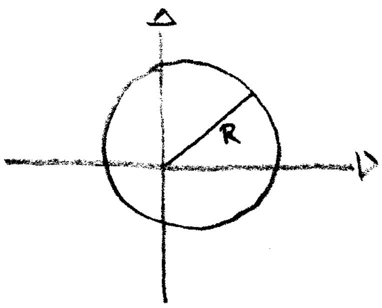
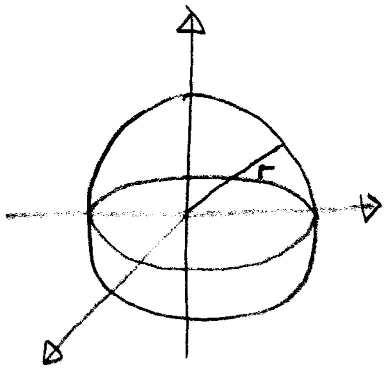
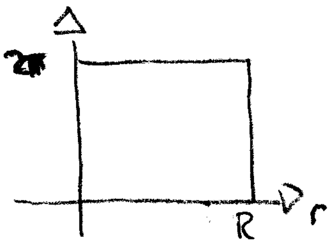
Wichtiges bei Problemen mit sphärischer Symmetrie
(POLARKOORDINATEN)







$$r = \sqrt{x^2 + y^2}$$

$$\varphi = \arctan\left(\frac{y}{x}\right)$$

$$\frac{\partial r}{\partial x} = \frac{\cancel{1}}{\cancel{2}} \frac{1}{\sqrt{x^2+y^2}} \cancel{2} x = \frac{x}{r} = \frac{\cancel{r} \cos\varphi}{\cancel{r}} = \underline{\underline{\cos\varphi}}$$

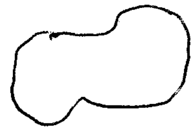
$$\frac{\partial \varphi}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \underbrace{(-1)\frac{y}{x^2}}_{\frac{d}{dx}\left(\frac{y}{x}\right)} = (-1)\frac{\cancel{x^2}}{x^2+y^2} \frac{y}{\cancel{x^2}} = \frac{-y}{r^2} = \frac{-\sin\varphi}{r}$$

analog: $\dfrac{\partial \varphi}{\partial y} = \dfrac{\cos\varphi}{r}$

Integration von Funktionen mehrerer Veränderlicher

(i)

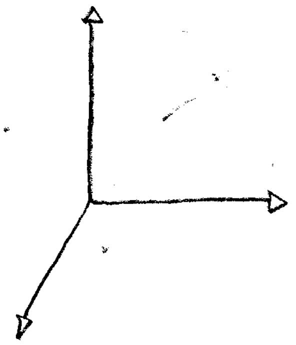
$$\rho : \underset{\substack{\cap \\ \mathbb{R}^3}}{G} \longrightarrow \mathbb{R}$$ Ladungsdichteverteilung



G

$$Q = \int_G \rho(x,y,z)\, dx\, dy\, dz$$ (Gesamtladung)

(ii)

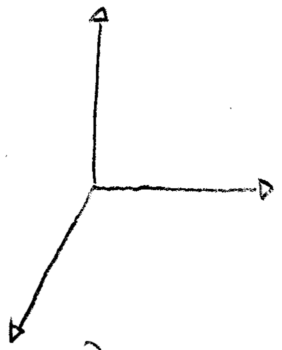


E elektrisches Feld

$E : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$$\Psi = \int_F \underline{E} \cdot \underline{n}\, d\omega$$

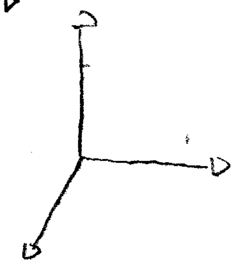
Normalenvektor an F



$=: \partial G$

Gauß:

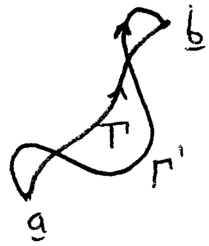
$$\int_F \underline{E}\,\underline{n}\, d\omega = \int_G \operatorname{div} \underline{E}\, dx\, dy\, dz$$



$= \partial G$

Rand eines Gebiets

(iii)

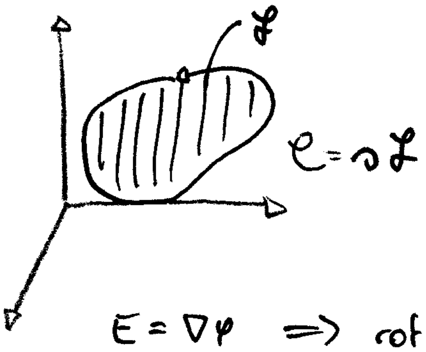


$$U = \int_\Gamma \underline{E}\, d\underline{x}$$

$$\underline{E} = \nabla \varphi \;:\; \oint_C \underline{E} \cdot d\underline{x} = 0$$

[...]

Stokes'scher Satz



$$\oint_{\mathcal{C}} \underline{H}\, d\underline{x} = \int_{\mathcal{F}} \operatorname{rot} \underline{H} \cdot \underline{n}\, d\omega$$

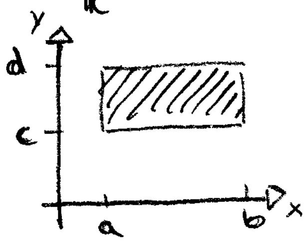
$$\operatorname{rot} \underline{H} = \nabla \times \underline{H}$$

$$E = \nabla \varphi \implies \operatorname{rot} E = \underbrace{\operatorname{rot} \nabla}_{=0} \varphi \quad \text{falls } \varphi \text{ glatt}$$

PARAMETERINTEGRALE

$f : \mathcal{G} \longrightarrow \mathbb{R}$ stetig, $\mathcal{G} \subset \mathbb{R}^2$

1)

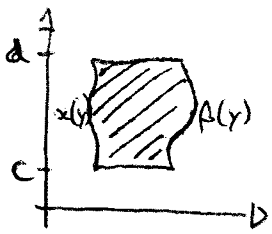


$\mathcal{G} = (a,b) \times (c,d)$

$$\int_{\mathcal{G}} f(x,y)\, dx\, dy = \int_c^d \left\{ \int_a^b f(x,y) dx \right\} dy$$

Reihenfolge kann vertauscht werden!

2)



$$\int_{\mathcal{G}} f(x,y)\, dx\, dy = \int_c^d \underbrace{\left(\int_{\alpha(y)}^{\beta(y)} f(x,y)\, dx \right)}_{= F(y)} dy$$

Def.: Funktionen der Art:

$$F(y) = \int_{\alpha(y)}^{\beta(y)} f(x,y) dx$$

heißt Parameterintegral

(Skript S 2[illegible] Satz 52)

Beweis:

(i) $\underbrace{|F(y+h) - F(y)|}_{\text{z.Z.} \downarrow\, 0 \text{ für } h \to 0} = \left| \int_a^b \big(f(x,y+h) - f(x,y) \big) dx \right|$

$$\leq \int_a^b \underbrace{|f(x,y+h) - f(x,y)|}_{o(h)}\, dx$$

$$\leq (b-a)\, o(h) \longrightarrow 0 \quad \text{für } h \to 0$$

(ii) $$F(y+h) - F(y) = \int_a^b \underbrace{f(x,y+h) - f(x,y)}_{f_y^*(x,y+\tau h)h}\, dx \qquad \tau \in (0,1)$$

Mittelwertsatz:

$$\Rightarrow \frac{F(y+h)-F(y)}{h} = \int_a^b \underbrace{f_y^*(x,y+\tau h)}\, dy$$

$h \to 0$ aus der Stetigkeit

$F'(y)$ $\qquad\qquad$ $f_y^*(x,y)$

$$\Rightarrow F'(y) = \int_a^b f_y(x,y)\, dx$$

LEIBNITZ - REGEL

gut für allgemeine Parameterintegrale

Beispiel:

- $$\frac{d}{dx} \int_0^{x \sin x} e^{-t^2}\, dt \overset{\text{Leibnitzregel}}{=} e^{-(x\sin x)^2} \frac{d}{dx}(x \sin x) = e^{-(x \sin x)^2}\left(\sin x + x \cos x\right)$$

- $$\frac{d}{dx} \int_0^{x \sin x} x t e^{-t^2}\, dt = \int_0^{x \sin x} t e^{-t^2}\, dt + x t e^{-t^2}\Big|_{t = x \sin x}$$

$$\boxed{t e^{-t^2} = \frac{d}{dt}\left(-\frac{1}{2} e^{-t^2}\right)}$$

$$= -\frac{1}{2} e^{-t^2}\Big]_0^{x \sin x} \cancel{+ x^2 \sin x}\, e^{-(x \sin x)^2}$$

$*$

$$= -\frac{1}{2}\left(e^{-(x \sin x)^2} - 1\right) + \cancel{x} \sin x) e^{-(x \sin x)^2}$$

$\frac{d}{dx}(x$

$*$ $\frac{d}{dx}(x \sin x)$