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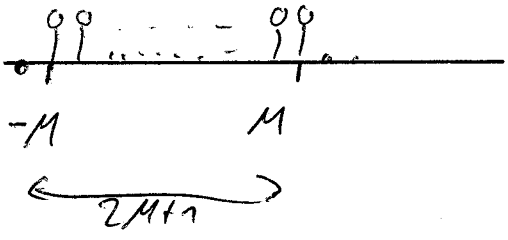
4.7.)

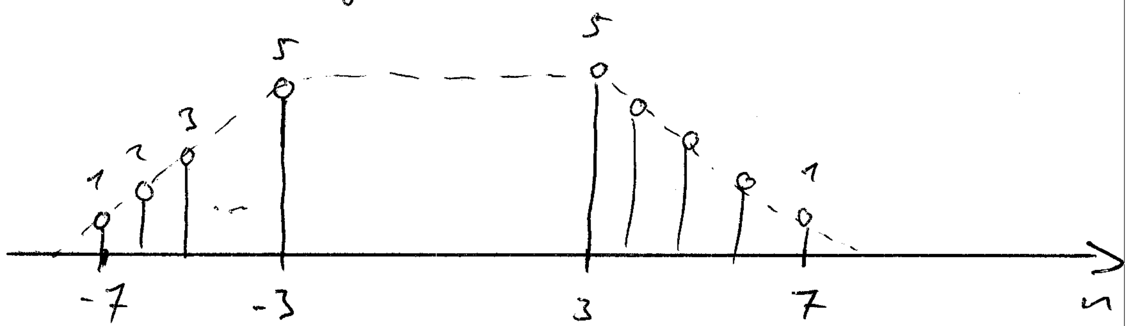


$s_1(n)$: $M=2 \Rightarrow$ Länge 5

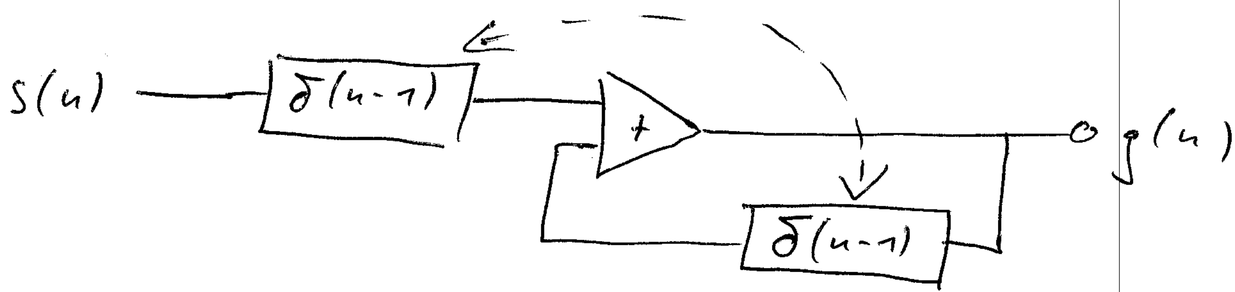
$s_2(n)$: $M=5 \Rightarrow$ Länge 11

Faltungsergebnis $g(n) = s_1(n) * s_2(n)$

Länge $5 + 11 - 1 = 15$



4.8.)

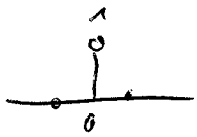


Zeitverzögerung um 1 Abtaststück

$g(n) = \delta(n-1) + g(n-1)$

Eingang $\delta(n) \rightarrow$ Ausgang $h(n)$

kausal, da $h(n) = 0 \quad n<0$



$n=0$: $h(0) = \delta(n-1) + h(-1) = 0$

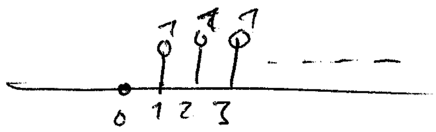
$n=1$: $h(1) = \delta(0) + h(0) = 1$

$n=2$: $h(2) = \delta(1) + h(1) = 1$

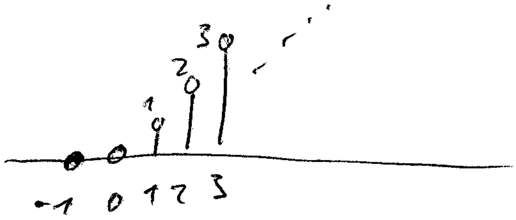
$n=3$: $h(3) = \delta(2) + h(2) = 1$

$n>3$: $h(n) = 1$

$h(n) = \varepsilon(n-1)$



b.) $s(n) = \varepsilon(n) \Rightarrow g(n) = \sum_{m=-\infty}^{n} h(m)$



wächst für $n \to \infty$ über alle Grenzen

$\to$ System instabil

$\left[\sum_{n=-\infty}^{\infty} |h(n)| \to \infty \right]$

4.9.) $\Rightarrow$ zum Selberrechnen

4.10.) $h(n) = \left(\frac{1}{5}\right)^n \varepsilon(n) = b^n \varepsilon(n)$ mit $b = \frac{1}{5}$

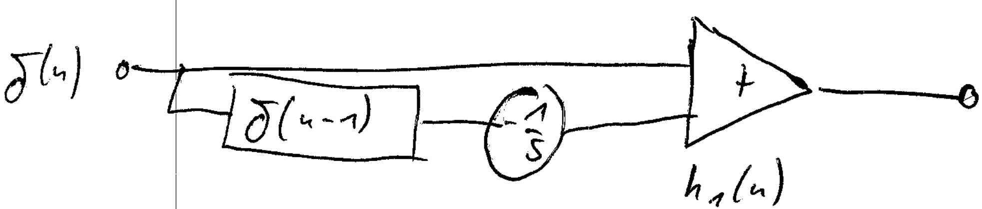
a) $h(n) + a\,h(n-1) = b^n \varepsilon(n) + a\,b^{n-1} \varepsilon(n-1)$

$= \underbrace{\delta(n) + b^n \varepsilon(n-1)}_{b^n \cdot \varepsilon(n)} + a\,b^{n-1} \varepsilon(n-1) \overset{!}{=} \delta(n)$

$\Rightarrow b^n + a\,b^{n-1} \overset{!}{=} 0 \Rightarrow a = -b = -\frac{1}{5}$

b.) $h(n) * h_1(n) = \delta(n)$

aus a.): $h(n) * \underbrace{\left[\delta(n) - \frac{1}{5}\,\delta(n-1)\right]}_{h_1(n)} = \delta(n)$



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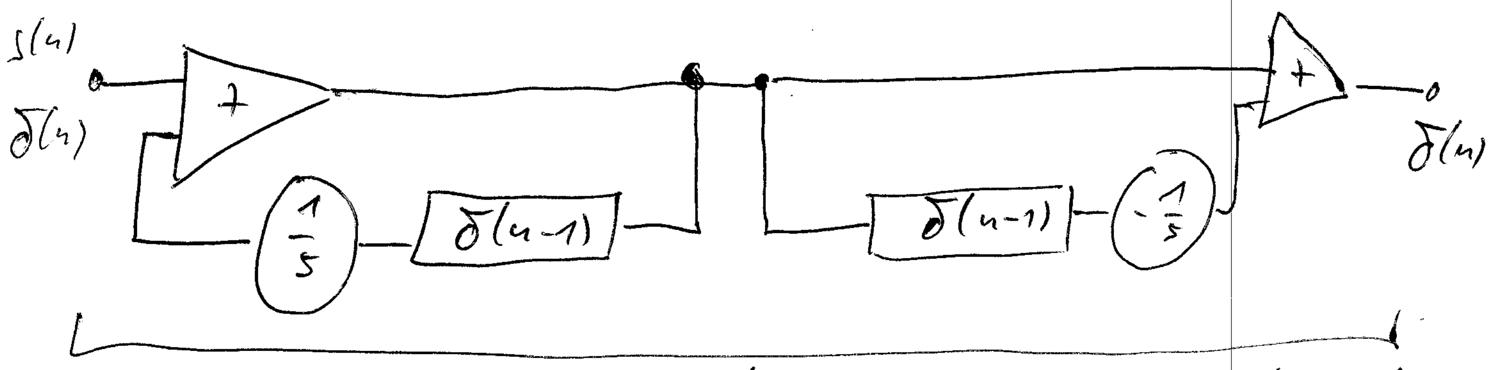
c.) Gesamtsystem



Gesamtsystem $\delta(n) \rightarrow s(n) * \underbrace{h(n) * h_1(n)}_{\delta(n)} = s(n)$

FIR-System hebt die Wirkung des IIR-Systems auf!

4.11.)

$$g(n) = \frac{1}{4} g(n-1) + s(n)$$

$$s(n) = \delta(n) \quad \Rightarrow \quad g(n) = h(n)$$

$$h(n) = \delta(n) + \frac{1}{4} h(n-1)$$

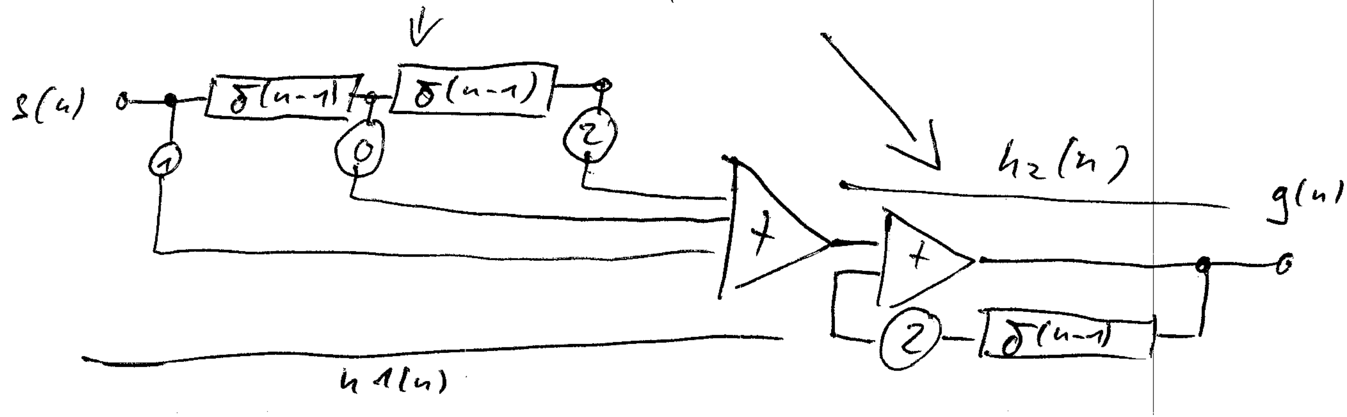
$$= \delta(n) + \frac{1}{4}\left[\delta(n-1) + \frac{1}{4} h(n-2)\right]$$

$$= \delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{4^2} h(n-2)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k \delta(n-k) = \left(\frac{1}{4}\right)^k \cdot \varepsilon(n)$$

4.12.)

$$g(n) = \underbrace{s(n) + 2s(n-2)}_{\text{FIR-Filter}} + \underbrace{2g(n-1)}_{\text{IIR-Filter}}$$



b.) FIR: $h_1(n) = \delta(n) + 2 \cdot \delta(n-2)$

IIR: $h_2(n) = 2^n \cdot \varepsilon(n)$

Gesamt: $h(n) = h_1(n) * h_2(n)$

$$= h_2(n) * [\delta(n) + 2\,\delta(n-2)]$$
$$= h_2(n) + 2 \cdot h_2(n-2)$$
$$= 2^n \varepsilon(n) + 2(2^{n-2}\varepsilon(n-2))$$
$$= 2^n [\varepsilon(n) + 2 \cdot 2^{-2} \varepsilon(n-2)]$$
$$= 2^n [\varepsilon(n) + \tfrac{1}{2} \varepsilon(n-2)]$$

Anmerkung: $h(n) \to \infty$ für $n \to \infty$

instabiles System!

4.13.) $s_a(t) = s(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$$S_a(f) = S(f) * \frac{1}{T} \sum_{n=-\infty} \delta(t - \frac{k}{T})$$

$r = \frac{1}{T}$ Abtastrate

Alternativ mit Siebeigenschaft

$$s_a(t) = \sum_{n=-\infty}^{\infty} s(nT) \cdot \delta(t - nT)$$

$$S_a(f) = \sum_{n=-\infty}^{\infty} s(nT) \cdot e^{-j2\pi f nT}$$

mit Normierung $T = 1 \Leftrightarrow r = 1$

$$S_a(f) = \sum_{n=-\infty}^{\infty} s(n) e^{-j2\pi f n}$$

$$s(n) = \int_{-1/2}^{1/2} S_a(f) \cdot e^{j2\pi f n}\, df$$

GET 4 GÜ 10 „Siebeigenschaft" 23. JUNI 2010

a.) $\delta(n-n_0) \circ\!\!-\!\!\bullet \sum_{n=-\infty}^{\infty} \delta(n-n_0) e^{-j2\pi f n} = e^{-j2\pi f n_0}$

b.) $s(n-n_0) \circ\!\!-\!\!\bullet \sum_{n=-\infty}^{\infty} s(n-n_0) e^{-j2\pi f n} \underset{n-n_0=m}{=} \sum_{m=-\infty}^{\infty} s(m) \cdot e^{-j2\pi f (m+n_0)}$

$= \underbrace{\sum_{m=-\infty}^{\infty} s(m) \cdot e^{-j2\pi f m}}_{S_a(f)} \cdot e^{-j2\pi f n_0}$

Alternativ: $s(n-n_0) = s(n) * \delta(n-n_0)$

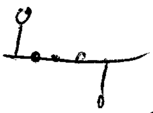
$\circ\!\!\!|\!\!\!\bullet$ => mit a.)

$S_a(f) \cdot F\{\delta(n-n_0)\} = S_a(f) \cdot e^{-j2\pi f n_0}$

c.) $\delta(n-1) + \delta(n+1) \circ\!\!-\!\!\bullet \underbrace{e^{-j2\pi f} + e^{j2\pi f}}_{\text{mit a.) mit } n_0 = \pm 1} = 2\cos(2\pi f)$

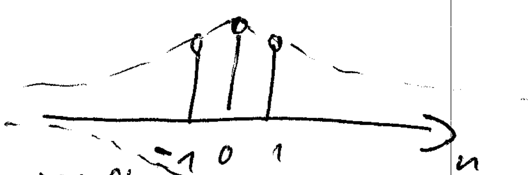
gerade

d.) $\delta(n+2) - \delta(n-2) \underset{\text{mit a.)}}{\bullet\!\!-\!\!\bullet} e^{j4\pi f} - e^{-j4\pi f} = 2j \sin(4\pi f)$



ungerade

$0 < a < 1$



e.) $s(n) = a^{|n|}$

$S_a(f) = \sum_{n=-\infty}^{\infty} a^{|n|} e^{-j2\pi f n} = \underbrace{\sum_{n=0}^{\infty} (a \cdot e^{-j2\pi f})^n}_{\text{kausal}} + \underbrace{\sum_{n=-\infty}^{-1} (a\, e^{j2\pi f})^{-n}}_{\text{antikausal}}$

$\sum_{n=-\infty}^{-1} (a\, e^{j2\pi f})^{-n} = \sum_{n=1}^{\infty} (a\, e^{j2\pi f})^n$

mit $\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$ für $|q|<1$

$\Rightarrow$ für $|a|<1$

$$S_a(f) = \frac{1}{1-ae^{-j2\pi f}} + \frac{1}{1-ae^{j2\pi f}} - 1$$

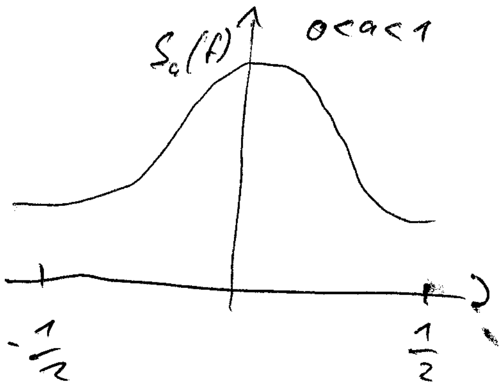
$\Downarrow$ weil die rechte Summe $1 \ldots \infty$

$$= \frac{1-ae^{j2\pi f} + 1 - ae^{-j2\pi f}}{1-ae^{j2\pi f}-ae^{-j2\pi f}+a^2} - 1$$

$$= \frac{2-2a\cos(2\pi f)}{1+a^2-2a\cos(2\pi f)} - 1$$

$$= \frac{2-\cancel{2a\cos(2\pi f)}-1-a^2\cancel{+2a\cos(2\pi f)}}{1+a^2-2a\cos(2\pi f)}$$

$$= \frac{1-a^2}{1+a^2-2a\cos(2\pi f)}$$



Nenner minimal bei $f=0$
maximal bei $f=\pm\frac{1}{2}$

f.) $s(n)-s(n-1) \circ\!\!-\!\!\bullet\ S_a(f)\cdot\left(1-e^{-j2\pi f}\right)$

$$= S_a(f)\cdot 2j\sin(\pi f)\,e^{-j\pi f}$$

g.) $s(n)+s(n-1) \circ\!\!-\!\!\bullet\ S_a(f)\cdot\left(1+e^{-j2\pi f}\right) = S_a(f)\, 2\cos(\pi f)\cdot e^{-j\pi f}$

h.) $n\cdot s(n) \circ\!\!-\!\!\bullet$

$$s(n) \circ\!\!-\!\!\bullet\ S_a(f) \Rightarrow \frac{d}{df}S_a(f) = \sum_{n=-\infty}^{\infty} -j2\pi\,[n\cdot s(n)]\,e^{-j2\pi f n}$$

$$\frac{d}{df}e^{-j2\pi f n} = -j2\pi n\, e^{-j2\pi f n}$$

$$-j2\pi\ [redacted]\ n\cdot s(n) \circ\!\!-\!\!\bullet\ \frac{d}{df}S_a(f) \quad \Big|\cdot j\frac{1}{2\pi}$$

$$\Rightarrow n(s(n)) \circ\!\!-\!\!\bullet\ j\frac{1}{2\pi}\frac{d}{df}S_a(f)$$

i.) $s(n) = \left(\frac{1}{2}\right)^{n-1} \varepsilon(n-1) = \left[\left(\frac{1}{2}\right)^n \varepsilon(n)\right] * \delta(n-1)$

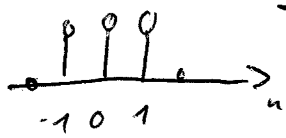
$$S_a(f) = \left(\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j2\pi nf}\right) \cdot e^{-j2\pi f} \quad \text{mit} \sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

$|q| < 1$

$$= \frac{1}{1-\frac{1}{2}e^{-j2\pi f}} e^{-j2\pi f} = \frac{1}{e^{j2\pi f} - \frac{1}{2}}$$

$s(n)$

4.14.)



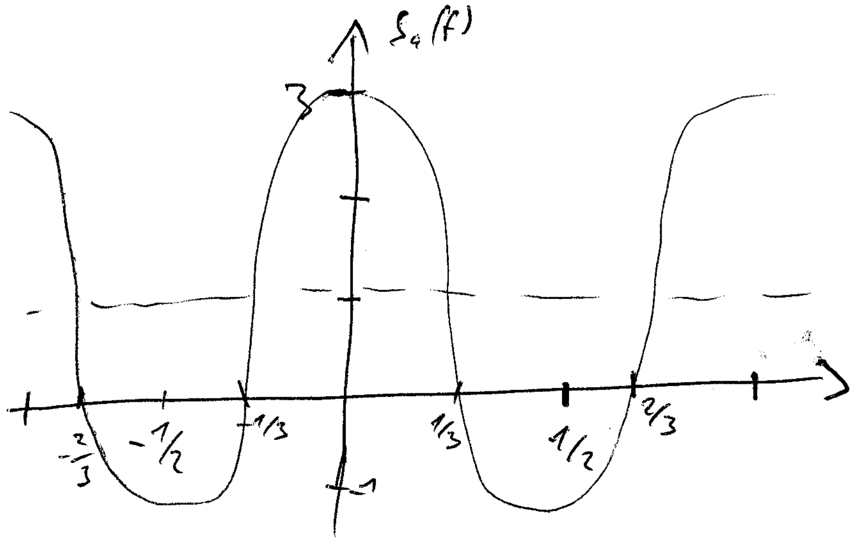
a.) $S_a(f) = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j2\pi fn}$

$$= [\delta(n-1) + \delta(n) + \delta(n+1)] e^{-j2\pi fn}$$

$$= e^{-j2\pi f} + 1 + e^{j2\pi f} = 1 + 2\cos(2\pi f)$$

$S_a(f) = 0$ bei $1 + 2\cos(2\pi f) = 0 \Rightarrow \cos(2\pi f) = -\frac{1}{2}$

$f = \frac{1}{3}$ und $f = \frac{2}{3}$



b.) $$S(z) = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} = \left[\delta(n+1) + \delta(n) + \delta(n-1)\right] z^{-n}$$

$$= z^{1} + 1 + z^{-1}$$

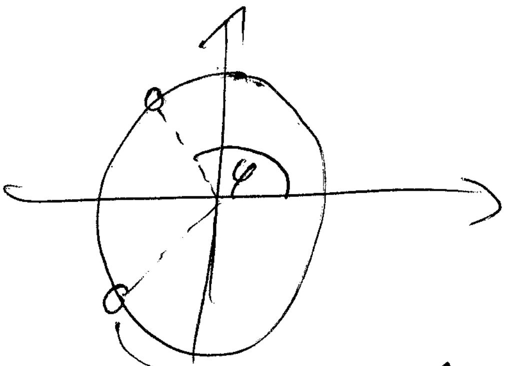
$$= \frac{z^2 + z + 1}{z}$$

Nullstellen $z_{n1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 1} = -\frac{1}{2} \pm \frac{j}{2}\sqrt{3}$

Polstelle $z_p = 0$

$$|z_n|^2 = \left(\frac{1}{2}\right) + \frac{3}{2^2} = 1$$

Nullstellen auf Einheitskreis



$$\varphi = \frac{2\pi}{3} \mathrel{\hat{=}} f = \frac{1}{3}$$

$$\hat{=}\ f = \frac{2}{3}$$

c.) $$S(z)\Big|_{z=e^{j2\pi f}} = S_a(f)$$

(auf Einheitskreis)

$$z^{1} + 1 + z^{-1}\Big|_{z=e^{j2\pi f}} = 1 + 2\cos(2\pi f)$$