ET IV Kleingruppe 1

Aufgabe 1.4

$$u_1(t) = L \cdot \frac{di(t)}{dt} + u_2(t)$$

$$u_2(t) = R \cdot i(t) \quad \Rightarrow i(t) = \frac{u_2(t)}{R}$$

$$\Rightarrow u_1(t) = \frac{L}{R} \cdot \frac{du_2(t)}{dt} + u_2(t)$$

$$u_1(t) = B \cdot e^{j2\pi f t}$$

$$u_2(t) = B \cdot H(f) \cdot e^{j2\pi f t}$$

$$\Rightarrow B \cdot e^{j2\pi f t} = \frac{L}{R} \frac{d}{dt}\left(B \cdot H(f) \cdot e^{j2\pi f t}\right) + B \cdot H(f) \cdot e^{j2\pi f t}$$

$$\Leftrightarrow B \cdot e^{j2\pi f t} = \frac{L}{R} j 2\pi f \cdot B \cdot H(f) \cdot e^{j2\pi f t} + B \cdot H(f) e^{j2\pi f t}$$

$$\Leftrightarrow 1 = H(f) \cdot \left[j \frac{L}{R} \cdot 2\pi f + 1 \right]$$

$$\Rightarrow H(f) = \frac{1}{1 + j 2\pi f \frac{L}{R}}$$

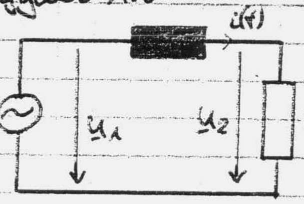
$$|H(f)| = \frac{1 - j 2\pi f \frac{L}{R}}{1 + (2\pi f \frac{L}{R})^2} = \frac{1}{1 + (2\pi f \frac{L}{R})^2} - j \frac{2\pi f \frac{L}{R}}{1 + (2\pi f \frac{L}{R})^2}$$

$$\varphi(f) = \arctan\left(\frac{\mathrm{Im}\{H(f)\}}{\mathrm{Re}\{H(f)\}}\right)$$

$$= \arctan\left(-\frac{L}{R} \cdot 2\pi f\right) = -\arctan(2\pi f T) \quad , \text{mit } T = \frac{L}{R}$$

Zur Übereinstimmung muss gelten: $\frac{L}{R} = R \cdot C$

Aufgabe 1.6



$$, R' = R \| R_2 = \frac{R \cdot R_2}{R + R_2}$$

$$u_1(t) = L \cdot \frac{di(t)}{dt} + u_2(t)$$

Rechenweg siehe Aufgabe 1.4, ersetze dabei R durch R'

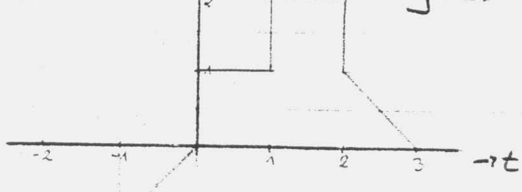
$$\Rightarrow H(f) = \frac{1}{1 + j 2\pi f \frac{L}{R'}}$$

$$|H(f)| = \frac{1}{1 + (2\pi f \frac{L}{R'})^2} - j \frac{2\pi f \frac{L}{R'}}{1 + (2\pi f \frac{L}{R'})^2}$$

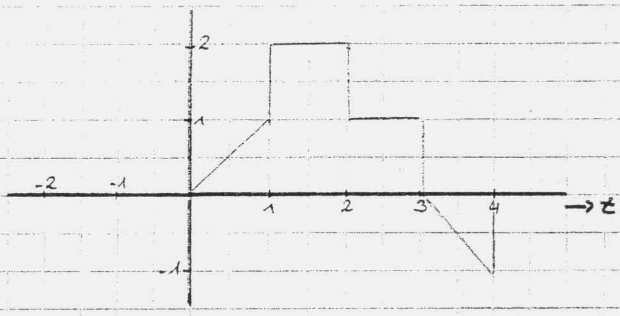
$$\varphi(f) = -\arctan(2\pi f T) \quad , \text{mit } T = \frac{L}{R'}$$

Aufgabe 2.3

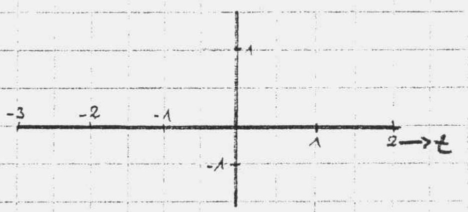
a) $s(t-1) = s\left(\frac{t-1}{1}\right)$ → Verschiebung des Signals um 1 Einheit nach recht



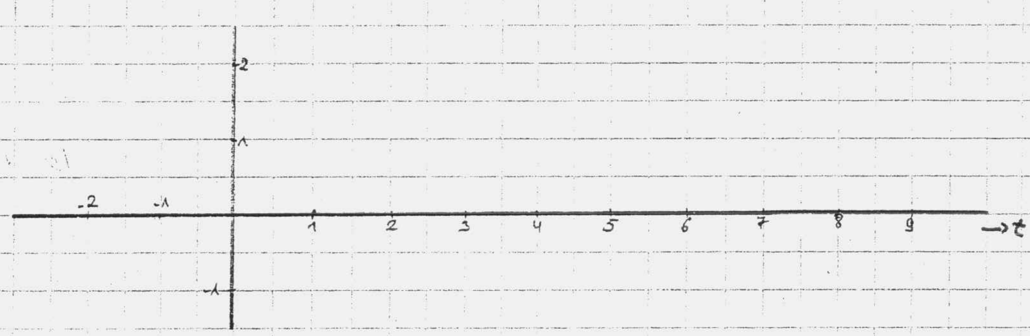
b) $s(2-t) = s\left(-\frac{t-2}{1}\right)$ → Spiegelung des Signals und dann um 2 Einheiten nach rechts verschieben



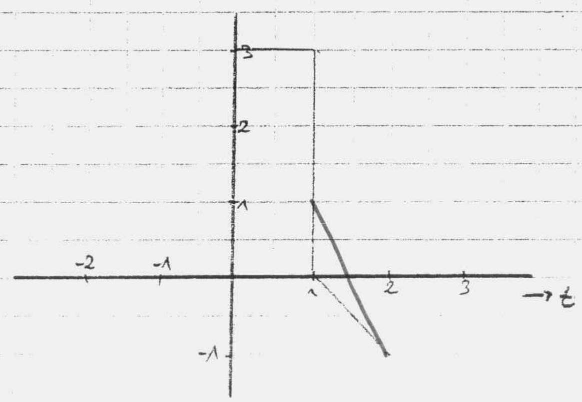
c) $s(2t+1) = s\left(\frac{t+1/2}{1/2}\right)$ → Stauchen mit Faktor 2, dann um $\frac{1}{2}$ Einheit nach links verschieben



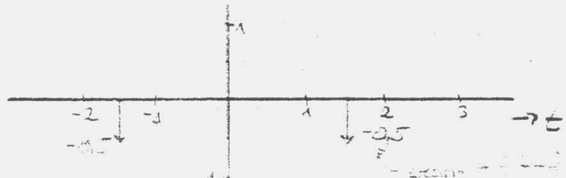
d) $s(4-\frac{t}{2}) = s\left(-\frac{t-8}{2}\right)$ → Spiegelung, mit Faktor 2 dehnen und um 8 Einheiten nach rechts verschieben



e) $[s(t)+s(-t)]\cdot\varepsilon(t)$ → Original und Spiegelung addieren, wobei $[s(t)+s(-t)]\cdot\varepsilon(t) = 0$, für $t < 0$



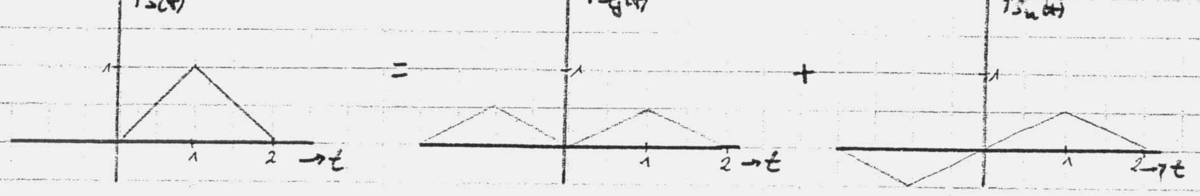
f) $s(t)\left[\delta(t+\frac{3}{2}) - \delta(t-\frac{3}{2})\right]$ → Multiplikation des Signals mit zwei Dirac-Impulsen



ET IV Hausaufgabe 1

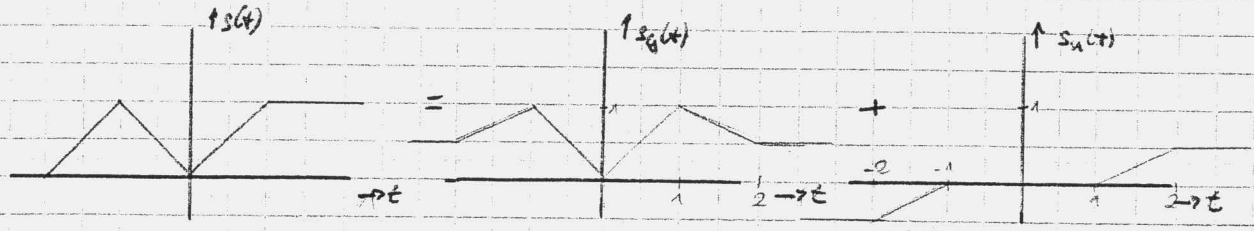
Aufgabe 2.4

i) $s(t) = \Lambda(t-1) = \begin{cases} t & , 0 \le t \le 1 \\ 2-t & , 1 < t \le 2 \\ 0 & , \text{sonst} \end{cases}$



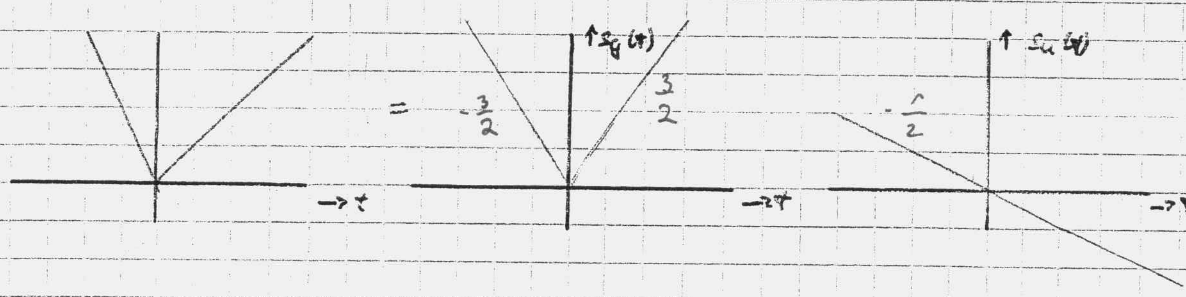
ii)

$$s(t) = \begin{cases} 0 & , \text{sonst} \\ 2+t & , -2 \le t \le -1 \\ |t| & , -1 < t < 1 \\ 1 & , 1 \le t \end{cases}$$

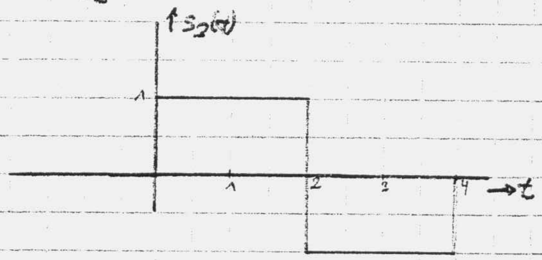


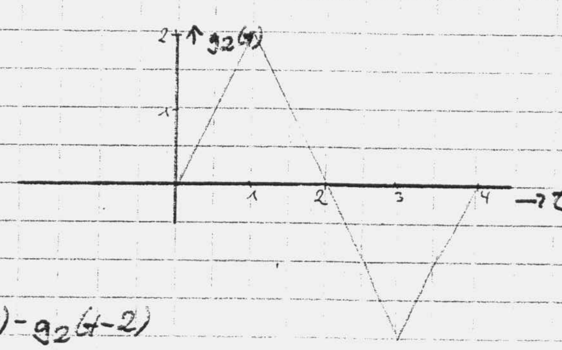
iii)

$$s(t) = \begin{cases} -2t & , t \le 0 \\ t & , t > 0 \end{cases}$$

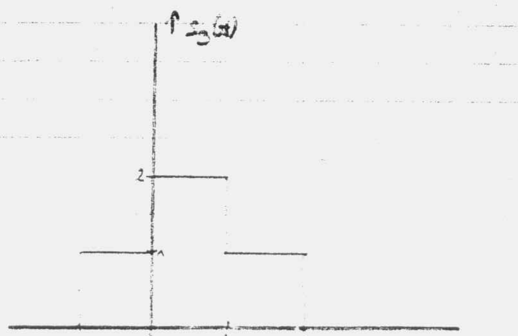


Aufgabe 2.5

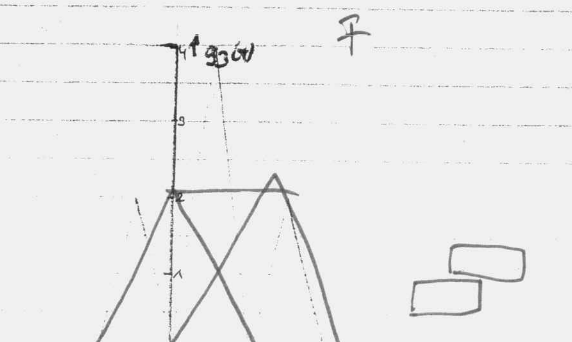




$s_2(t) = s_1(t) - s_1(t-2) \quad \overset{\text{LTI-System}}{\Rightarrow} \quad g_2(t) = g_1(t) - g_2(t-2)$



F



$s_3(t) = s_1(t+1) + s_1(t) \quad \overset{\text{LTI-System}}{\Rightarrow} \quad g_3(t) = g_1(t+1) + g_1(t)$

ET IV Kleingruppe 2

Aufgabe 2.6

$$\delta(2t) = \lim_{T_0 \to 0} \frac{1}{T_0} \operatorname{rect}\left(\frac{2t}{T_0}\right) \stackrel{!}{=} \frac{1}{2}\delta(t)$$

$$\delta(2t) = \lim_{T_0 \to 0} \frac{1}{T_0} \operatorname{rect}\left(\frac{2t}{T_0}\right)$$

$$= \lim_{T_0 \to 0} \frac{1}{T_0} \operatorname{rect}\left(\frac{t}{T_0/2}\right)$$

Substitution: $T = \frac{T_0}{2}$

$$= \lim_{2T \to 0} \frac{1}{2T} \operatorname{rect}\left(\frac{t}{T}\right) = \frac{1}{2} \lim_{2T \to 0} \frac{1}{T} \operatorname{rect}\left(\frac{t}{T}\right)$$

$$= \frac{1}{2}\delta(t)$$

Aufgabe 2.7

$$g(t) = t^2 \cdot \frac{d^2 s(t)}{dt^2} + t \cdot \frac{ds(t)}{dt}$$

Linearität ist gegeben, falls gilt: $\operatorname{Tr}\left\{\sum_i a_i s_i(t)\right\} = \sum_i a_i g_i(t)$

$$\operatorname{Tr}\left\{\sum_i a_i s_i(t)\right\} = t^2 \frac{d^2 \sum_i a_i s_i(t)}{dt^2} + t \cdot \frac{d \sum_i a_i s_i(t)}{dt}$$

$$= t^2 \sum_i a_i \frac{d^2 s_i(t)}{dt^2} + t \sum_i a_i \frac{d s_i(t)}{dt}$$

$$= \sum_i a_i \left(t^2 \frac{d^2 s_i(t)}{dt^2} + t \frac{d s_i(t)}{dt}\right)$$

$$= \sum_i a_i g_i(t)$$

→ Das System ist linear

Zeitinvarianz ist gegeben, falls gilt: $\operatorname{Tr}\{s(t-\tau_0)\} = g(t-\tau_0)$

$$\operatorname{Tr}\{s(t-\tau_0)\} = t^2 \frac{d^2 s(t-\tau_0)}{dt^2} + t \frac{d s(t-\tau_0)}{dt} \neq g(t-\tau_0) = (t-\tau_0)^2 \frac{d^2 s(t-\tau_0)}{dt^2} + (t-\tau_0) \frac{d s(t-\tau_0)}{dt}$$

→ Das System ist nicht zeitinvariant

⇒ Es liegt kein LTI-System vor

Hausaufgabe 2

Aufgabe 2.10

a) i) für $\alpha = \beta$

$$g(t) = s(t) * h(t)$$
$$= \int_{-\infty}^{\infty} e^{-\beta\tau} \varepsilon(\tau) \cdot e^{-\beta(t-\tau)} \varepsilon(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} e^{-\beta\tau} \varepsilon(\tau)\, e^{-\beta t+\beta\tau} \varepsilon(t-\tau)\, d\tau$$
$$= \int_{0}^{\infty} e^{-\beta\tau+\beta\tau}\, e^{-\beta t} \varepsilon(t-\tau)\, d\tau$$
$$= e^{-\beta t} \int_{0}^{\infty} \varepsilon(t-\tau)\, d\tau$$
$$= e^{-\beta t} \cdot t \cdot \varepsilon(t)$$

ii) für $\alpha \neq \beta$

$$g(t) = s(t) * h(t)$$
$$= \int_{-\infty}^{\infty} e^{-\alpha\tau} \varepsilon(\tau)\, e^{-\beta(t-\tau)} \varepsilon(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} e^{-\alpha\tau} \varepsilon(\tau)\, e^{-\beta t+\beta\tau} \varepsilon(t-\tau)\, d\tau$$
$$= \int_{0}^{\infty} e^{\tau(\beta-\alpha)}\, e^{-\beta t} \varepsilon(t-\tau)\, d\tau$$
$$= e^{-\beta t} \int_{0}^{t} e^{\tau(\beta-\alpha)} \varepsilon(t)\, d\tau$$
$$= e^{-\beta t} \varepsilon(t) \cdot \frac{1}{\beta-\alpha} \left. e^{\tau(\beta-\alpha)} \right]_0^t = e^{-\beta t} \varepsilon(t) \frac{1}{\beta-\alpha} \left[e^{t\beta} e^{-t\alpha} - 1 \right]$$
$$= \frac{1}{\beta-\alpha} \varepsilon(t) \left[e^{-t\alpha} - e^{-t\beta} \right]$$

b) $g(t) = s(t) * h(t)$

$$s(t) = \varepsilon(t) - 2\varepsilon(t-2) + \varepsilon(t-5)$$
$$= \varepsilon(t) * \left[\delta(t) - 2\delta(t-2) + \delta(t-5) \right]$$

$g_1(t) = h(t) * \varepsilon(t)$ siehe Assoziativgesetz

$$= \int_{-\infty}^{\infty} e^{2\tau} \underbrace{\varepsilon(1-\tau)}_{\tau<1} \cdot \underbrace{\varepsilon(t-\tau)}_{\tau<t}\, d\tau$$

i) $t > 1$: $= \int_{-\infty}^{1} e^{2\tau}\, d\tau = \frac{1}{2} e^{2}$

ii) $t \leq 1$: $= \int_{-\infty}^{t} e^{2\tau}\, d\tau = \frac{1}{2} e^{2t}$

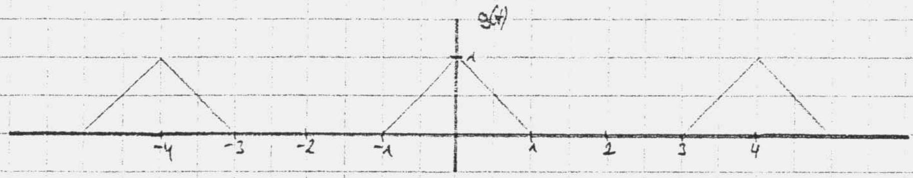
$$\Rightarrow g_1(t) = \frac{1}{2} e^{2} \varepsilon(t-1) + \frac{1}{2} e^{2t} \varepsilon(1-t)$$

$$g(t) = g_1(t) * \left[\delta(t) - 2\delta(t-2) + \delta(t-5) \right]$$

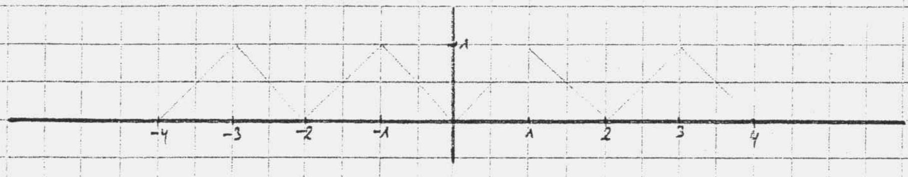
hier nicht weiter berechnen, da sinnvolle Lösung nicht möglich

Aufgabe 2.11

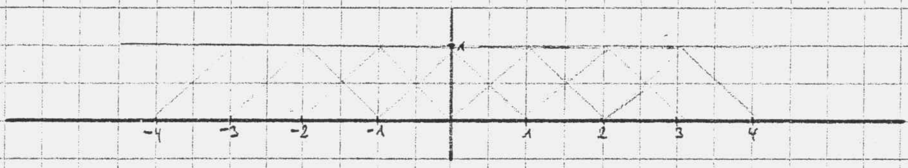
a)



b)



c)



-2.12
2.14

05.05.09

ET IV Kleingruppe 3

Aufgabe 2.10 c)

a) $s(t) = \sin(\pi t) \cdot \mathrm{rect}\left(\frac{t-1}{2}\right)$

$h(t) = 2\,\mathrm{rect}\left(\frac{t}{2}-1\right)$

$g(t) = s(t) * h(t)$

$= \int_{-\infty}^{\infty} \sin(\pi\tau)\,\mathrm{rect}_{i)}\left(\frac{\tau-1}{2}\right) \cdot 2\,\mathrm{rect}\left(\frac{t-\tau}{2_{ii)}}-1\right) d\tau$

i) $-\frac{1}{2} \le \frac{\tau-1}{2} \le \frac{1}{2}$ ii) $-\frac{1}{2} \le \frac{t-\tau}{2}-1 \le \frac{1}{2}$

$0 \le \tau \le 3$ $t-1 \ge \tau \ge t-3$

Fallunterscheidung:

- $g(t) = 0$, $t-1 \le 0 \Leftrightarrow t \le 1$
- $g(t) = \int_0^{t-1} 2\sin(\pi\tau)\,d\tau = -\frac{2}{\pi}\left[\cos(\pi\tau)\right]_0^{t-1}$
  $= -\frac{2}{\pi}\left[\cos(\pi(t-1)) - 1\right]$, $0 \le t-1 \le 2 \Leftrightarrow 1 \le t \le 3$
- $g(t) = \int_{t-3}^{2} 2\sin(\pi\tau)\,d\tau = -\frac{2}{\pi}\left[\cos(\pi\tau)\right]_{t-3}^{2}$
  $= -\frac{2}{\pi}\left[\cos(\pi(t-1)) - 1\right]$, $0 \le t-3 \le 2 \Leftrightarrow 3 \le t \le 5$
- $g(t) = 0$, $t > 5$

b) $s(t) = a\cdot t + b$

$h(t) = \frac{4}{3}\,\mathrm{rect}\left(t-\frac{1}{2}\right) - \frac{1}{3}\,\delta(t-2)$

$g(t) = s(t) * h(t)$

$= [a\cdot t + b] * \left[\frac{4}{3}\,\mathrm{rect}\left(t-\frac{1}{2}\right) - \frac{1}{3}\,\delta(t-2)\right]$

$= [a\cdot t+b] * \left[\frac{4}{3}\,\mathrm{rect}\left(t_{i)}-\frac{1}{2}\right)\right] - [a\cdot t+b] * \left[\frac{1}{3}\,\delta(t-2)\right]$

i) $-\frac{1}{2} \le t-\tau-\frac{1}{2} \le \frac{1}{2}$

$t \ge \tau \ge t-1$

$= \int_{t-1}^{t} \frac{4}{3}(a\cdot\tau+b)\,d\tau - \frac{1}{3}\,s(t-2)$ Faltung mit Dirac-Impuls $\hat{=}$ Verschiebung

$= \frac{4}{3}\left[a\left(\frac{t^2}{2} - \frac{(t-1)^2}{2}\right) + b\right] - \frac{1}{3}[a\cdot(t-2)+b]$ zum Ort des Dirac-Impulses

$= \frac{4}{3}\left[a\left(\frac{t^2}{2} - \frac{t^2}{2} + t - \frac{1}{2}\right) + b\right] - \frac{1}{3}[at - 2a + b]$

$= \frac{4}{3}\left[at - \frac{a}{2} + b\right] - \frac{1}{3}at + \frac{2}{3}a - \frac{1}{3}b$

$= \frac{4}{3}at - \frac{2}{3}a + \frac{4}{3}b - \frac{1}{3}at + \frac{2}{3}a - \frac{1}{3}b$

$= a\cdot t + b = s(t)$ Sonderfall

Periode

c) $s(t) = [\mathrm{rect}(t) - \mathrm{rect}(t-1)] * \sum_{n=-\infty}^{\infty} \delta(t-2n)$

$h(t) = (1-t) \cdot \mathrm{rect}(t - \frac{1}{2})$

$g(t) = \underbrace{h(t) * \mathrm{rect}(t)}_{g_1(t)} \; \alpha [\beta \delta(t) - \delta(t-1)] * \sum_{n=-\infty}^{\infty} \delta(t-2n)$

$g_1(t) = \int_{-\infty}^{\infty} h(\tau) \, \mathrm{rect}(t-\tau) \, d\tau$

$= \int_{0}^{t+\frac{1}{2}} (1-\tau) \, d\tau = \frac{1}{2}(t - t^2 + \frac{3}{4}) \qquad -\frac{1}{2} t \le \frac{1}{2}$

$g_1(t) = \int_{t-\frac{1}{2}}^{1} (1-\tau) \, d\tau = \frac{1}{2}(\frac{9}{4} - 3t + t^2 \qquad \frac{1}{2} \le t \le \frac{3}{2}$

Hausaufgabe 3

2.12

a) $\int_{-\infty}^{t} e^{-(t-\tau)} s(\tau-2)\,d\tau$ muss auf die Form $\int_{-\infty}^{\infty} s(\tau)\,h(t-\tau)\,d\tau$ gebracht werden

$= \int_{-\infty}^{\infty} e^{-(t-\tau)}\,\varepsilon(t-\tau)\,s(\tau-2)\,d\tau$

nur den Integranden umformen

$s(t-2) * e^{-t}\,\varepsilon(t)$

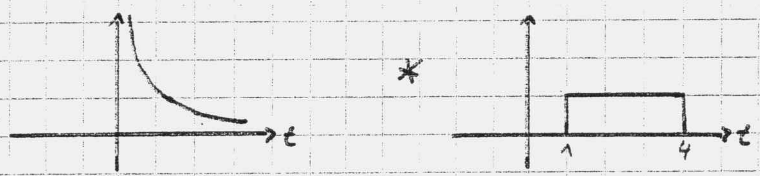
$= s(t) * \delta(t-2) * e^{-t}\,\varepsilon(t)$

$= s(t) * \underbrace{\left[\delta(t-2) * e^{-t}\,\varepsilon(t)\right]}_{}$

$h(t) \hat{=}$ Impulsantwort

b) $g(t) = \left[\mathrm{rect}\left(\frac{t}{3}\right) * \delta\left(t-\frac{1}{2}\right)\right] * \left[\delta(t-2) * e^{-t}\,\varepsilon(t)\right]$

$= e^{-t}\,\varepsilon(t) * \left[\mathrm{rect}\left(\frac{t}{3}\right) * \delta(t-2{,}5)\right]$

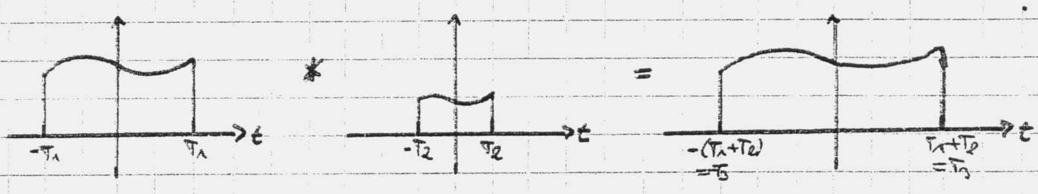


$t<1: \quad g(t) = 0$

$1<t<4: \quad g(t) = \int_{0}^{t-1} e^{-t}\cdot 1\,dt = 1-e^{-t+1}$

$4<t \quad : \quad g(t) = \int_{t-4}^{t-1} e^{-t}\cdot 1\,dt = e^{-t+1} - e^{-t+4}$

2.14



ET IV Kleingruppe 4

2.15

a) $g(t) = s(t) * h(t)$

$= s(t) * \delta(t) * h(t)$

$= s(t) * \varepsilon'(t) * h(t)$ $\quad \varepsilon'(t) = \delta(t)$

$= s(t) * [h(t) * \varepsilon'(t)] = s(t) * h_2(t)$

b) $g(t) = s'(t) * h(t)$

$= s(t) * \delta'(t) * h(t)$

$= s(t) * [h(t) * \delta'(t)] = s(t) * h'(t)$

c) $g(t) = \left[\int_{-\infty}^{t} s(\tau)\, d\tau\right] * h'(t)$

$= \left[\int_{-\infty}^{\infty} s(\tau)\, \varepsilon(t-\tau)\, d\tau\right] * h'(t)$

$= s(t) * \varepsilon(t) * h(t) * \delta'(t)$

$= [s(t) * \delta'(t)] * [h(t) * \varepsilon(t)]$

$= s'(t) * \left[\int_{-\infty}^{\cancel{\infty}\, t} h(\tau)\, \cancel{\varepsilon(t-\tau)}\, d\tau\right] = s'(t) * \left[\int_{-\infty}^{t} h(\tau)\, d\tau\right.$

2.13

$g(t) = s(t) * h(t)$

$= \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$

$= \int_{-\infty}^{\infty} \mathrm{rect}(\tau)\, e^{j\omega_0 (t-\tau)}\, d\tau$

$= \int_{-1/2}^{1/2} e^{j\omega_0 (t-\tau)}\, d\tau$

$= \int_{-1/2}^{1/2} e^{j\omega_0 t}\, e^{-j\omega_0 \tau}\, d\tau$

$= e^{j\omega_0 t} \cdot \left[-\frac{1}{j\omega_0}\, e^{-j\omega_0 \tau}\right]_{-1/2}^{1/2}$

$= -\frac{e^{j\omega_0 t}}{j\omega_0} \left[e^{-j\omega_0/2} - e^{j\omega_0/2}\right]$

$= e^{j\omega_0 t}\, \frac{1}{j\omega_0} \left[e^{j\omega_0/2} - e^{-j\omega_0/2}\right]$

$= e^{j\omega_0 t} \cdot \sin\left(\frac{\omega_0}{2}\right) \cdot \frac{2}{\omega_0}$ $\qquad \frac{1}{2i}\left[e^{ix} - e^{-ix}\right] = \sin(x)$

$= e^{j\omega_0 t}\, \mathrm{si}\left(\frac{\omega_0}{2}\right) \stackrel{!}{=} 0$ $\qquad$ [redacted]

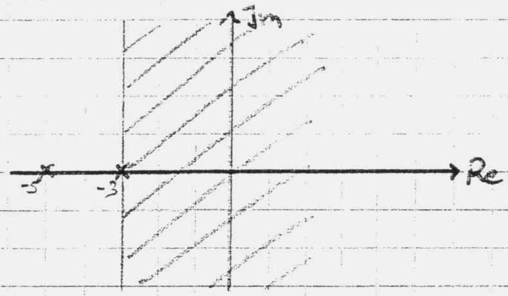
$\Rightarrow \mathrm{si}\left(\frac{\omega_0}{2}\right) = 0$

$\Leftrightarrow \frac{\omega_0}{2} = k \cdot \pi$

$\Rightarrow \omega_0 = 2 \cdot k \cdot \pi \quad , k \neq 0$

Hausaufgabe 4

Aufgabe 3.1



$p_{P1} = -\beta = -3$

$p_{P2} = -5$

→ Rechtsseitige Zeitfunktion

$Re\{p\} > -3$, damit ein gültiges Pol-Nullstellendiagramm vorliegt

$\Rightarrow \beta = 3$

$$S(p) = \frac{1}{p+5} + \frac{1}{p+\beta}$$
$$= \frac{1}{p+5} + \frac{1}{p+3}$$

Aufgabe 3.3

$$H(p) = \frac{2(p+2)}{(p+3)(p+4)} \overset{!}{=} A_0 + \sum_{i=1}^{2} \frac{A_i}{p-p_{Pi}}$$

$p_{P1} = -3$; $p_{P2} = -4$

$$A_0 = \lim_{p\to\infty} H(p) = 0$$

$$A_1 = \lim_{p\to p_{P1}} [H(p)\cdot(p-p_{P1})] = \lim_{p\to -3} \frac{2(p+2)}{p+4} = -2$$

$$A_2 = \lim_{p\to p_{P2}} [H(p)\cdot(p-p_{P2})] = \lim_{p\to -4} \frac{2(p+2)}{p+3} = 4$$

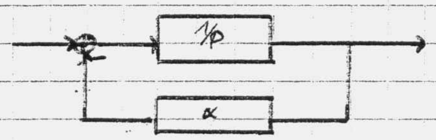
$$\Rightarrow H(p) = \frac{-2}{p+3} + \frac{4}{p+4} = -2H_1(p) + 4H_2(p)$$

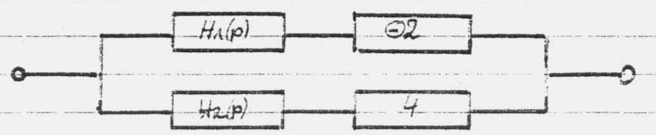
Rechtsseitige Zeitfunktion, da $Re\{p\} > -3$

allgemein: $\frac{1}{p+\alpha} \overset{\mathcal{L}}{\longleftrightarrow} e^{-\alpha t}\cdot\varepsilon(t)$

$$\Rightarrow h(t) = \varepsilon(t)\left[-2e^{-3t} + 4e^{-4t}\right]$$

$H(p) = \frac{1}{p+\alpha}$





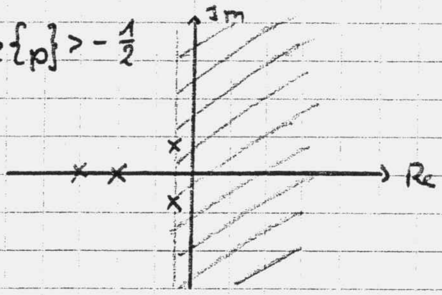
Tiefpass $H(f) = \frac{1}{1+2\pi j f RC}$

ET IV Kleingruppe 5

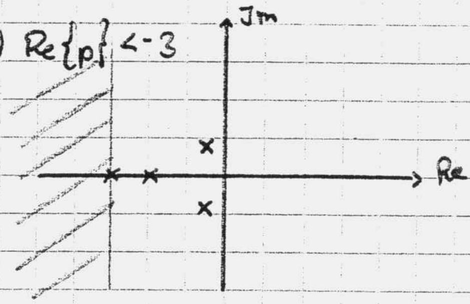
Aufgabe 3.2

$$S(p) = \frac{p-1}{(p+2)(p+3)(p^2+p+1)} = \frac{p-1}{(p+2)(p+3)(p+\frac{1}{2} \pm i\sqrt{3/4})}$$

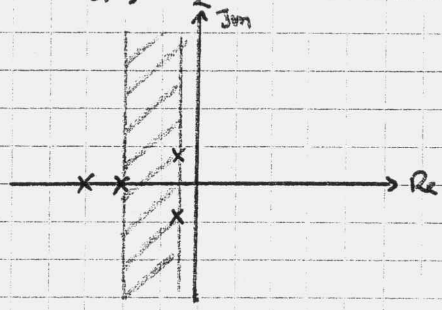
i) $Re\{p\} > -\frac{1}{2}$



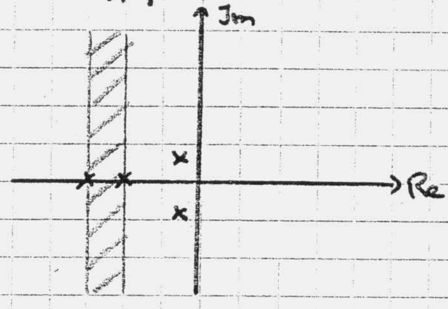
ii) $Re\{p\} < -3$



iii) $-2 < Re\{p\} < -\frac{1}{2}$



iv) $-3 < Re\{p\} < -2$



Aufgabe 3.4

$$g(t) = \beta e^{-t} \varepsilon(t) + \alpha\beta e^{t} \varepsilon(-t)$$

$$\mathcal{L}\{g(t)\} = G(p) = \frac{\beta}{p+1} - \frac{\alpha\beta}{p-1} \qquad , -1 < Re\{p\} < 1$$

$$= \frac{\beta(p-1) - \alpha\beta(p+1)}{p^2-1} \stackrel{!}{=} \frac{p}{p^2-1}$$

$$\Rightarrow \beta(p-1) - \alpha\beta(p+1) \stackrel{!}{=} p$$

$$\Leftrightarrow \beta p - \beta - \alpha\beta p - \alpha\beta = \beta$$

$$\Rightarrow \beta - \alpha\beta \stackrel{!}{=} 1 \qquad (1)$$

$$-\beta - \alpha\beta \stackrel{!}{=} 0 \qquad (2)$$

aus (1) $\beta(1-\alpha) = 1$

$$\Leftrightarrow \beta = \frac{1}{1-\alpha} \qquad , \alpha \neq 1$$

einsetzen in (2) $-\frac{1}{1-\alpha} - \alpha \frac{1}{1-\alpha} = 0$

$$\Leftrightarrow -1 - \alpha = 0$$

$$\Rightarrow \underline{\alpha = -1} \qquad \Rightarrow \underline{\beta = \frac{1}{2}}$$

Hausaufgabe 5

Aufgabe 3.5

aus a): Signal ist gerade ⇒ es liegt ein zweiseitiges Signal vor

aus b) + c): $S(p)$ hat 4 Polstellen und $p_{P1} = \frac{1}{2} e^{j\frac{\pi}{4}}$

$s(t)$ ist reell ⇒ $p_{P2} = p_{P1}^* = \frac{1}{2} e^{-j\frac{\pi}{4}}$

$s(t)$ ist gerade ⇒ $S(p)$ ist gerade

$\Rightarrow p_{P3} = -p_{P4}$

$p_{P4} = p_{P3}^* = -p_{P2}$

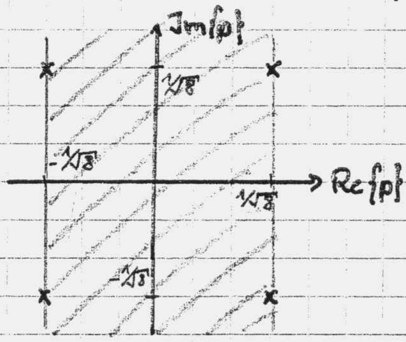
$$\Rightarrow S(p) = \frac{A}{(p-p_{P1})(p-p_{P2})(p-p_{P3})(p-p_{P4})}$$

$$= \frac{A}{(p^2 - \frac{p}{\sqrt{2}} + \frac{1}{4})(p^2 + \frac{p}{\sqrt{2}} + \frac{1}{4})}$$

aus d): $\int_{-\infty}^{\infty} s(t)\, dt = 4$

$$\int_{-\infty}^{\infty} s(t)\, dt = S(0) = \int_{-\infty}^{\infty} s(t)\, e^{-pt}\, dt \quad , \text{für } p = 0$$

$\Rightarrow S(0) = 4$, also $A = \frac{1}{4}$



Aufgabe 3.7

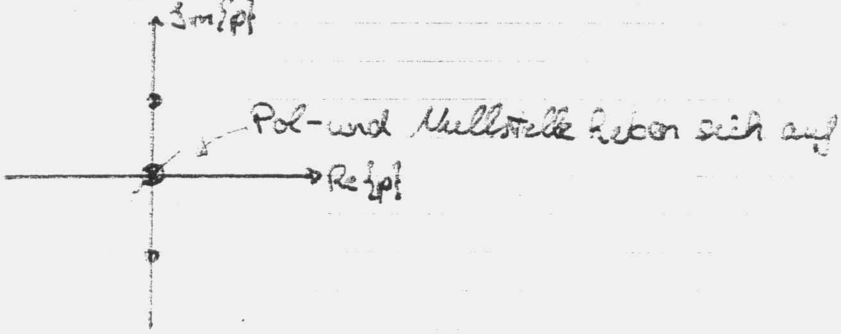
a) $s(t) = \text{rect}(t - \frac{1}{2}) = \varepsilon(t) - \varepsilon(t-1)$

$$\frac{1}{p} - \frac{1}{p} e^{-pt_0}$$

$$= \frac{1}{p}(1 - e^{-pt_0}) \quad , t_0 = 1$$

Polstelle $p_{P1} = 0$

Nullstellen $p_N = 2\pi \cdot i$, da dann $e^{-pt_0} = 1$



→ Konvergenzgebiet ist gesamte p-Ebene

j) $s(t) = \delta(3t) + \varepsilon(3t)$

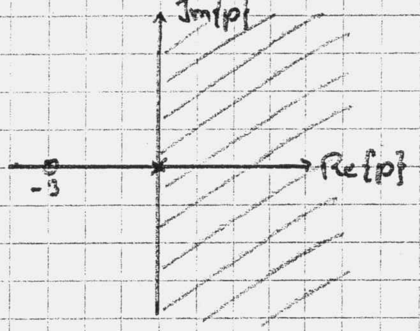
$s\left(\frac{t}{T}\right) \circ\!\!-\!\!\bullet\ |T| \cdot S(p \cdot T)$

$\left|\frac{1}{3}\right| \cdot 1 + \left|\frac{1}{3}\right| \frac{1}{p \cdot \frac{1}{3}}$

$= \frac{1}{3}\left(1 + \frac{3}{p}\right) = \frac{1}{3}\left(\frac{p+3}{p}\right)$

Polstelle $p_{P1} = 0$

Nullstelle $p_{N1} = -3$



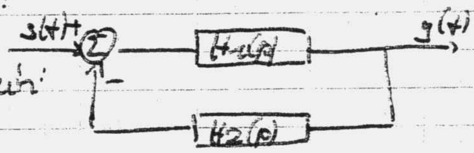
→ Konvergenzgebiet $Re\{p\} > 0$

ET IV Kleingruppe

Aufgabe 3.6

Hinweis:

allgemein:



$$H(p) = \frac{H_1(p)}{1 + H_1(p)\,H_2(p)}$$

zur Aufgabe: $H_1(p) = \frac{2/p}{1 + 4 \cdot 2/p} = \frac{2/p}{1 + 8/p} = \frac{2}{p+8}$

$$H_2(p) = \frac{1/p}{1 + 2 \cdot 1/p} = \frac{1/p}{1 + 2/p} = \frac{1}{p+2}$$

$$\cancel{\Rightarrow H(p) = \frac{H_1(p)}{1 - H_1(p)\,H_2(p)}}$$

$$= \frac{2/p+8}{1 - \cancel{2/p+8 \cdot 1/p+2}}$$

$$= \frac{2/p+8}{1 - \frac{2}{p^2+10p+16}} = \frac{3p+12}{p^2+10p+16} = \frac{G(p)}{S(p)}$$

$$G(p) = S(p) \cdot H(p)$$

$$= S(p) \cdot (H_1(p) + H_2(p))$$

$$= S(p) \cdot \left[\frac{2}{p+8} + \frac{1}{p+2} \right]$$

$$G(p)\left[p^2 + 10p + 16\right] = S(p)\left[3p + 12\right]$$

$$\frac{d^2 g(t)}{dt^2} + 10\frac{d g(t)}{dt} + 16 g(t) = 3\frac{d s(t)}{dt} + 12 s(t)$$

Transformation von $H(p)$:

$$H(p) = \frac{2}{p+8} + \frac{1}{p+2} \quad \bullet\!\!-\!\!\circ \quad (2e^{-8t} + e^{-2t}) \cdot \varepsilon(t) = h(t)$$

Aufgabe 3.8

$p^2 - p - 2 = 0$

$p = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2}$

$p_{P1} = -1 \quad p_{P2} = 2$

a) DGL transformieren:

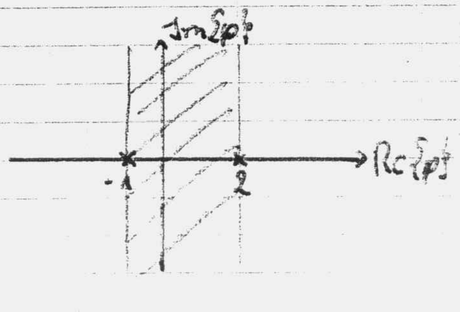
$$\frac{d^2 g(t)}{dt^2} - \frac{d g(t)}{dt} - 2 g(t) = s(t)$$

$$p^2 G(p) - p\,G(p) - 2G(p) = S(p)$$

$$\Leftrightarrow G(p)\left[p^2 - p - 2\right] = S(p)$$

$$\Leftrightarrow \frac{G(p)}{S(p)} = \frac{1}{p^2 - p - 2} = H(p)$$

$$H(p) = \frac{1}{(p+1)(p-2)} \longrightarrow \text{Partialbruchzerlegung} \quad -\tfrac{1}{3} \quad \tfrac{1}{3}$$



Polstellen: $p_{P1} = -1 \qquad p_{P2} = 2$

Nullstelle: p_0 im Unendlichen

b) i) stabil $\Rightarrow$ KB enthält Im-Achse

$$-1 < \mathrm{Re}\{p\} < 2$$

$$\Rightarrow h(t) = -\frac{1}{3} e^{2t} \varepsilon(-t) - \frac{1}{3} e^{-t} \varepsilon(t)$$

ii) kausal $\Rightarrow$ rechtsseitiges Signal

$\rightarrow$ KB: $\mathrm{Re}\{p\} > 2$

$$\Rightarrow h(t) = \frac{1}{3} e^{2t} \varepsilon(t) - \frac{1}{3} e^{-t} \varepsilon(t)$$

iii) weder stabil, noch kausal

$\rightarrow$ KB $\mathrm{Re}\{p\} < -1$

$$\Rightarrow h(t) = -\frac{1}{3} e^{2t} \varepsilon(-t) + \frac{1}{3} e^{-t} \varepsilon(-t)$$

Hft: 4.4 a) [a, d, e] + b)

ET IV Hausaufgabe 5

Aufgabe 4.4

a) (a) $c_k = \frac{1}{T}\int_0^T s(t)\, e^{-j2\pi k f_0 t}\, dt$

$= \frac{1}{2}\int_{-1}^{1} t\, e^{-j\pi k t}\, dt$

$= \frac{1}{2}\,\frac{1}{(-j\pi k)^2}\, e^{-j\pi k t}\left[-j\pi k t - 1\right]\Big|_{-1}^{1}$

$= \frac{-1}{2(\pi k)^2}\left[e^{-j\pi k}(-j\pi k - 1) - e^{j\pi k}(j\pi k - 1)\right]$

$= \frac{1}{2(\pi k)^2}\left[j\pi k\,(e^{-j\pi k} + e^{j\pi k}) + (e^{-j\pi k} - e^{j\pi k})\right]$

$= \frac{1}{2(\pi k)^2}\left[j\pi k \cdot 2\cos(\pi k) - 2j\sin(\pi k)\right]$

$= j\left(\frac{\cos(\pi k)}{\pi k} - \underbrace{\frac{\sin(\pi k)}{(\pi k)^2}}_{=0}\right) = j\,\frac{(-1)^k}{\pi k} \qquad , k \neq 0$

für $k = 0$: $c_0 = \frac{1}{2}\int_{-1}^{1} t \cdot e^0\, dt = 0$

(d) $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - T\cdot n) - \left[2\sum_{n=-\infty}^{\infty} \underbrace{\delta(t - T\cdot n) * \delta(t-1)}\right]$

aus Formelsammlung

Zeitverschiebung $s(t - \underset{=1}{t_0}) \Rightarrow c_k\, e^{-j2\pi k f_0 t_0}$

$c_k = \frac{1}{2} - \left[2 \cdot \frac{1}{2} e^{-j\pi k}\right]$

$= \frac{1}{2} - (-1)^k$

(e) $s(t) = \left[\mathrm{rect}(t) * \sum_{n=-\infty}^{\infty} \delta(t - T\cdot n)\right] * \left[\delta(t + \tfrac{3}{2}) - \delta(t - \tfrac{3}{2})\right]$

$\circ\!\!-\!\!\bullet$

$S(f) = \mathrm{si}(\pi f) \cdot \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f - \tfrac{k}{T}) \cdot (1 \cdot e^{j3\pi f} - e^{-j3\pi f})$

$= \mathrm{si}(\pi f) \cdot \frac{1}{3} \cdot j\,\sin(3\pi f) \cdot \sum_{k=-\infty}^{\infty} \delta(f - \tfrac{k}{T})$

$= \sum_{k=-\infty}^{\infty} \mathrm{si}(\pi f) \cdot \frac{1}{3} \cdot j \cdot \sin(3\pi f) \cdot \delta(f - \tfrac{k}{T})$

Siebeigenschaft

$= \sum_{k=-\infty}^{\infty} \mathrm{si}(\tfrac{\pi}{T}\cdot k) \cdot \frac{1}{3} \cdot j \cdot \sin(3\pi \tfrac{k}{T})\, \delta(f - \tfrac{k}{T})$

$S(f) = \quad c_k \quad \cdot \delta(f - \tfrac{k}{T})$

$\Rightarrow$ Koeffizienten $= c_k$

$\hookrightarrow c_k = \mathrm{si}(\tfrac{\pi}{T}\cdot k) \cdot \frac{1}{3} \cdot j \cdot \sin(3\pi \tfrac{k}{T})$

b) $c_k = \frac{1}{T}\int_0^T s(t)\, e^{-j2\pi k f_0 t}\, dt$

$= \frac{1}{2}\int_{-1}^{1} e^{-t}\, e^{-j2\pi k f_0 t}\, dt$

$= \frac{1}{2}\int_{-1}^{1} e^{-t(1 + j\pi k)}\, dt$

$= \frac{1}{2}\,\frac{1}{-(1 + j\pi k)}\left[e^{-(1 + j\pi k)} - e^{(1 + j\pi k)}\right]$

$= \frac{1}{2}\,\frac{1}{-(1 + j\pi k)}\left[e^{-1}(-1)^k - e^{1}(-1)^k\right]$

$= \frac{1}{2}\,\frac{1}{-(1 + j\pi k)}\,(-1)^k\,(e^{-1} - e^{1})$

ET IV Kleingruppe 6

Aufgabe 4.1

$$s(t) = \underbrace{c_0}_{=0} + \sum_{k=1}^{\infty} 2\,\mathrm{Re}\{c_k\, e^{j2\pi k f_0 t}\}$$

$$f_0 = \frac{1}{T} = \frac{1}{8}$$

$$\Rightarrow s(t) = 2\mathrm{Re}\{2\,e^{j2\pi k \frac{1}{8} t}\} + 2\mathrm{Re}\{4j\, e^{j2\pi k \frac{1}{8} t}\}$$

$$= 2\mathrm{Re}\{2\,e^{j\frac{1}{4}\pi \cdot 1 t}\} + 2\mathrm{Re}\{4j\, e^{j\frac{1}{4}\pi 3t}\}$$

$$= 4\cos(\tfrac{\pi}{4}t) - 8\sin(\tfrac{3}{4}\pi t)$$

$$= 4\cos(\tfrac{\pi}{4}t) + 8\cos(\tfrac{3}{4}\pi t + \tfrac{\pi}{2})$$

Aufgabe 4.3

a) $\rightarrow s(t) = c_0 + 2\sum_{k=1}^{\infty}[\mathrm{Re}\{c_k\}\cos(k2\pi f_0 t) - \mathrm{Im}\{c_k\}\sin(k2\pi f_0 t)]$

$-s(t) = s(-t) \rightarrow \mathrm{Re} = 0$

$\Rightarrow s(t) = -2\sum_{k=1}^{\infty}[\mathrm{Im}\{c_k\}\sin(2\pi k f_0 t)]$

b) $\rightarrow T = 2$

$\Rightarrow s(t) = -2\sum_{k=1}^{\infty}[\mathrm{Im}\{c_k\}\sin(\pi k t)]$

c) $\rightarrow c_k = c_{-k} = 0$ für $|k| > 1$

$\Rightarrow s(t) = -2\,[\mathrm{Im}\{c_k\}\sin(\pi t)]$

d) $\rightarrow \frac{1}{2}\int_{-1}^{1} s^2(t)\,dt = \sum_{k=-\infty}^{\infty}|c_k|^2 \overset{!}{=} 1$ Parseval-Theorem

$\Rightarrow |c_1|^2 + |c_1|^2 = 1 \quad \Rightarrow |c_1|^2 = \frac{1}{2}$

$\Rightarrow s_{1,2} = \pm\sqrt{2}\,\sin(\pi t)$

keine weiteren Lösungen möglich

Aufgabe 4.6

a) $s(t-t_0) + s(t+t_0)$

$$\Rightarrow d_k = c_k \cdot e^{-j2\pi k f_0 t_0} + c_k\, e^{j2\pi k f_0 t_0}$$

$$= c_k\,(e^{-j2\pi k f_0 t_0} + e^{j2\pi k f_0 t_0})$$

$$= c_k \cdot 2 \cdot \cos(2\pi k f_0 t_0)$$

$$\Rightarrow s(3t-1) = \sum_{k=-\infty}^{\infty} c_k\, e^{j2\pi k f_0 (3t-1)}$$

$$= \sum_{k=-\infty}^{\infty} \underbrace{c_k\, e^{-j2\pi k f_0}}_{d_k}\, e^{j6\pi k f_0 t}$$

$\hookrightarrow$ neue Grundfrequenz $f_1 = 3 f_0$

Hausaufgabe 6

Aufgabe 4.13

a) $s(t) = [e^{-at} \cdot \cos(2\pi f_0 t)] \cdot \varepsilon(t) \qquad , a>0$

$$s(t) = e^{-at} \cdot \varepsilon(t) \cdot \cos(2\pi f_0 t)$$

$$\circ\!\!-\!\!\bullet$$

$$S(f) = \frac{1}{a+j2\pi f} * \left[\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right]$$

$$= \frac{1}{2}\left[\frac{1}{a+j2\pi(f-f_0)} + \frac{1}{a+j2\pi(f+f_0)}\right]$$

b) $s(t) = \sum_{k=-\infty}^{\infty} \delta(t-2n) * [2\delta(t) + \delta(t-1)]$

$$\circ\!\!-\!\!\bullet$$

$$S(f) = \frac{1}{2}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{2}\right) \cdot \left[2 + e^{-j2\pi f}\right]$$

$$= \frac{1}{2}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{2}\right) \cdot \left[2 + e^{-j2\pi \frac{k}{2}}\right]$$

$$= \frac{1}{2}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{2}\right) \cdot \left[2 + e^{-jk\pi}\right]$$

$$= \frac{1}{2}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{2}\right) \cdot \left[2 + (-1)^k\right]$$

i) $s(t) = (1-t^2)\,\mathrm{rect}\left(t-\frac{1}{2}\right)$

$$S(f) = \int_{-\infty}^{\infty} (1-t^2)\,\mathrm{rect}\left(t-\frac{1}{2}\right) e^{-j2\pi f t}\,dt$$

$$= \int_0^1 (1-t^2)\, e^{-j2\pi f t}\,dt$$

$$= \int_0^1 e^{-j2\pi f t}\,dt - \int_0^1 \underset{u}{t^2}\, \underset{v'}{e^{-j2\pi f t}}\,dt$$

$$= -\frac{1}{j2\pi f} e^{-j2\pi f t}\Big|_0^1 - t^2 \frac{-1}{j2\pi f} e^{-j2\pi f t}\Big|_0^1 + \int_0^1 2t\, \frac{-1}{j2\pi f}\, e^{-j2\pi f t}\,dt$$

$$= \cancel{-\frac{1}{j2\pi f} e^{-j2\pi f}} + \frac{1}{j2\pi f} + \cancel{\frac{1}{j2\pi f} e^{-j2\pi f}} - \frac{1}{j\pi f}\int_0^1 t\, e^{-j2\pi f t}\,dt$$

$$= \frac{1}{j2\pi f} - \frac{1}{j\pi f}\,\frac{1}{(-j2\pi f)^2}\, e^{-j2\pi f t}\,(+j2\pi f t + 1)\Big|_0^1$$

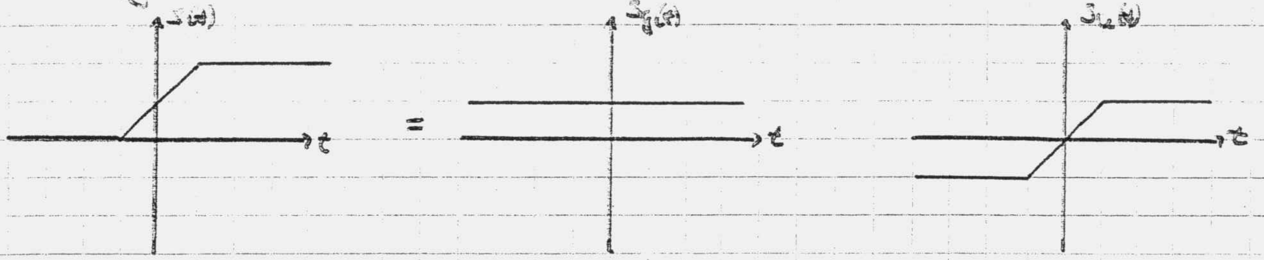
$$= \frac{1}{j2\pi f} - \frac{1}{j\pi f}\,\frac{1}{4\pi^2 f^2}\, e^{-j2\pi f}\,(j2\pi f + 1) + \frac{1}{j\pi f}\,\frac{1}{4\pi^2 f^2}\,(0+1)$$

$$= \frac{1}{j2\pi f} - \frac{j2\pi f}{j4\pi^3 f^3}\, e^{-j2\pi f} - \frac{1}{j4\pi^3 f^3}\, e^{-j2\pi f} + \frac{1}{j4\pi^3 f^3}$$

$$= \frac{1}{j2\pi f} - \frac{2}{4\pi^2 f^2}\, e^{-j2\pi f} - \frac{}{j(2\pi f)^3}\,\left(2e^{-j2\pi f} - 2\right)$$

ET IV Kleingruppe 7

Aufgabe 4.9



b) $\cdot\, s(t) = \mathrm{rect}(t) * \varepsilon(t) = \int_{-\infty}^{t} \mathrm{rect}(\tau)\, d\tau$

a)

$$S(f) = \frac{1}{j2\pi f}\, si(\pi f) + \frac{1}{2}\, si(0) \cdot \delta(f)$$

$$= \frac{1}{j2\pi f}\, si(\pi f) + \frac{1}{2}\, \delta(f)$$

$\cdot\, s(t) = Re\{S(f)\} + j\, Im\{S(f)\}$

$$s_g(t) = \frac{1}{2}$$

$$Re\{S(f)\} = \frac{1}{2}\,\delta(f)$$

c) $s_u(t) = s(t) - \frac{1}{2}$

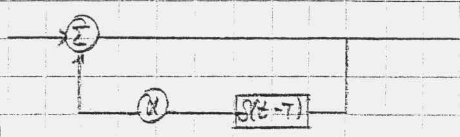
$$\frac{d}{dt} s_u(t) = \mathrm{rect}(t)$$

$$j\, Im\{S(f)\} \cdot (j2\pi f) = si(\pi f)$$

$$j\, Im\{S(f)\} = \frac{si(\pi f)}{j2\pi f}$$

Aufgabe 5.1

a) $g(t) = s(t) + \alpha\, g(t-T)$



$$G(p) = S(p) + \alpha\, G(p)\, e^{-pT} \qquad | : S(p)$$

$$\Rightarrow H(p) = 1 + \alpha\, H(p) \cdot e^{-pT}$$

$$h(t) = \delta(t) + \alpha\, h(t-T) \qquad (*)$$

$$h(t) = \delta(t) + \alpha\, \delta(t-T) + \alpha^2\, \delta(t-2T) + \dots$$

$$= \sum_{k=0}^{\infty} \alpha^k\, \delta(t-kT)$$

b) $\int_0^{\infty} |h(t)|\, dt = \sum_{k=0}^{\infty} \alpha^k$ (wegen $\delta(t)$)

$$= \begin{cases} \frac{1}{1-\alpha} & \text{für } |\alpha| < 1 \\ \infty & \text{für } |\alpha| > 1 \end{cases} \quad \rightarrow \text{instabil}$$

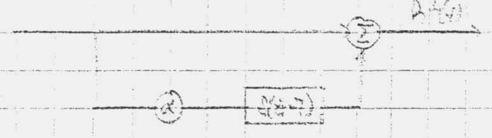
c) $g(t) * h^I(t) \overset{!}{=} s(t) * h(t) * h^I(t) \overset{!}{=} s(t)$

weil Faltung mit $\delta(t)$ → keine Veränderung

$$\Rightarrow h(t) * h^I(t) = \delta(t) \overset{(*)}{=} h(t) - \alpha\, h(t-T)$$

$$= h(t) * [\delta(t) - \alpha\, \delta(t-T)]$$

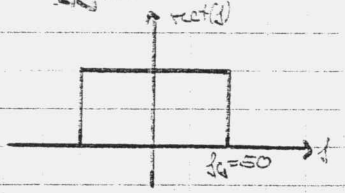
$\Rightarrow h^{I}(t) = \delta(t) - \alpha\,\delta(t-T)$



5.2 ; 6.3

Hausaufgabe 7

Aufgabe 5.2



$T = \frac{1}{6} \quad \Rightarrow f_0 = \frac{1}{T} = 6$

$c_k = 0 \quad$, wenn $k \cdot f_0 > f_g = 50 \quad \Rightarrow k \cdot 6 > 50$

$\Rightarrow c_k = 0 \quad$ für $k > 8$

Aufgabe 6.3

$w(t) = s_1(t) \cdot s_2(t) \xleftrightarrow{\mathcal{F}} W(f) = S_1(f) * S_2(f)$

$W(f) = 0 \quad$ für $|f| > f_1 + f_2 = f_g$

Nyquist-Rate $f_N = \frac{1}{T} = 2f_g$

$\Rightarrow T = \frac{1}{2f_g} = \frac{1}{2f_1 + 2f_2}$

→ keine Überlappung im Spektrum $W_a(f)$

ET IV Kleingruppe 8

Aufgabe 5.5

$$h(t) = \frac{\sin(4(t-1))}{(t-1)} = 4\ si(4(t-1))$$

$$= 4\ si(4t) * \delta(t-1)$$

siehe Fourier-Transformation

$$2\pi f_G \stackrel{!}{=} 4 \Rightarrow f_G = \frac{2}{\pi}$$

$$H(f) = 4 \cdot \frac{1}{2} \cdot \frac{\pi}{2}\ rect(\pi \frac{f}{4}) \cdot e^{-2j\pi f} \quad \hat{=} \text{ ein Tiefpass}$$

a) $s_1(t) = \cos(6t + \frac{\pi}{2})$

Grundfrequenz hier: $f_1 = \frac{3}{\pi} > f_G = \frac{2}{\pi}$

Da die Grundfrequenz größer ist als die Frequenz des Tiefpasses

$\Rightarrow g_1(t) = 0$

b) $s_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$

$$2\pi f_2 \stackrel{!}{=} 3k \Rightarrow f_2 = \frac{3k}{2\pi}$$

$$S_2(f) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \left[\frac{j}{2}\delta(f+f_2) - \frac{j}{2}\delta(f-f_2)\right]$$

Für $k \geq 2$ ist $f_2 = \frac{3k}{2\pi} > f_G = \frac{2}{\pi}$

Daher wird alles geblockt, außer für $k=1$

$$S_2(f) = \frac{1}{2}\left[\frac{j}{2}\delta(f+f_2) - \frac{j}{2}\delta(f-f_2)\right]$$

$$s_2(t) = \frac{1}{2}\sin(3t)$$

? $\Rightarrow g_2(t) = \frac{\pi}{2}\sin(3(t-1))$

c) $s_3(t) = 4\ si(4(t+1)) = 4\ si(4t) * \delta(t+1)$

$$2\pi f_G \stackrel{!}{=} 4 \Rightarrow f_G = \frac{2}{\pi}$$

$$S_3(f) = \pi\ rect(\pi \frac{f}{4})\ e^{j2\pi f}$$

$$G_3(f) = S_3(f) \cdot H(f)$$

$$G_3(f) = \pi \cdot rect(\pi \frac{f}{4})\ e^{j2\pi f} \cdot \pi\ rect(\pi \frac{f}{4})\ e^{-j2\pi f}$$

$$G_3(f) = \pi^2 \cdot rect(\pi \frac{f}{4})$$

$$g_3(t) = \pi\ si(4t)$$

d) $s_4(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2 = \frac{4}{\pi}\ si^2(2t)$

$$S_4(f) = rect(\frac{\pi}{2} f) * rect(\frac{\pi}{2} f) = \frac{2}{\pi}\ \Lambda(\frac{\pi}{2} f)$$

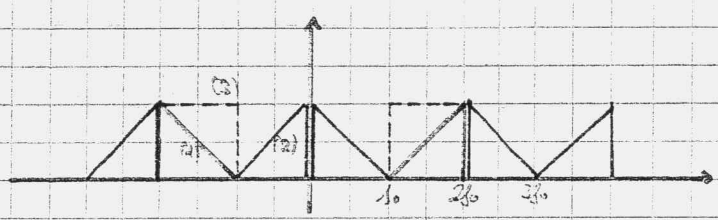
$$G_4(f) = S_4(f) \cdot H(f)$$

$$= \frac{2}{\pi}\ \Lambda(\frac{\pi}{2} f) \cdot \pi\ rect(\pi \frac{f}{4})\ e^{-j2\pi f}$$

$$G_4(f) = 2\Lambda\left(\frac{T}{2}f\right) e^{-j2\pi f}$$

$$g_4(t) = T\, s_4(t-1)$$

Aufgabe 6.1



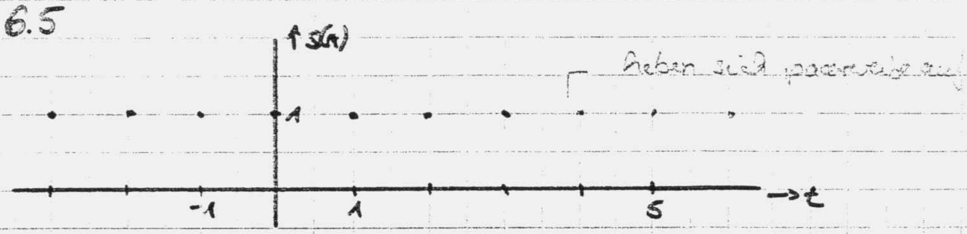
Rückgewinnung durch Bandpass:

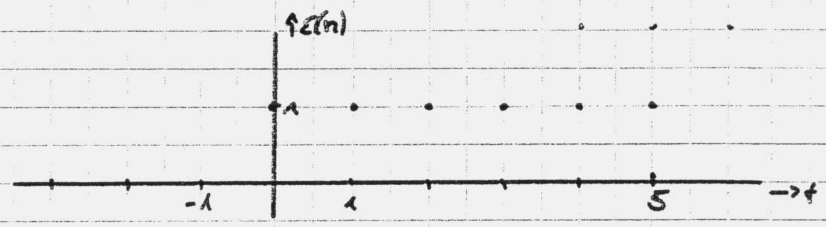
$$H(f) = T\left[\mathrm{rect}\left(\frac{f}{f_0}+1{,}5\right)+\mathrm{rect}\left(\frac{f}{f_0}-1{,}5\right)\right]$$

6.5; 6.6

Hausaufgabe 8

Aufgabe 6.5



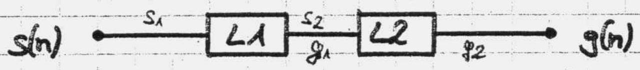


$s(n)=\varepsilon(-(n-3)) = \varepsilon(-n+3)$

$\Rightarrow m = -1 \quad ; n_0 = -3$

Aufgabe 6.6

a)



$\Rightarrow s_2 = g_1$

$g(n) = g_2(n) = s_2(n-2) + \frac{1}{2} s_2(n-3)$

$= g_1(n-2) + \frac{1}{2} g_1(n-3)$

$= 2s_1(n-2) + 4s_1(n-3) + s_1(n-3) + 2s_1(n-4)$

$= 2s_1(n-2) + 5s_1(n-3) + 2s_1(n-4)$

$= s_1(n) * [2\delta(n-2) + 5\delta(n-3) + 2\delta(n-4)]$

$= s(n) * h(n)$

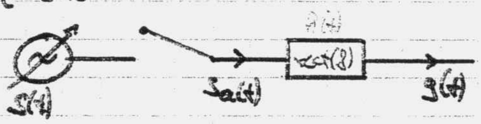
$\Rightarrow h(n) = 2\delta(n-2) + 5\delta(n-3) + 2\delta(n-4)$

b) $h(n)$ ändert sich nicht, da gilt:

$h_1(n) * h_2(n) = h_2(n) * h_1(n)$

ET IV Kleingruppe 9

Aufgabe 6.2

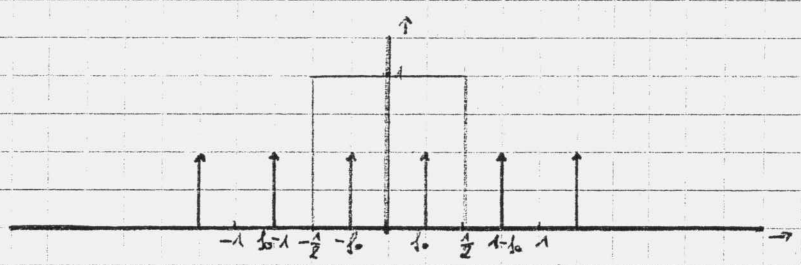


$g(t) = s_a(t) * h(t)$ $\qquad s(t) = \cos(2\pi f_0 t)$

$s_a(t) = s(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-n)$

$G(f) = \left\{ \left[\frac{1}{2}\delta(f+f_0) + \frac{1}{2}\delta(f-f_0) \right] * \sum_{k=-\infty}^{\infty} \delta(f-k) \right\} \cdot \mathrm{rect}(f)$

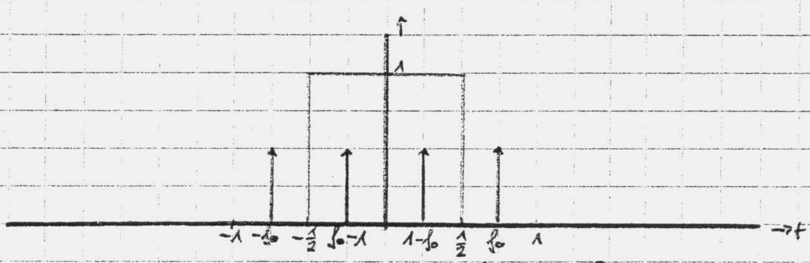
$f_0 = \frac{1}{4}$



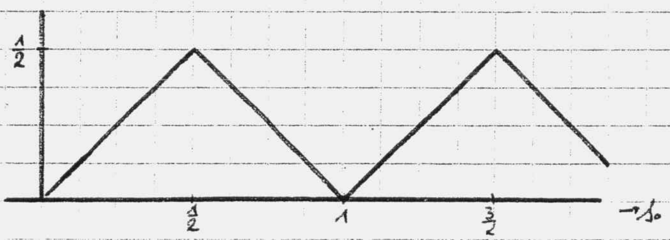
TP
$s_a(t)$
$s(t)$

$\rightarrow g(t) = \cos(2\pi f_0 t) = s(t)$ $\quad ; f_0 < \frac{1}{2}$

$f_0 = \frac{3}{4}$



$\rightarrow g(t) = \cos(2\pi(1-f_0)t)$ $\quad \frac{1}{2} < f_0 < \frac{3}{4}$



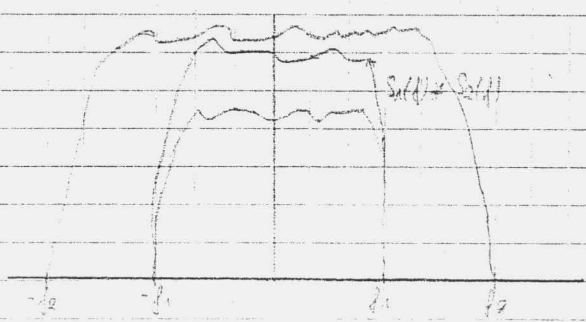
Aufgabe 6.4

$g(t) = s_1(t) \cdot s_2(t)$

$f_G = f_{G1} = 500$

$f_S \geq 2 f_G$

$\Rightarrow T \leq \frac{1}{2 f_{G1}} = 10^{-3}$



6.7
6.8

Hausaufgabe 10

Aufgabe 6.13

a) $s(n) = \left(\frac{1}{2}\right)^{n+1} \varepsilon(n+3)$

$= \left(\frac{1}{2}\right)^{-2} \left(\frac{1}{2}\right)^{n+3} \varepsilon(n+3)$

$= \left[\left(\frac{1}{2}\right)^{n} \varepsilon(n)\right] * \delta(n+3) \cdot 4$

$S(z) = 4 \cdot z^{3} \frac{1}{1-\left(\frac{1}{2}\right) z^{-1}} = \frac{4z^{4}}{z-\frac{1}{2}}$

KB: $|z| > \frac{1}{2}$

Polstelle: $z_p = \frac{1}{2}$

Nullstellen: $z_n = 0$ (4-fach)

b) $s(n) = 2^{n} \varepsilon(-n) + \left(\frac{1}{4}\right)^{n} \varepsilon(n-1)$

$= \left[2 \cdot 2^{n} \cdot \varepsilon(-n-1)\right] * \delta(n-1) + \left[\frac{1}{4} \left(\frac{1}{4}\right)^{n} \varepsilon(n)\right] * \delta(n-1)$

$S(z) = -2 \frac{1}{1-2z^{-1}} z^{-1} + \frac{1}{4} \frac{1}{1-\frac{1}{4}z^{-1}} z^{-1} = \frac{\frac{7}{4}z}{(z-\frac{1}{4})(2-z)}$

KB: $\frac{1}{4} < |z| < 2$

Polstellen: $z_{p1} = \frac{1}{4}$; $z_{p2} = 2$

Nullstelle: $z_n = 0$

c) $s(n) = \left(\frac{1}{3}\right)^{n} \cdot \varepsilon(n) * \delta(n-2)$

$S(z) = \frac{1}{1-\frac{1}{3}z^{-1}} z^{-2} = \frac{1}{z(z-\frac{1}{3})}$

KB: $|z| > \frac{1}{3}$

Polstellen: $z_{p1} = 0$; $z_{p2} = \frac{1}{3}$

Aufgabe 6.14

$S(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}} = \frac{1-z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}$

$= -\frac{1}{2} \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{3}{2} \frac{1}{1+\frac{1}{2}z^{-1}}$

$s(n) = -\frac{1}{2} \left(\frac{1}{2}\right)^{n} \varepsilon(n) + \frac{3}{2} \left(-\frac{1}{2}\right)^{n} \varepsilon(n)$, da rechtsseitige Funktion

ET IV Kleingruppe 11

Aufgabe 6.15

aus a): reell ⇒ Pole und Nullstellen treten nur konjugiert komplex auf

aus b)+d): $z_{p1} = \frac{1}{2} e^{j\frac{\pi}{3}} \Rightarrow z_{p2} = \frac{1}{2} e^{-j\frac{\pi}{3}}$

aus c): $X(z) = K \cdot \frac{z^2}{(z - \frac{1}{2} e^{j\frac{\pi}{3}})(z - \frac{1}{2} e^{-j\frac{\pi}{3}})}$

aus e):
$$X(1) = \frac{K}{(1 - \frac{1}{2} e^{j\frac{\pi}{3}})(1 - \frac{1}{2} e^{-j\frac{\pi}{3}})}$$
$$= \frac{K}{1 - \frac{1}{2} e^{j\frac{\pi}{3}} - \frac{1}{2} e^{-j\frac{\pi}{3}} + \frac{1}{4}}$$
$$= \frac{K}{1 - \cos(\frac{\pi}{3}) + \frac{1}{4}} = \frac{K}{1 - \frac{1}{2} + \frac{1}{4}} = \frac{K}{3/4} \stackrel{!}{=} \frac{8}{3} \Rightarrow K = 2$$

$$\Rightarrow X(z) = \frac{2z^2}{(z - \frac{1}{2} e^{j\frac{\pi}{3}})(z - \frac{1}{2} e^{-j\frac{\pi}{3}})}, \quad KB: |z| > \frac{1}{2}$$

Aufgabe 6.16

a)+b) Teil 1

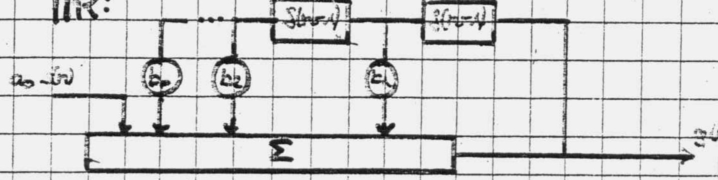
$$H_1(z) = \frac{1}{1 - \frac{5}{3} z^{-1} - \frac{37}{36} z^{-2} - \frac{5}{18} z^{-3} + \frac{1}{36} z^{-4}} = \frac{G_1(z)}{S_1(z)}$$
$$= \frac{\sum_{q=0}^{Q} a_q z^{-q}}{1 - \sum_{p=1}^{P} b_p z^{-p}}$$

$$\Rightarrow b_1 = \frac{5}{3}, \quad b_2 = \frac{37}{36}, \quad b_3 = \frac{5}{18}, \quad b_4 = -\frac{1}{36}$$

$$a_0 = 1$$

$$g_1(n) = \sum_{q=0}^{Q} a_q s_1(n-q) + \sum_{p=1}^{P} b_p g_1(n-p)$$
$$= s_1(n) + \frac{5}{3} g_1(n-1) - \frac{37}{36} g_1(n-2) + \frac{5}{18} g_1(n-3) - \frac{1}{36} g_1(n-4)$$

IIR:

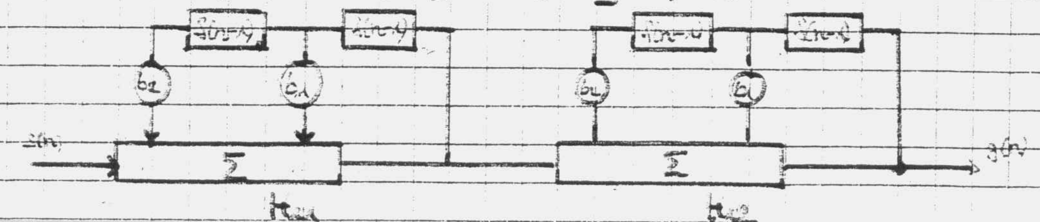


c) nach Ermittlung der Polstellen

$$H_1(z) = \frac{1}{1 - z^{-1} + \frac{1}{4} z^{-2}} \cdot \frac{1}{1 - \frac{2}{3} z^{-1} + \frac{1}{9} z^{-2}} = H_{11}(z) \cdot H_{12}(z)$$

für $H_{11}(z)$: $b_1 = 1, \quad b_2 = -\frac{1}{4}, \quad a_0 = 1$

$H_{12}(z)$: $b_1 = \frac{2}{3}, \quad b_2 = -\frac{1}{9}, \quad a_0 = 1$



d) $H_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1-\frac{1}{3}z^{-1}} \cdot \frac{1}{1-\frac{1}{3}z^{-1}} = H_{13}(z) \cdot H_{14}(z)$

für $H_{13}(z)$: $b_1 = \frac{1}{2}$, $a_0 = 1$

$H_{14}(z)$: $b_1 = \frac{1}{3}$, $a_0 = 1$

a) + b) Teil 2

$$H_2(z) = \frac{1}{1-\frac{3}{2}z^{-1}+2z^{-2}-\frac{5}{4}z^{-3}+\frac{1}{2}z^{-4}} = \frac{G_2(z)}{S_2(z)}$$

$$g_2(n) = s_2(n) + \frac{3}{2}g_2(n-1) - 2g_2(n-2) + \frac{5}{4}g_2(n-3) - \frac{1}{2}g_2(n-4)$$

$$H_2(z) = \frac{1}{1-z^{-1}+\frac{1}{2}z^{-2}} \cdot \frac{1}{1-\frac{1}{2}z^{-1}+z^{-2}}$$

$$= \frac{1}{1-\frac{1+i}{2}z^{-1}} \cdot \frac{1}{1-\frac{1-i}{2}z^{-1}} \cdot \frac{1}{1-\frac{1+i\sqrt{15}}{2}z^{-1}} \cdot \frac{1}{1-\frac{1-i\sqrt{15}}{2}z^{-1}}$$

6.14 a)–d)

7.1