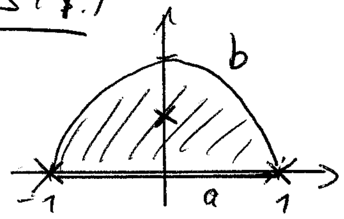
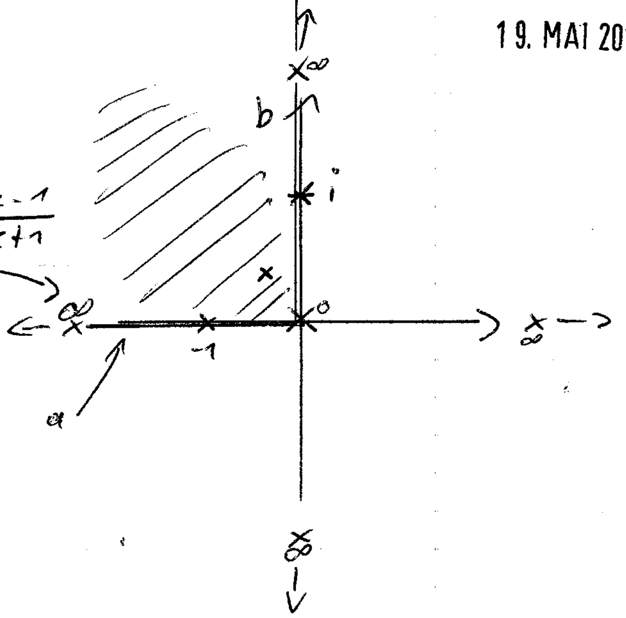
19. MAI 2010

AM4 KGÜ III

B14.)



$f(z) = \frac{z-1}{z+1}$



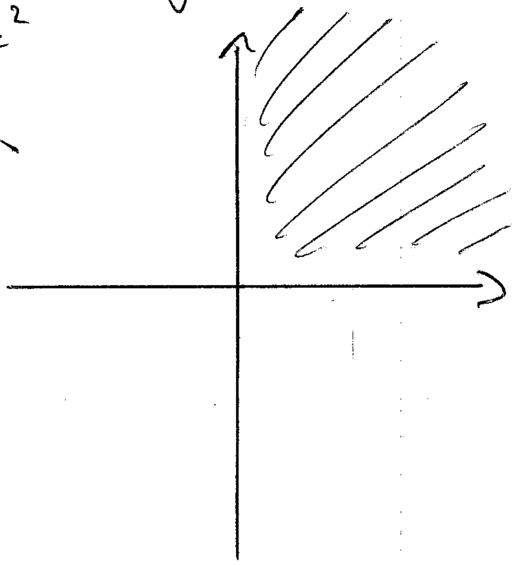
$f(1) = 0 \qquad f(-1) = \infty$

$f(0) = -1$

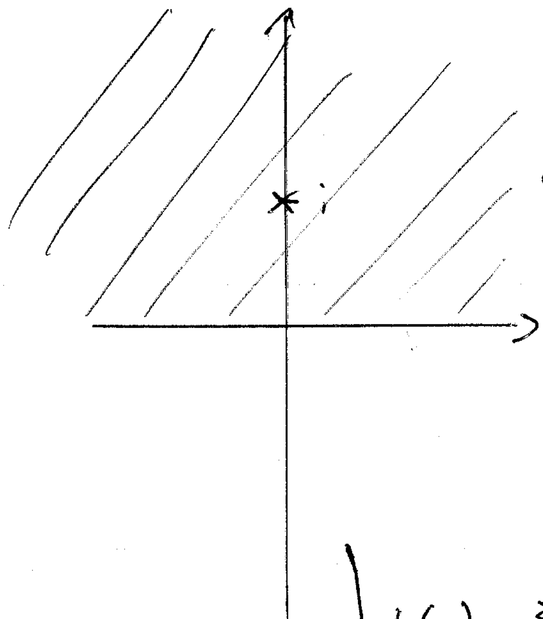
$f(i) = i$

$f\left(\frac{i}{2}\right) = -\frac{3}{5} + \frac{4}{5} i$

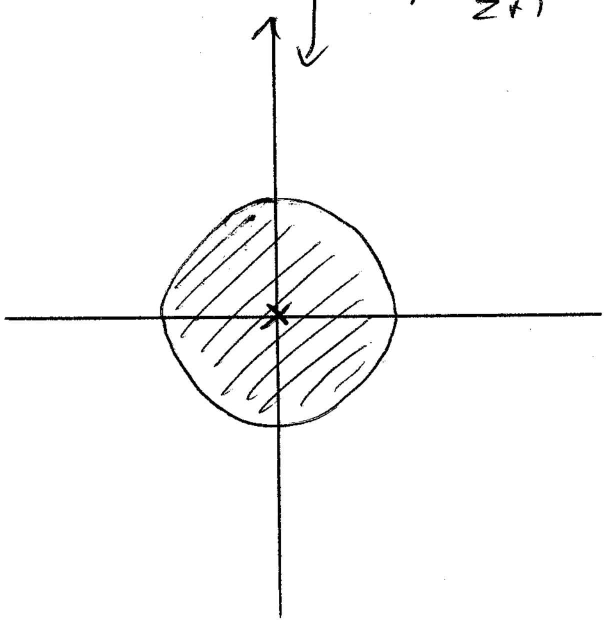
$g(z) = (-i) \cdot z$



$h(z) = z^2$



$k(z) = \frac{z-i}{z+i}$!



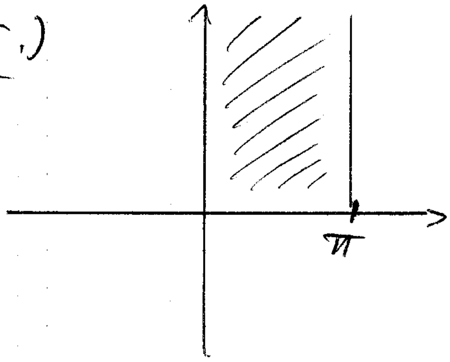
(ungefähres Klausurniveau)

$$k(z) = k(h(g(f(z))))$$

$$= \frac{(z-1)^2 - i(z+1)^2}{(z-1)^2 + i(z+1)^2}$$

ist gesuchte konforme Abb.

B 15.)



$f(z) = e^{iz}$

$\approx$ e^z für waagerechte Streifen

e^{iz} für senkrechte Streifen

$z = x + iy$

$\Rightarrow f(z) = e^{ix} \cdot e^{-y}$

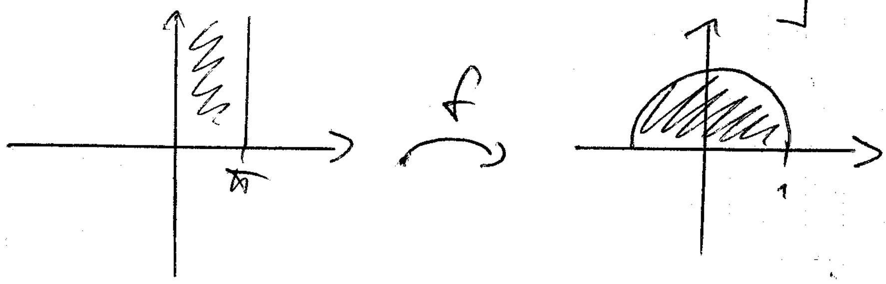
wegen $0 \leq Re(z) < 2\pi$ ist f injektiv

surjektiv (, da e^z) $\Rightarrow$ bijektiv

wegen $Im(z) \geq 0 \Rightarrow |f(z)| \leq 1$

wegen $0 \leq Re(z) < \pi \Rightarrow \arg(f(z)) < \pi$

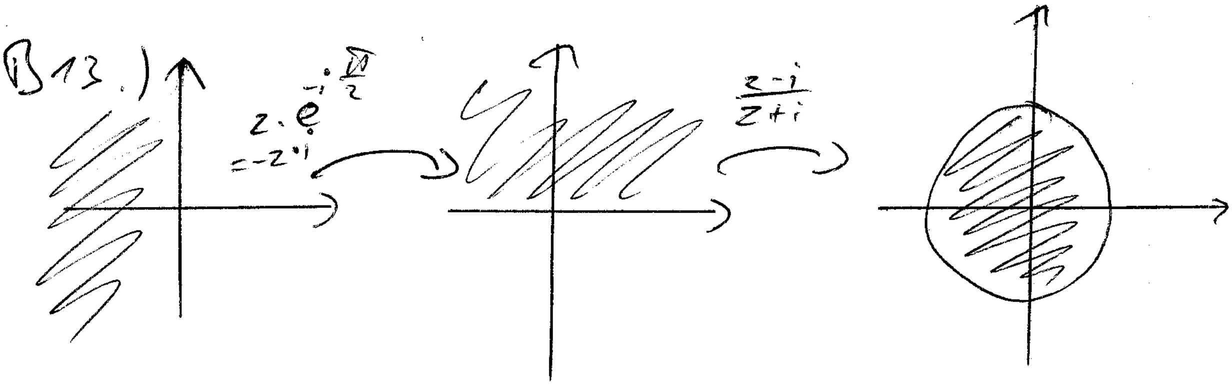
$\Rightarrow$



weiter wie B 14.):

$$n(z) = l(f(z)) = \frac{(e^{iz}-1)^2 - i(e^{iz}+1)^2}{(e^{iz}-1)^2 + i(e^{iz}+1)^2}$$

B13.)



$\Rightarrow \varphi(z) = \underline{\underline{\frac{z+1}{z-1}}}$

B16.)

$$\int_\Gamma f(z)\,dz = \int_{t_0}^{t_1} f(\gamma(t)) \cdot \gamma'(t)\,dt$$

wobei $\gamma(t)$ Parametrisierung von Γ
wobei alles hinreichend gutartig

a$_1$) $\int_{\Gamma_1} (z^2+1)\,dz$ mit Γ_1 als Strecke von i nach $1-i$

$t \mapsto (1-t)\cdot i + t\cdot(1-i)$
$= i + t(1-2i)$

$$= \int_0^1 \{[i+t(1-2i)]^2 + 1\} \cdot (1-2i)\,dt$$

$$= (1-2i) \cdot \int_0^1 \cancel{-1} + 2t(1-2i)\cdot i + t^2 \underbrace{(1-2i)^2}_{1-2i-4} \cancel{+1}\,dt$$

$$= (1-2i) \cdot \int_0^1 4t - 3t^2 + i(2t - 4t^2)\,dt$$

$$= (1-2i) \cdot (2 - 1 + i(1 - \tfrac{4}{3})) = \underline{\underline{\tfrac{1}{3} - i\tfrac{7}{3}}}$$

b.) $\int_{\Gamma_2} |z|^2 dz$ Γ_1 ist von 1 nach i führender Einheitskreisbogen
$t \mapsto e^{it}$

$$= \int_0^{\pi/2} |e^{2it}| \cdot i e^{it} \, dt$$
$$= e^{it} \Big|_0^{\pi/2} \underline{\underline{= i - 1}}$$

c.) $\int_{\Gamma_3} \bar{z} \, dz$ Γ_3 ist die im negativen Sinn durchlaufene Ellipse
$\{ z = x + iy : b^2x^2 + a^2y^2 = a^2b^2 ; a,b > 0 \}$
$t \mapsto a \cdot \sin(t) + i b \cdot \cos(t) \quad t \in [0, 2\pi)$

$$= \int_0^{2\pi} (a \cdot \sin(t) - ib\cos(t))(a \cdot \cos(t) - ib\sin(t)) \, dt$$
$$= \int_0^{2\pi} (a^2 - b^2)\sin(t)\cos(t) - iab\underbrace{(\sin^2(t) + \cos^2(t))}_{} dt$$
$$= (a^2 - b^2)\left[\frac{1}{2}\sin^2(t)\right]_0^{2\pi} - iabt\Big|_0^{2\pi} \qquad \int u u' dx = \frac{1}{2} u^2$$
$$\underline{\underline{= -2\pi a b i}}$$

B17.) K der von $-i\pi$ nach $i\pi$ führende Kreisbogen
$$t \mapsto \pi e^{it} \qquad t \in \left[-\frac{\pi}{2} ; +\frac{\pi}{2}\right]$$

$$\int_K z \cdot \cos(|z|) dz = \int_{-\pi/2}^{\pi/2} \pi e^{it} \cos(\pi) \, i\pi e^{it} \, dt$$
$$= \int_{-\pi/2}^{\pi/2} -i\pi^2 e^{2it} \, dt = -\pi^2 \left(\frac{1}{2} e^{2it}\right)_{-\pi/2}^{\pi/2}$$
$$= -\frac{\pi^2}{2}\left(e^{i\pi} - e^{-i\pi}\right) \underline{\underline{= 0}}$$