{B}}_{=0}\, d\vec{s}$$

$$+ \int_{c_1} (\vec{v} \times \vec{B})\, d\vec{s} + \int_{c_3} (\vec{v} \times \vec{B})\, d\vec{s}$$

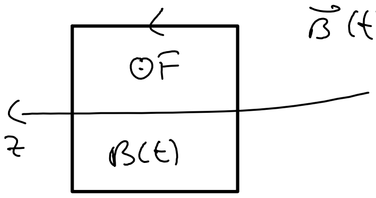
$$\vec{v} = \dot{\vec{r}} = \frac{a}{2}\omega \begin{pmatrix} \cos \omega t \\ -\sin \omega t \\ 0 \end{pmatrix}$$

$$c_1: \vec{v} \times \vec{B} = \frac{a}{2}\omega \begin{pmatrix} \cos \omega t \\ -\sin \omega t \\ 0 \end{pmatrix} \times \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix} = +\frac{a}{2}\omega B \sin(\omega t)$$

$$\int_{C_1} (\vec{v} \times \vec{B})\, d\vec{s} = \frac{ab}{2}\,\omega B \sin(\omega t) = \int_{C_3} (\vec{v} \times \vec{B})\, d\vec{s}$$

$$\underline{U_{ind} = ab\,\omega B \sin \omega t}$$

(B) mitbewegtem Beobachter



$$\vec{B}(t) = B \begin{pmatrix} \cos(\omega t) \\ -\sin(\omega t) \\ 0 \end{pmatrix}$$

$$U_{ind} = -\frac{d\Phi}{dt} = -\int_{F_C} \frac{d\vec{B}}{dt}\, d\vec{F} = -\int \omega B \begin{pmatrix} -\sin(\omega t) \\ -\cos(\omega t) \\ 0 \end{pmatrix} dF\, \vec{e}_x$$

$$d\vec{F} = dF\, \vec{e}_x \qquad \frac{d\vec{B}}{dt} = \omega B \begin{pmatrix} -\sin \omega t \\ -\cos \omega t \\ 0 \end{pmatrix}$$

$$= +ab\,\omega B \sin(\omega t)$$

10 Zeitabhängige Felder

10.1 Harmonische Zeitabhängigkeit

HEZ: Harmonisch eingeschwungener Zustand

$$E_x(\vec{r},t) = \hat{E}_x(\vec{r}) \cos(\omega t + \varphi_x(\vec{r}))$$

$$= Re\{\hat{E}_x e^{j\varphi_x} e^{j\omega t}\} = Re\{\underline{E}_x e^{j\omega t}\}$$

$$\frac{dE_x}{dt} = Re\{j\omega \underline{E}_x e^{j\omega t}\}$$

$$\vec{E}(\vec{r},t) = Re\{\underline{\vec{E}}\, e^{j\omega t}\}$$

10.2 HEZ

$$\text{rot}\, \underline{\vec{H}} = \underline{\vec{J}} + j\omega \underline{\vec{D}}$$

$$\text{rot}\, \underline{\vec{E}} = -j\omega \underline{\vec{B}}$$

$$\text{div}\, \underline{\vec{D}} = \underline{\varrho}$$

$$\text{div}\, \underline{\vec{B}} = 0$$

$$\text{div rot}\, \underline{\vec{H}} = \text{div}\, \underline{\vec{J}} + j\omega \underbrace{\text{div}\, \underline{\vec{D}}}_{\underline{\varrho}} = 0$$

$$\underline{\underline{\text{div}\, \underline{\vec{J}} = -j\omega \underline{\varrho}}}$$

$$\text{div rot}\, \underline{\vec{E}} = -j\omega\, \text{div}\, \underline{\vec{B}} = 0$$

$$\Rightarrow \text{div}\, \underline{\vec{B}} = 0$$

lineare, isotrope und zeitinvariante Elemente

ε, μ, σ stückweise konstant über dem Ort

$$\text{rot}\, \vec{H} = (\sigma \vec{E} + j\omega \varepsilon \vec{E}) = (\sigma + j\omega \varepsilon) \vec{E}$$

$$\text{rot}\,\underline{\vec{H}} = (\sigma\underline{\vec{E}} + j\omega\varepsilon\underline{\vec{E}}) = (\sigma + j\omega\varepsilon)\underline{\vec{E}}$$

$$\text{rot}\,\underline{\vec{E}} = -j\omega\mu\underline{\vec{H}}$$

$$\text{div}\,\underline{\vec{H}} = 0$$

$$\text{div}\,\underline{\vec{E}} = \frac{\rho}{\varepsilon} = 0$$

$$\text{div}\,\text{rot}\,\underline{\vec{H}} = 0 = \text{div}\left[(\sigma + j\omega\varepsilon)\underline{\vec{E}}\right] = (\sigma + j\omega\varepsilon)\,\text{div}\,\underline{\vec{E}}$$

$$\Rightarrow \text{div}\,\underline{\vec{E}} = 0$$

Zeitbereich:
$$\text{rot}\,\vec{H} = \sigma\vec{E} + \varepsilon\dot{\vec{E}}$$
$$\text{rot}\,\vec{E} = -\mu\dot{\vec{H}}$$
$$\text{div}\,\vec{H} = 0$$
$$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon} \quad (= 0 \Rightarrow \rho = 0)$$

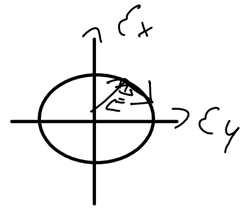
HEZ: $\underline{\varepsilon}(\omega), \underline{\mu}(\omega), \underline{\sigma}(\omega)$

Polarisation:

$$\underline{\vec{E}} = \vec{E}_{re} + j\vec{E}_{im}$$

$$\vec{E}(t) = \text{Re}\{\underline{\vec{E}}\,e^{j\omega t}\} = \text{Re}\{(\vec{E}_{re} + j\vec{E}_{im})(\cos(\omega t) + j\sin(\omega t))\}$$
$$= \vec{E}_{re}\cos(\omega t) - \vec{E}_{im}\sin(\omega t)$$

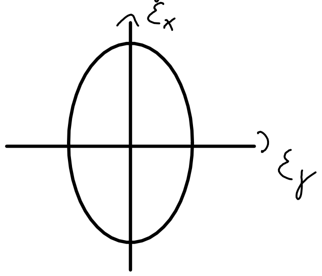
$$\vec{E}_{re} = E_x\,\vec{e}_x \qquad E_{im} = E_y\,\vec{e}_y$$



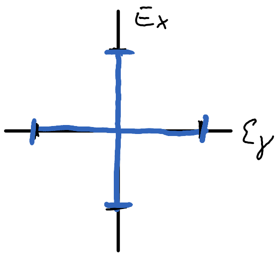
a) $|E_x| = |E_y| \Rightarrow$ zirkular polarisierte Welle

$$\vec{E}(t) = \begin{pmatrix} E_x\cos(\omega t) \\ E_y\sin(\omega t) \end{pmatrix} = |E_x| \cdot \begin{pmatrix} \cos(\omega t) \\ -\sin(\omega t) \end{pmatrix}$$

b) $|E_x| \neq |E_y| \Rightarrow$ elliptisch polarisiert



c) $|E_x| = 0 \vee |E_y| = 0$



10.3 Wellenausbreitung

unbewegt, homogen, isotrop, linear

ε, μ, σ bzw. $\underline{\varepsilon}, \underline{\mu}, \underline{\sigma}$

A) $\text{rot}\,\vec{H} = \sigma\vec{E} + \varepsilon\dot{\vec{E}} \quad |\frac{\partial}{\partial t} \Rightarrow \mu\,\text{rot}\,\dot{\vec{H}} = \mu\sigma\dot{\vec{E}} + \mu\varepsilon\ddot{\vec{E}}$

$\text{rot}\,\vec{E} = -\mu\dot{\vec{H}} \qquad \text{rot rot}\,\vec{E} = -\mu\,\text{rot}\,\dot{\vec{H}}$

$\underbrace{\text{rot rot}}\,\vec{E} = -\mu(\sigma\dot{\vec{E}} + \varepsilon\ddot{\vec{E}})$

$\nabla\times(\nabla\times\vec{E}) = \cancel{\nabla(\underbrace{\nabla\vec{E}}_{\frac{\sigma}{\varepsilon}=0})} - \Delta\vec{E}$

Telegraphengleichung:

$\Delta\vec{E} - \mu\sigma\dot{\vec{E}} - \mu\varepsilon\ddot{\vec{E}} = \vec{0}$

$\Delta\vec{H} - \mu\sigma\dot{\vec{H}} - \mu\varepsilon\ddot{\vec{H}} = \vec{0}$

HEZ: $\Delta\vec{\underline{E}} \underbrace{- j\omega\mu(\sigma + j\omega\varepsilon)}_{\underline{k}^2}\vec{\underline{E}} = \vec{0}$

$\Delta\vec{\underline{E}} + \underline{k}^2\vec{\underline{E}} = \vec{0} \qquad \underline{k}^2 = \omega^2\varepsilon\mu - j\omega\mu\sigma$

Helmholtz-Gl.

identisch H-Feld

Spezialfall $\sigma = 0$

$\underline{k}^2 = \omega^2\varepsilon\mu$

$k = \omega\sqrt{\varepsilon\mu}$

C: Zeitbereich $\quad \sigma = 0$

$\Delta\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \vec{0} = \Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$

$c = \frac{1}{\sqrt{\varepsilon\mu}} \qquad = \Delta\vec{E} - \frac{\partial^2\vec{E}}{\partial(ct)^2}$

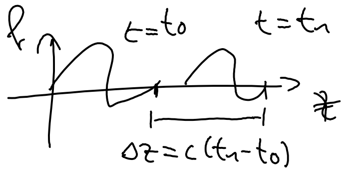
D: D'Alembertsche Lösung

$\vec{E} = E_x(z,t)\,\vec{e}_x$

$\Delta E_x - \frac{\partial^2 E_x}{\partial(ct)^2} = 0$

$$E_x(z,t) = f(ct-z) + g(ct+z)$$

f, g : beliebig, aber 2-fach stetig differenzierbar



Phase: $ct - z = \text{const}$

$z(t) = ct \quad \text{const}$

$$\frac{dz}{dt} = c$$