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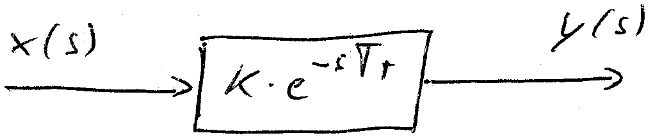
A18.) Totzeitglied

$$y(t) = k \cdot x(t - T_t)$$

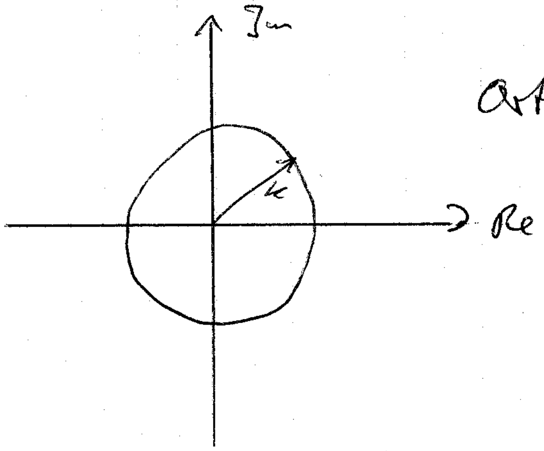
T_t : Totzeit

$$G(s) = k \cdot e^{-sT_t}$$
$$y(s) = k \cdot X(s)\, e^{-sT_t}$$



a.) $G(i\omega) = k \cdot e^{-i\omega T_t}$
$$= k \cdot (\cos(\omega T_t) + i \sin(\omega T_t))$$

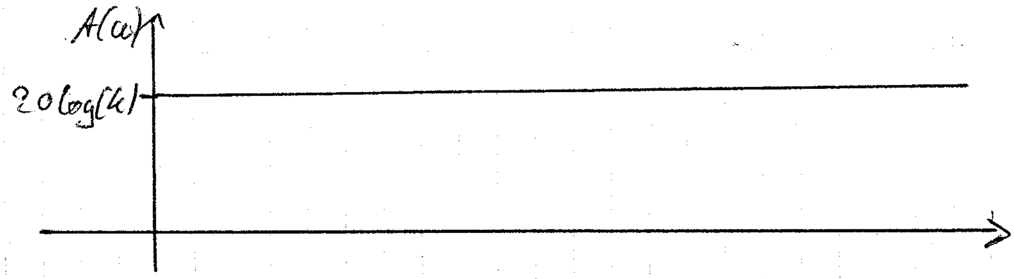
Ortskurve



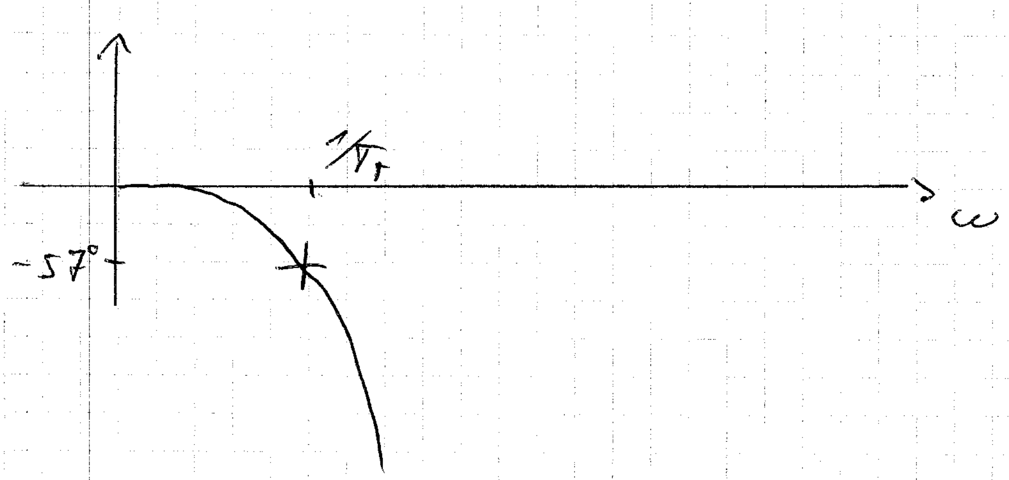
$|G(i\omega)| = k$ $\qquad$ $|G(i\omega)|_{dB} = 20 \log(k)$

$\varphi_{rad}(\omega) = -\omega T_t$
$\varphi(\omega) = -\omega T_t \cdot \frac{180°}{\pi}$

} Umrechnung in Grad erforderlich



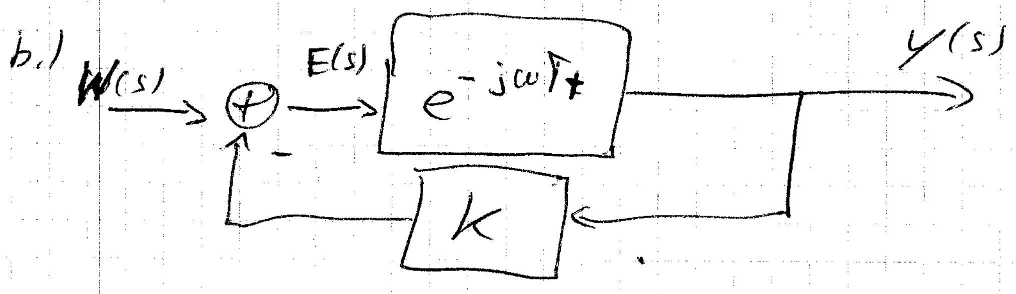
$\varphi(\omega = \frac{1}{T_t}) = -57°$



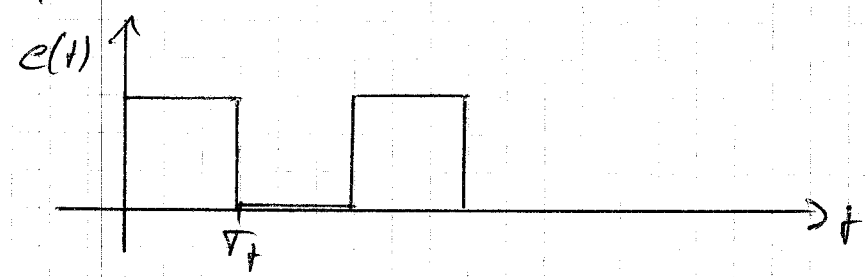
Totzeitglied hat sehr starken Phasenabfall im hohen Frequenzbereich.

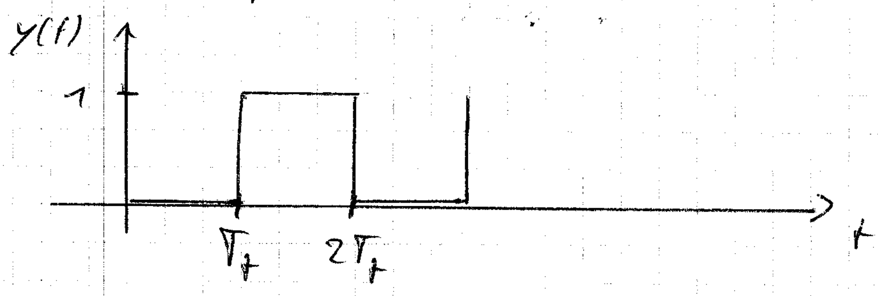
=> beim Reglerentwurf ist darauf zu achten, dass ω_D links (> 1 Dekade) von $\frac{1}{T_t}$ liegt.

b.)



$y(t) = K\,x(t - T_t)$





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c.) System instabil für $K > 1$

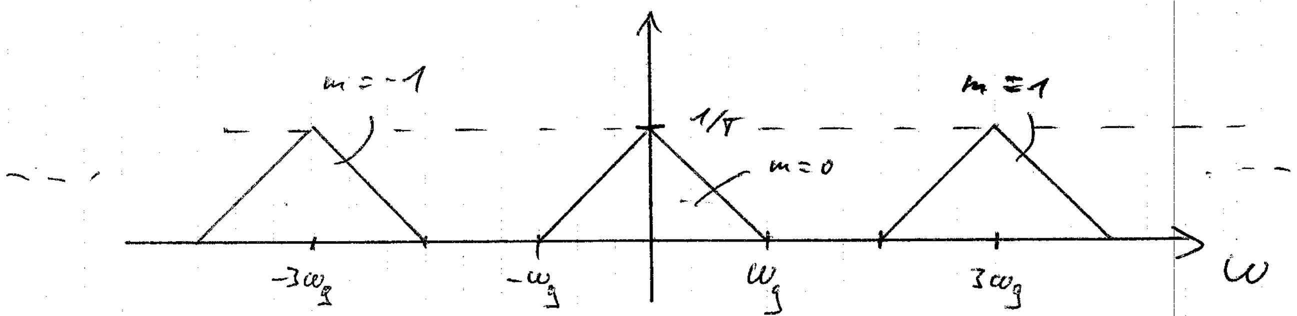
A37.) Ideale Abtastung

a.)
$$x_a(t) = x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t - kT)$$
$$= \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t - kT)$$

b.)
$$x_a(i\omega) = \left[X(\omega) * \frac{2\pi}{T} \sum_{m=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi m}{T}\right)\right] \cdot \frac{1}{2\pi}$$
$$= \frac{1}{T} \sum_{m=-\infty}^{\infty} X\left(\omega - \frac{2\pi m}{T}\right)$$

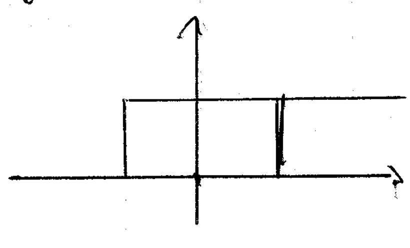
$$\omega_a = \frac{2\pi}{T}$$

c.) $f_a = 3 \cdot f_g$ $\quad \omega_a = 3\,\omega_g$

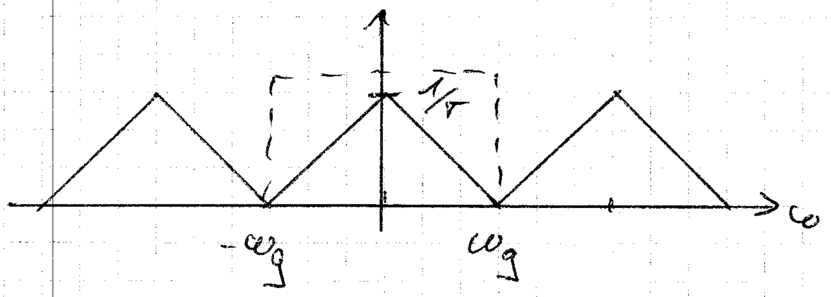


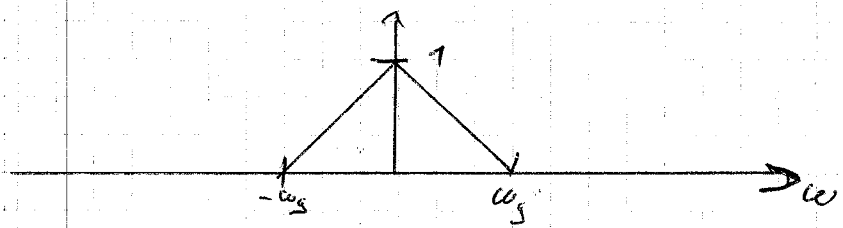
d.) $\omega_a \geq 2\omega_g$

=> Eingangssignal kann wieder gewonnen werden, da keine Überlappungen im Spektrum



e.) $\omega_a = 2\omega_g$





Tiefpass: $H(i\omega) = \text{rect}\left(\frac{\omega}{2\omega_g}\right) \cdot T$

$$T = \frac{2\pi}{\omega_a} \overset{\omega_a = 2\omega_g}{=} \frac{\pi}{\omega_g}$$

Durch Tiefpassfilterung kann das Spektrum des ursprünglichen Signals exakt wiedergewonnen werden.

f.) $\hat{X}(i\omega) = X_a(i\omega) \frac{\pi}{\omega_g} \cdot \text{rect}\left(\frac{\omega}{2\omega_g}\right)$

$$\hat{x}(t) = x_a(t) * \frac{\pi}{\omega_g} \cdot \frac{\omega_g}{\pi} \cdot si(\omega_g t)$$

$$= \left[\sum_{n=-\infty}^{\infty} x(nT) \cdot \delta(t - nT)\right] * si(\omega_g t)$$

$$= \sum_{n=-\infty}^{\infty} x(nT) \cdot si(\omega_g(t - nT))$$

$\rightarrow$ Im Zeitbereich Interpolation

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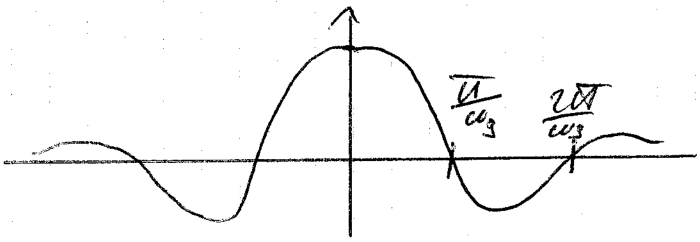
→ im Zeitbereich Interpolation
→ für exakte Rekonstruktion reichen die Abtastwerte $x(nT)$

$$si(\omega_g t) = \frac{\sin(\omega_g t)}{\omega_g t}$$

$$si(0) = 1$$

$$si(\omega_g t) = 0 \quad \text{für } \omega_g t = n\pi \; ; \; n \neq 0$$

$$t = \frac{n\pi}{\omega_g}$$



g.: $$X(i\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$$

z.zg.: $$X_a(i\omega) = \sum_{n=-\infty}^{\infty} x(nT) \cdot e^{-i\omega nT}$$

$$X_a(i\omega) = \int_{-\infty}^{\infty} x_a(t) e^{-i\omega t} dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT)\, e^{-i\omega t} dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT)\, e^{-i\omega nT} \cdot dt$$

$$= \underline{\underline{\sum_{n=-\infty}^{\infty} x(nT) e^{-i\omega nT}}} \underbrace{\int_{-\infty}^{\infty} \delta(t - nT)\, dt}_{=1}$$