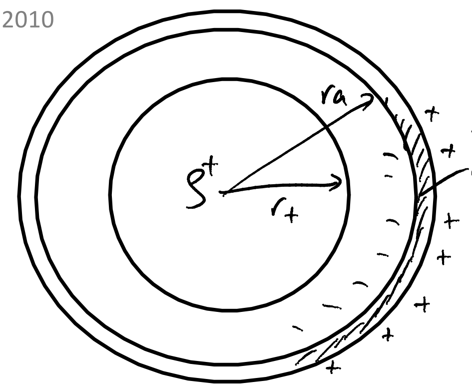
GGÜ 1

Freitag, 29. Oktober 2010
10:00

A 2

Kugel 1:

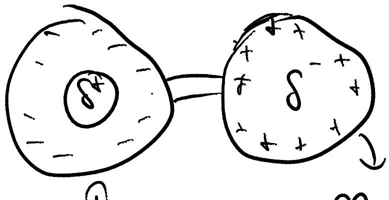


$\operatorname{div} \vec{D} = \rho$

$\oint_{F(V)} \vec{D}\, d\vec{F} = \int_V \rho\, dV$

$\int_{V_m} \rho\, dV = 0$, durch Feldfreiheit im idealen Leiter

Kugel 2 analog



äußere Oberfläche ungeladen $\quad t>0$

$t=\Delta t$ f. $\Delta t \to 0$

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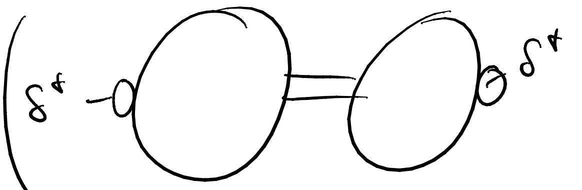


der Fall, wenn a) nicht gelten würde)

a) Außenraum feldfrei f. $t>0$

→ Gesamtladung unter der idealen Metalloberfläche für beide Kugeln identisch $= 0$

für beide Kugeln wünschen $= 0$

$\frac{\rho_+}{\rho_-}$ gesucht

$$\int_{\substack{r_+ \\ \text{Kugel 1}}} \rho_+ \, dV + \int_{\substack{\text{Kugel 2} \\ r_a}} \rho_- \, dV = 0$$

$$\Rightarrow \frac{4}{3}\pi r_+^3 \rho_+ + \frac{4}{3}\pi r_a^3 \rho_- = 0 \qquad r_+ = \frac{1}{2} r_a$$

$$\Rightarrow \frac{4}{3}\pi \frac{r_a^3}{8} \cdot \rho_+ + \frac{4}{3}\pi r_a^3 \rho_- = 0$$

$$\Rightarrow \frac{\rho_+}{\rho_-} = -8$$

b) $\operatorname{rot} \vec{H} = \vec{J} + \dot{\vec{D}} \qquad / \operatorname{div}(\dots)$

$$\operatorname{div}(\operatorname{rot} \vec{H}) = 0 = \operatorname{div} \vec{J} + \operatorname{div} \dot{\vec{D}}$$

$$\operatorname{div} \vec{D} = \rho$$

$$\Rightarrow \operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

c) $\vec{J} = \sigma_{Kugel} \vec{E} = \sigma_{Kugel} \frac{1}{\varepsilon} \vec{D}$

K-Gl. $\operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} = 0$

$$\Rightarrow \sigma_{Kugel} \cdot \frac{1}{\varepsilon} \operatorname{div} \vec{D} + \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \sigma_{Kugel} \cdot \frac{1}{\varepsilon} \rho + \frac{d\rho}{dt} = 0$$

$$\Rightarrow \frac{\dot{\rho}}{\rho} = -\frac{\sigma_{Kugel}}{\varepsilon} = -\frac{1}{\tau} \qquad \tau = \frac{\varepsilon}{\sigma_{Kugel}}$$

Lösung Kugel 1:

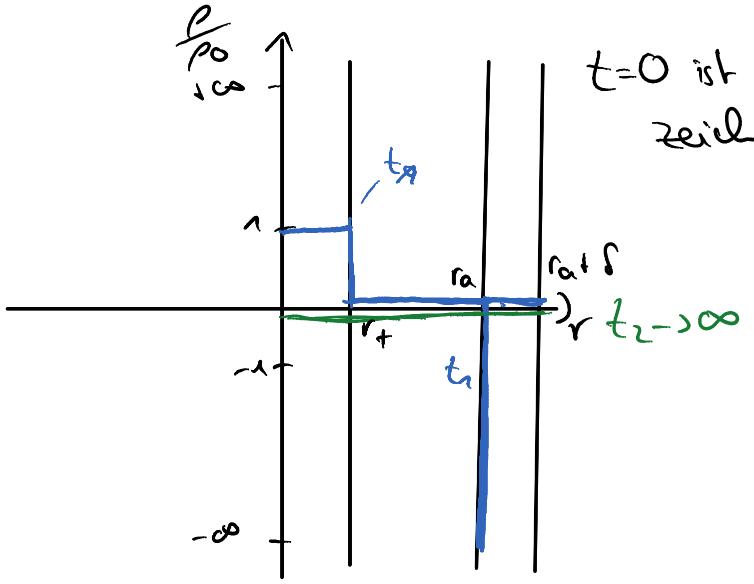
$$\rho(t) = \rho_+ e^{-t/\tau} \quad \text{f.} \quad 0 \le r \le r_+$$

$$\rho(t) \equiv 0 \quad \text{f.} \quad r_+ < r < r_a$$

Kugel 2:

$$\rho(t) = \rho_- e^{-t/\tau} \quad \text{f.} \quad 0 \le r < r_a$$

$-\frac{1}{8}\rho_+$



t=0 ist nicht zeichenbar!

d

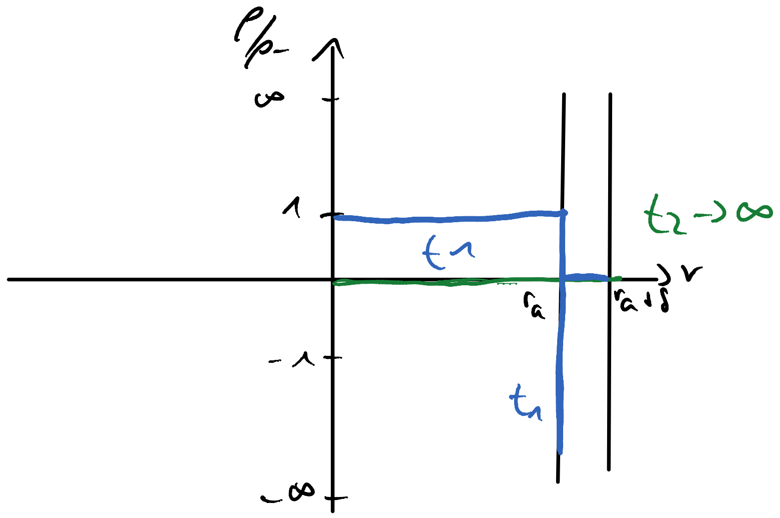
$\rho \neq 0$

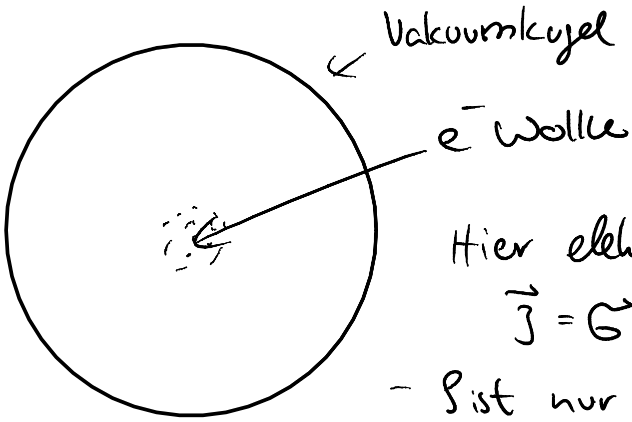
$d=0$

$\frac{Q}{V\to 0}$

⇒ Oberflächenladungsdichte

$\Rightarrow \rho \overset{\neq 0}{=} \pm\infty$





Hier elektrischen Leiter mit

$\vec{J} = \sigma \vec{E}$

- ρ ist nur Quelle des $\vec{D}$-Feldes und das Reservoir für Ladung ρ trägt nicht den Strom

- ohmsche Gesetz definiert den Strom

– Umnsere Vorzeichen definiert den Strom

e) $I_1 = -\frac{d}{dt}\int_{\substack{\text{Kugel}\\ r_F}} \rho(r,t)\, dV = -\frac{d}{dt}\int_{\substack{r_+\\ \text{Kugel}}} \rho(t)\, dV$

$= -\frac{4}{3}\pi r_+^3\ \rho_+ \frac{d}{dt}\left(e^{-t/\tau}\right) = -\frac{4}{3}\pi r_+^3\ \rho_+\left(-\frac{1}{\tau}\right)e^{-t/\tau}$

$= \frac{4}{3}\pi r_+^3\ \frac{\rho_+}{\tau} e^{-t/\tau} = \frac{4}{3}\pi r_a^3\ \frac{1}{8}\ \frac{\rho_+}{\tau}\exp\left(-\frac{t}{\tau}\right)$

$\forall t > 0$

Kugel 2: $I_2 = -\frac{d}{dt}\int_{\substack{\text{Kugel}\\ r_F}} \rho(r,t) dV = -\frac{d}{dt}\left[\rho_- e^{-\frac{t}{\tau}}\right]\cdot \frac{4}{3}\pi r_F^3$

Nur Abnahme von ρ_- innerhalb von r_F zählt!

$= \frac{1}{8}\rho_+ \frac{4}{3}\pi r_F^3\cdot\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}} \qquad \| \ \rho_- = -\frac{1}{8}\rho_+$

$= -\frac{\rho_+}{6}\pi r_F^3\frac{1}{\tau}\ e^{-\frac{t}{\tau}} = -\frac{\rho_+}{6}\cdot\frac{\pi}{\tau}\cdot\frac{27}{64} r_a^3 \exp\left(-\frac{t}{\tau}\right)$

$\frac{I_1}{I_2} = -\frac{64}{27}$

f) $I_m(t) = 0 \quad t > 0$

Aufgabe 1

σ_0 im Medium

$\sigma_A = 0$ im Außenbereich

$\phi(G_1, G_2) = 0V \qquad \phi(G_3) = U$

a) $\vec{E} = -\operatorname{grad}\phi$

Rot $\vec{E} = 0 \Rightarrow \vec{E}_t(x=0^-) = \vec{E}_t(x=0^+)$

$\frac{\vec{J}_t(x=0^-)}{\sigma_A} = \frac{\vec{J}_t(0^+)}{\sigma_0}$

$\Rightarrow \vec{J}_t(x=0^-) = \vec{0} = \frac{\sigma_A}{\sigma_0}\vec{J}_t(x=0^+)$

Div $\vec{J} = 0$

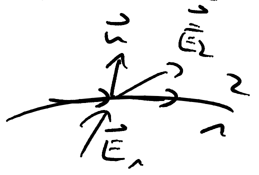
$\Rightarrow J_n(x=0^-) = J_n(x=0^+)$

$\sigma_A E_n(x=0^-) = \sigma_0 E_n(x=0^+)$

$J_n(x=0^+) = 0 \wedge E_n(x=0) = 0$

Einschub:

$\operatorname{Rot} \vec{E} = \vec{n} \left(\vec{E}_2 - \vec{E}_1\right) = 0$



$\frac{\partial}{\partial z} \phi = 0$

$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \Rightarrow$ bei $x=0$ $\vec{E} = \begin{pmatrix} E_n \\ E_t \end{pmatrix}$

$E_n\big|_{x=0} = -\frac{\partial}{\partial x} \phi = 0$

b) $\phi(x,y) = f(x)\, g(y)$

Laplacegl. $\Delta \phi = 0$

Poissongl. $\Delta \phi = -\rho_V$

$\Delta \phi = \frac{\partial^2}{\partial x^2} \phi + \frac{\partial^2}{\partial y^2} \phi = 0 \qquad | : \phi$

$\frac{\Delta \phi}{\phi} = \frac{\frac{\partial^2}{\partial x^2} f(x)}{f(x)} + \frac{\frac{\partial^2}{\partial y^2} g(y)}{g(y)} = 0 = -k_x^2 - k_y^2 = 0$

$k_x^2 = -k_y^2$

$$\left[\begin{array}{l} f_n(x) = A_{xn} \cos(k_{xn} x) + B_{xn} \sin(k_{xn} x) \\ g_n(y) = A_{yn} \cos(k_{yn} y) + B_{yn} \sin(k_{yn} y) \\ \qquad k_{xn}, k_{yn} \neq 0 \end{array}\right]$$

für $k_{xn} = k_{yn} = 0$

$f_0(x) = A_{x0} + B_{x0}\, x$

$g_0(y) = A_{y0} + B_{y0}\, y$

c) G_1: $\phi(y=0) = 0 \Rightarrow g(y=0) = 0$

$A_{yn} = 0 \qquad n \in \mathbb{N}_0$

$$G_4: \left.\frac{\partial \phi}{\partial x}\right|_{x=0} = 0 \Rightarrow \left.\frac{\partial f(x)}{\partial x}\right|_{x=0} = 0$$

$$B_{xn} = 0 \quad n \in \mathbb{N}_0$$

$$\phi = \underbrace{A_{x0} B_{y0}}_{c_0} y + \sum_{n=1}^{\infty} \underbrace{A_{xn} B_{yn}}_{c_n} \sin(k_{yn} y) \cos(k_{xn} x)$$

d) G_3 $\quad \phi(y=b) = U \qquad 0<x<a$

$$= c_0 b + \sum_{n=1}^{\infty} c_n \cdot \cos(k_{xn} x) \sin(k_{yn} b)$$

$$\Rightarrow c_0 b = U \qquad c_0 = \frac{U}{b}$$

$$c_n \neq 0$$

$$\Rightarrow \sin(k_{yn} b) = 0 \qquad \forall n$$

$$k_{yn} = \frac{n\pi}{b} \qquad k_{xn} = \pm j \frac{n\pi}{b}$$

$$\phi = \frac{U}{b} y + \sum_{n=1}^{\infty} c_n \underbrace{\cos\left(\pm j \frac{n\pi}{b} x\right)}_{\cosh\left(\frac{n\pi}{b} x\right)} \sin\left(\frac{n\pi}{b} y\right)$$

G_2 $\quad \phi(x=a, y) = 0$

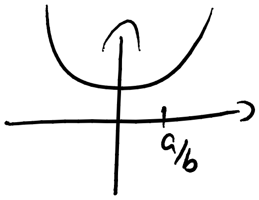
$$\int_0^b \phi(x=a,y) \sin\left(\frac{m\pi}{b} y\right) = \int_0^b 0 \, dy = 0$$

$$= \int_0^b \frac{U}{b} y \sin\left(\frac{m\pi}{b} y\right) dy + \underbrace{\sum_{n=1}^{\infty} c_n \cosh\left(\frac{n\pi}{b} a\right) \cdot \underbrace{\int_0^b \sin\left(\frac{n\pi}{b} y\right) \sin\left(\frac{m\pi}{b} y\right) dy}_{\frac{b}{2} \delta_{m,n}}}_{c_m \cosh\left(\frac{m\pi}{b} a\right) \frac{b}{2}}$$

$$= -\frac{Ub}{b} \frac{\cos(m\pi)}{\frac{m\pi}{b}} = -\frac{Ub \, (-1)^m}{m\pi}$$

$$c_m = \frac{2U \, (-1)^m}{m\pi \cosh\left(m\pi \frac{a}{b}\right)}$$

$$c_n = \frac{2U\,(-1)^{\dots}}{n\pi\cosh\left(n\pi\frac{a}{b}\right)}$$



$$\lim_{n\to\infty}(c_n) = 0$$

$$n \in \mathbb{N}^+$$