Summary Modeling and Simulation of Complex Power Systems WS 2013/14[1]
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1 Natural Coupling

- circuit composed of b branches and n nodes
- calculation of $2b$ unknowns (voltage and current of each branch)

explicit variable: known, e.g. $x = 2$

implicit variable: unknown, to calculate, e.g. $x^2 = 2^x$

1.1 Nodal Analysis (NA)

- one node («0») as reference potential (known)
- voltage of each of $n-1$ nodes $\vec{x}$ to calculate
- $n-1$ equations to solve
- best option for circuit simulation
- information of electronic circuit fully contained in voltage
- $\mathbf{G} \cdot \vec{x} = \vec{s}$ with nodal conductance matrix $\mathbf{G}$ and current injection vector $\vec{s}$

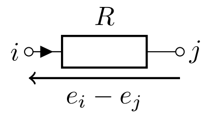
1.1.1 by inspection

build nodal conductance matrix $\mathbf{G}$ and current injection vector $\vec{s}$ from node equations

1.1.2 matrix stamp

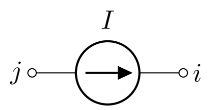
construction of the nodal conductance matrix $\mathbf{G}$ and current injection vector $\vec{s}$ by components

resistance R between nodes i and j:



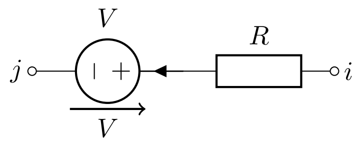
$$\mathbf{G}_{sub} = \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{bmatrix} \frac{1}{R} & \frac{-1}{R} \\ \frac{-1}{R} & \frac{1}{R} \end{bmatrix} \end{array}$$

ideal current source I between nodes i and j: (injecting in node i)



$$\vec{s}_{sub} = \begin{array}{c} i \\ j \end{array} \begin{bmatrix} I \\ -I \end{bmatrix}$$

real voltage source V with in series resistance R between nodes i and j: (with + at node i)

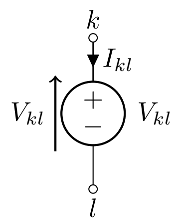


$$\mathbf{G}_{sub} = \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{bmatrix} \frac{1}{R} & \frac{-1}{R} \\ \frac{-1}{R} & \frac{1}{R} \end{bmatrix} \end{array} \text{ and } \vec{s}_{sub} = \begin{array}{c} i \\ j \end{array} \begin{bmatrix} \frac{V}{R} \\ -\frac{V}{R} \end{bmatrix}$$

1.2 Modified Nodal Analysis (MNA)

- extension of NA for modeling ideal voltage sources
- important to represent real devices like controlled power supplies
- add one equation (voltage between nodes) and one unknown (current through the ideal voltage source)
- nodal conductance matrix $\mathbf{G}$ with $(n-1+v_{ideal}) \times (n-1+v_{ideal})$
- source vector $\vec{s}$ with $(n-1+v_{ideal}) \times 1$
- voltage of nodes and currents of ideal voltage sources $\vec{x}$ to calculate
- apply matrix stamp for every ideal voltage source

ideal voltage source V_{kl} between nodes k and l:



$$\mathbf{G}_{sub} = \begin{array}{c c} & \begin{array}{ccc} k & l & \; \end{array} \\ \begin{array}{c} k \\ l \\ \; \end{array} & \left[\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & -1 \\ \hline 1 & -1 & 0 \end{array}\right] \end{array}, \; \vec{x}_{sub} = \begin{array}{c} k \\ l \\ \; \end{array} \left[\begin{array}{c} 0 \\ 0 \\ \hline I_{kl} \end{array}\right] \text{ and } \vec{s}_{sub} = \begin{array}{c} k \\ l \\ \; \end{array} \left[\begin{array}{c} 0 \\ 0 \\ \hline V_{kl} \end{array}\right]$$

[1] please report errors to robert.uhl@rwth-aachen.de

1.3 Solution methods

- Gaussian elimination: solution $\vec{x}$ by processing $\mathbf{G}$ and $\vec{s}$
- LU factorization: useful for multiple solutions with fixed $\mathbf{G}$ and changing $\vec{s}$ (see resistive companion)

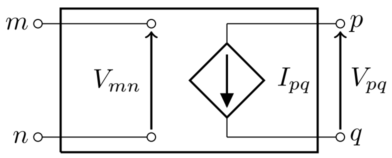
1.4 Controlled sources

- 4 terminal device
- 2 terminals for controlling variable
- 2 terminals for controlled variable

examples:
- small signal representation of transistors
- operational amplifiers
- ideal transformers

1.4.1 Voltage Controlled Current Source (VCCS)

- directly representable in nodal analysis
- characteristic equation: $I_{pq} = g \cdot V_{mn}$ with trans-conductance g and $V_{mn} = V_m - V_n$

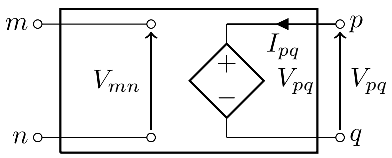


matrix stamp:

$$\mathbf{G}_{sub} = \begin{array}{cc} & \begin{array}{cc} m & n \end{array} \\ \begin{array}{c} p \\ q \end{array} & \begin{bmatrix} g & -g \\ -g & g \end{bmatrix} \end{array}$$

1.4.2 Voltage Controlled Voltage Source (VCVS)

- not directly representable in nodal analysis
- characteristic equation: $V_{pq} = \alpha \cdot V_{mn}$ with $V_{pq} = V_p - V_q$ and $V_{mn} = V_m - V_n$



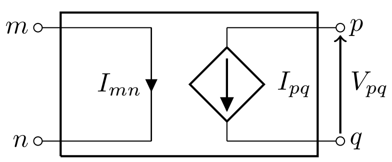
for nodal analysis:
1. matrix stamp for VCCS with gain $g = \frac{-\alpha}{R_{int}}$
2. matrix stamp for parallel small internal resistance R_{int} between nodes p and q

matrix stamp for modified nodal analysis:

$$\mathbf{G}_{sub} = \begin{array}{cc} & \begin{array}{ccccc} m & n & p & q & \end{array} \\ \begin{array}{c} m \\ n \\ p \\ q \\ \end{array} & \left[\begin{array}{cccc|c} & & & & 0 \\ & & & & 0 \\ & & & & 1 \\ & & & & -1 \\ \hline -\alpha & \alpha & 1 & -1 & 0 \end{array}\right] \end{array} \text{ and } \vec{x}_{sub} = \left[\begin{array}{c} \\ \hline I_{pq} \end{array}\right]$$

1.4.3 Current Controlled Current Source (CCCS)

- not directly representable in nodal analysis
- characteristic equation: $I_{pq} = \beta \cdot I_{mn}$



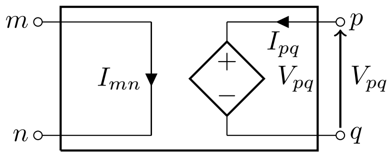
for nodal analysis:
1. matrix stamp for VCCS with gain $g = \frac{\beta}{R_{mn}}$
2. matrix stamp for small resistance R_{mn} between nodes m and n

matrix stamp for modified nodal analysis:

$$\mathbf{G}_{sub} = \begin{array}{cc} & \begin{array}{ccccc} m & n & p & q & \end{array} \\ \begin{array}{c} m \\ n \\ p \\ q \\ \end{array} & \left[\begin{array}{cccc|c} & & & & 1 \\ & & & & -1 \\ & & & & \beta \\ & & & & -\beta \\ \hline 1 & -1 & 0 & 0 & 0 \end{array}\right] \end{array} \text{ and } \vec{x}_{sub} = \left[\begin{array}{c} \\ \hline I_{mn} \end{array}\right]$$

1.4.4 Current Controlled Voltage Source (CCVS)

- not directly representable in nodal analysis
- characteristic equation: $V_{pq} = r \cdot I_{mn}$ with $V_{pq} = V_p - V_q$



for nodal analysis:

1. matrix stamp for VCCS with gain $g = \frac{\beta}{R_{mn} \cdot R_{int}}$
2. matrix stamp for small resistance R_{mn} between nodes m and n
3. matrix stamp for small resistance R_{int} between nodes p and q

matrix stamp for modified nodal analysis:

$$\mathbf{G}_{sub} = \begin{array}{c} \\ m \\ n \\ p \\ q \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} m & n & p & q & & \end{array} \\ \left[\begin{array}{cccc|cc} & & & & 1 & 0 \\ & & & & -1 & 0 \\ & & & & 0 & 1 \\ & & & & 0 & -1 \\ \hline 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -r & 0 \end{array} \right] \end{array} \text{ and } \vec{x}_{sub} = \left[\begin{array}{c} \\ \hline I_{mn} \\ I_{pq} \end{array} \right]$$

2 Linear circuits transient analysis - Resistive companion

- transformation of dynamic elements in a corresponding DC equivalent circuit
- represents an iteration of an integration method

2.1 Integration methods

- integration time step Δt
- smaller time step is more accurate/better
- explicit method:
  - predict the future value of the solution by using information from the previous steps
  - may diverge for large time step
  - faster calculation
- implicit method
  - requires knowledge of a value at the future time step
  - always numerically stable/will always converge
  - more effort for calculation

2.1.1 Euler Forward (EF)

- assumes function $x(t)$ is constant between $k \cdot \Delta t$ and $(k+1) \cdot \Delta t$ and equal to value $x(k \cdot \Delta t)$
- result: $y(k+1) = y(k) + x(k) \cdot \Delta t$
- explicit method

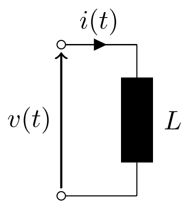
2.1.2 Euler Backward (EB)

- assumes function $x(t)$ is constant between $k \cdot \Delta t$ and $(k+1) \cdot \Delta t$ and equal to value $x((k+1) \cdot \Delta t)$
- result: $y(k+1) = y(k) + x(k+1) \cdot \Delta t$
- implicit method

2.1.3 Trapezoidal Rule (TR)

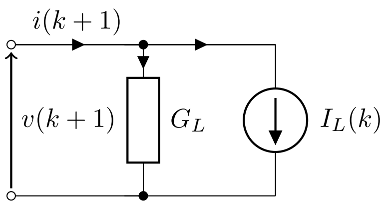
- assumes function $x(t)$ is linear between $k \cdot \Delta t$ and $(k+1) \cdot \Delta t$ with values $x(k \cdot \Delta t)$ and $x((k+1) \cdot \Delta t)$
- result: $y(k+1) = y(k) + \frac{x(k)+x(k+1)}{2} \cdot \Delta t$
- implicit method
- combination of Euler Forward and Euler Backward for better accuracy

2.2 Resistive companion formulation for an inductance



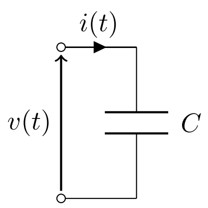
$v(t) = L \cdot \frac{di(t)}{dt}$ $\qquad \tau = \frac{L}{R}$

$i(t) = i(t_0) + \frac{1}{L} \cdot \int_{t_0}^{t} v(\tau)\, d\tau$



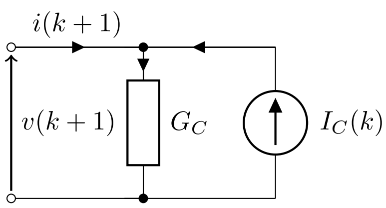
- Euler Forward: $i(k+1) = i(k) + \frac{\Delta t}{L} \cdot v(k) = I_L(k) + 0 \cdot v(k+1)$ (ideal current source)
- Euler Backward: $i(k+1) = i(k) + \frac{\Delta t}{L} \cdot v(k+1) = I_L(k) + G_L \cdot v(k+1)$
- Trapezoidal Rule: $i(k+1) = \frac{\Delta t}{2L} \cdot v(k+1) + \left(i(k) + \frac{\Delta t}{2L} \cdot v(k)\right) = G_L \cdot v(k+1) + I_L(k)$
  with $G_L = \frac{\Delta t}{2L}$ and $I_L(k) = i(k) + G_L \cdot v(k)$

2.3 Resistive companion formulation for a capacitance



$i(t) = C \cdot \frac{dv(t)}{dt}$ $\qquad \tau = R \cdot C$

$v(t) = v(t_0) + \frac{1}{C} \cdot \int_{t_0}^{t} i(\tau)\, d\tau$



- Euler Forward: $v(k+1) = v(k) + \frac{\Delta t}{C} \cdot i(k)$ (ideal voltage source)
- Euler Backward: $v(k+1) = v(k) + \frac{\Delta t}{C} \cdot i(k+1)$
  $\iff i(k+1) = \frac{C}{\Delta t} \cdot v(k+1) - \frac{C}{\Delta t} \cdot v(k) = G_C \cdot v(k+1) - I_C(k)$
- Trapezoidal Rule: $v(k+1) = v(k) + \frac{\Delta t}{2C} \cdot (i(k+1) + i(k))$
  $\iff i(k+1) = \frac{2C}{\Delta t} \cdot v(k+1) - \left(i(k) + \frac{2C}{\Delta t} \cdot v(k)\right) = G_C \cdot v(k+1) - I_C(k)$
  with $G_C = \frac{2C}{\Delta t}$ and $I_C(k) = i(k) + G_C \cdot v(k)$

3 Signal Coupling

3.1 State space

system state:
Amount of information at any time t_0 that, together with all inputs for $t \geq t_0$, uniquely determines the behaviour of the system for all $t \geq t_0$. State variables must be continuous and linearly representable.

state equations:
$\dot{\vec{x}}(t) = \vec{f}(\vec{x}, \vec{u}, t)$ explicit first order differential equations

output equations:
$\vec{y}(t) = \vec{g}(\vec{x}, \vec{u}, t)$

linear time-invariant system:
$\dot{\vec{x}}(t) = \mathbf{A} \cdot \vec{x}(t) + \mathbf{B} \cdot \vec{u}(t)$
$\vec{y}(t) = \mathbf{C} \cdot \vec{x}(t) + \mathbf{D} \cdot \vec{u}(t)$

3.2 Integration method - Predictor and corrector

improves accuracy compared with Euler-Forward or Euler-Backward

$\dot{x} = f(x, t)$
$t_k \rightarrow t_{k+1}$

predictor: *(like Euler Forward)*
$\hat{x}_{k+1} = x_k + dy_1 \cdot \Delta t$ with $dy_1 = f(x_k, t_k)$

corrector: *(like Trapezoidal Rule)*
$x_{k+1} = x_k + \frac{dy_1 + dy_2}{2} \cdot \Delta t$ with $dy_2 = f(\hat{x}_{k+1}, t_{k+1})$

3.3 Automatic state equations for circuits

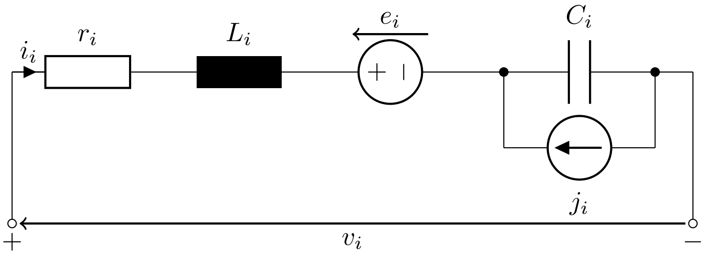
- combines signal coupling and natural coupling
- uses branch parameters and circuit topology (natural coupling)
- automatically generates state space model (signal coupling)
- numerical integration technique can be selected after formulation, which simplifies programming of variable time-step integration techniques
- simple development and implementation of simulation

node incidence matrix $\mathbf{A}_a$: $((n) \times (b))$
- each row corresponds to a node (n)
- each column corresponds to a branch (b) and contains two non-zero elements (1 and -1)
- for positive terminal of branch j connected to node i, $a_{ij} = 1$
- for negative terminal of branch j connected to node i, $a_{ij} = -1$
- $\mathbf{A}_a \cdot \vec{i}_{br} = \vec{0}$ with branch currents $\vec{i}_{br}$ (from *KCL*)

$$\mathbf{A}_a \mathrel{\widehat{=}} \tilde{\mathbf{A}}_a = \left[\begin{array}{c|c} \mathbf{I}_{(n-1)\times(n-1)} & \hat{\mathbf{A}}_{(n-1)\times(b-n+1)} \\ \hline \mathbf{0}_{(1)\times(n-1)} & \mathbf{0}_{(1)\times(b-n+1)} \end{array}\right] \qquad \textit{(by matrix operations)}$$

basic loop matrix $\mathbf{B}_b$: $((b - n + 1) \times (b))$
- each row corresponds to a mesh in the circuit (b - n + 1)
- each column corresponds to a branch (b)
- -1 and 1 represents the direction of the branch voltage in the mesh
- $\mathbf{B}_b \cdot \vec{v}_{br} = \vec{0}$ with branch voltages $\vec{v}_{br}$
- $\mathbf{B}_b^T = \begin{bmatrix} -\hat{\mathbf{A}} \\ \mathbf{I} \end{bmatrix}$
- $\vec{i}_{br} = \mathbf{B}_b^T \cdot \vec{i}_x$ with independent branch currents $\vec{i}_x$

branch model: *(not suitable for representing all systems)*



$v_i = r_i \cdot i_i + L_i \cdot \frac{di_i}{dt} + e_i + P_i \cdot \int (i_i + j_i) dt$

state variables: *(length of single sub-vectors is equal to number of branches)*

$$\vec{x} = \begin{bmatrix} \vec{q}_c \\ \vec{i}_x \end{bmatrix}$$

input variables:

$$\vec{u} = \begin{bmatrix} \vec{\jmath}_{br} \\ \vec{e}_{br} \end{bmatrix}$$

output variables:

$$\vec{y} = \begin{bmatrix} \vec{i}_{br} \\ \vec{v}_{br} \end{bmatrix}$$

resulting state space matrices:

$$\mathbf{A} = \begin{array}{c} \\ \dot{\vec{q}}_c \\ \dot{\vec{i}}_x \end{array} \begin{array}{c} \begin{array}{cc} \vec{q}_c & \vec{i}_x \end{array} \\ \left[\begin{array}{c|c} \mathbf{0} & \mathbf{M}^T \cdot \mathbf{B}_b^T \\ \hline -\mathbf{L}_x^{-1} \cdot \mathbf{P}_x & -\mathbf{L}_x^{-1} \cdot \left(\mathbf{r}_x + \frac{d}{dt}\mathbf{L}_x\right) \end{array}\right] \end{array} \qquad \mathbf{B} = \begin{array}{c} \\ \dot{\vec{q}}_c \\ \dot{\vec{i}}_x \end{array} \begin{array}{c} \begin{array}{cc} \vec{\jmath}_{br} & \vec{e}_{br} \end{array} \\ \left[\begin{array}{c|c} \mathbf{M}^T & \mathbf{0} \\ \hline \mathbf{0} & -\mathbf{L}_x^{-1} \cdot \mathbf{B}_b \end{array}\right] \end{array}$$

$$\mathbf{C} = \begin{array}{c} \\ \vec{i}_{br} \\ \vec{v}_{br} \end{array} \begin{array}{c} \begin{array}{cc} \vec{q}_c & \vec{i}_x \end{array} \\ \left[\begin{array}{c|c} \mathbf{0} & \mathbf{B}_b^T \\ \hline \mathbf{P}_{br} \cdot \mathbf{M} - \mathbf{L}_{br} \cdot \mathbf{B}_b^T \cdot \mathbf{L}_x^{-1} \cdot \mathbf{P}_x & \left(\mathbf{r}_{br} + \frac{d}{dt}\mathbf{L}_{br}\right) \cdot \mathbf{B}_b^T - \mathbf{L}_{br} \cdot \mathbf{B}_b^T \cdot \mathbf{L}_x^{-1} \cdot \left(\mathbf{r}_x + \frac{d}{dt}\mathbf{L}_x\right) \end{array}\right] \end{array}$$

$$\mathbf{D} = \begin{array}{c} \\ \vec{i}_{br} \\ \vec{v}_{br} \end{array} \begin{array}{c} \begin{array}{cc} \vec{\jmath}_{br} & \vec{e}_{br} \end{array} \\ \left[\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I} - \mathbf{L}_{br} \cdot \mathbf{B}_b^T \cdot \mathbf{L}_x^{-1} \cdot \mathbf{B}_b \end{array}\right] \end{array}$$

with:

$\mathbf{r}_x = \mathbf{B}_b \cdot \mathbf{r}_{br} \cdot \mathbf{B}_b^T$

$\mathbf{L}_x = \mathbf{B}_b \cdot \mathbf{L}_{br} \cdot \mathbf{B}_b^T$ **$\mathbf{L}_x$ has to be invertible!** *(minimum one branch with L)*

$\mathbf{P}_x = \mathbf{B}_b \cdot \mathbf{P}_{br} \cdot \mathbf{M}$

$\mathbf{M}$: $m_{ij} = 1$ if capacitor j is in the branch i, else $m_{ij} = 0$ $((b) \times (\text{capacitors}))$

$\vec{q}_c = \int \vec{i}_c \, dt$ $\vec{q}_c = \mathbf{M}^T \cdot \vec{q}_{br}$

diagonal resistance matrix $\mathbf{r}_{br}$:
diagonal entries correspond to branch resistances r_i

branch inductance matrix $\mathbf{L}_{br}$:

- self-inductances L_i (diagonal entries)
- mutual-inductances M_{ij} (off-diagonal entries)

branch potential coefficient matrix $\mathbf{P}_{br}$:
diagonal entries for reciprocal capacitances $\frac{1}{C_i}$ of corresponding branch i

4 Diakoptics

- dividing (tearing) the original network into a number of isolated subnetworks
- matrix of coefficients for each small network is inverted independently
- solution of the full network obtained from matrices of subnetworks
- no current or voltage sources in the removed branches, only basic components allowed
- removed branches must not form a closed loop or contain nodes not included in the remaining network
- as many variables as were in the original problem plus as many additional variables as there are removed branches

advantages:

- reduction of computational effort (n^3)
- reduction of needed storage
- economical
- solution is obtained directly by a number of definite steps without any approximations or iterations

disadvantages:

- n additional calculations for n removed branches, may overweight the advantages of this method
- decouple the network manually

notation:

- index α: nodal quantities
- index β: mesh quantities
- index ψ: removed branch quantities

4.1 mesh current analysis

1. known voltage sources $\vec{E}_\beta$ in branches
2. unknown mesh currents $\vec{\imath}_\beta$ *(to calculate)*
3. build equivalent network with equivalent voltage sources $\tilde{\vec{e}}_\beta$
4. build removed network with equivalent current sources $\tilde{\vec{\imath}}_\psi$ and voltages across removed branches $\vec{v}_\psi$
5. build $\tilde{\vec{\imath}}_\psi$ from mesh currents $\vec{\imath}_\beta$:
   $\tilde{\vec{\imath}}_\psi = \mathbf{C}_{\psi\beta} \cdot \vec{\imath}_\beta$ with connection matrix $\mathbf{C}_{\psi\beta}$
6. build $\tilde{\vec{e}}_\beta$ from voltages across removed branches $\vec{v}_\psi$:
   $\tilde{\vec{e}}_\beta = \mathbf{B}_{\beta\psi} \cdot \vec{v}_\psi = -\mathbf{C}^t_{\beta\psi} \cdot \vec{v}_\psi$
7. build mesh current equations of equivalent network:
   $\tilde{\mathbf{Z}}_{\beta\beta} \cdot \vec{\imath}_\beta = \vec{E}_\beta + \tilde{\vec{e}}_\beta$ with block diagonal impedance matrix $\tilde{\mathbf{Z}}_{\beta\beta}$ of divided subnetworks
8. build relation between currents and voltages of removed network:
   $\tilde{\vec{\imath}}_\psi = \mathbf{Y}_{\psi\psi} \cdot \vec{v}_\psi$
9. build fundamental equations of Diakoptics:
   $\tilde{\mathbf{Z}}_{\beta\beta} \cdot \vec{\imath}_\beta = \vec{E}_\beta - \mathbf{C}^t_{\beta\psi} \cdot \vec{v}_\psi$
   $\mathbf{Y}_{\psi\psi} \cdot \vec{v}_\psi = \mathbf{C}_{\psi\beta} \cdot \vec{\imath}_\beta$
10. build solution for unknown mesh currents $\vec{\imath}_\beta$:
    $\vec{\imath}_\beta = \tilde{\mathbf{Z}}^{-1}_{\beta\beta} \cdot \left(\vec{E}_\beta - \mathbf{C}^t_{\beta\psi} \cdot \tilde{\mathbf{Y}}^{-1}_{\psi\psi} \cdot \mathbf{C}_{\psi\beta} \cdot \tilde{\mathbf{Z}}^{-1}_{\beta\beta} \cdot \vec{E}_\beta\right)$
    with $\tilde{\mathbf{Y}}_{\psi\psi} = \mathbf{Y}_{\psi\psi} + \mathbf{C}_{\psi\beta} \cdot \tilde{\mathbf{Z}}^{-1}_{\beta\beta} \cdot \mathbf{C}^t_{\beta\psi}$

4.2 nodal voltage analysis

1. known current injections $\vec{I}_\alpha$ in nodes
2. unknown nodal voltages $\vec{v}_\alpha$ *(to calculate)*
3. build equivalent network with equivalent current sources $\tilde{\vec{\imath}}_\alpha$
4. build removed network with equivalent voltage sources $\tilde{\vec{e}}_\psi$ and currents through removed branches $\vec{\imath}_\psi$
5. build $\tilde{\vec{\imath}}_\alpha$ from currents through removed branches $\vec{\imath}_\psi$:
   $\tilde{\vec{\imath}}_\alpha = \mathbf{C}_{\alpha\psi} \cdot \vec{\imath}_\psi$ with connection matrix $\mathbf{C}_{\alpha\psi}$
6. build $\tilde{\vec{e}}_\psi$ from voltages across removed branches $\vec{v}_\alpha$:
   $\tilde{\vec{e}}_\psi = \mathbf{B}_{\psi\alpha} \cdot \vec{v}_\alpha = -\mathbf{C}^t_{\psi\alpha} \cdot \vec{v}_\alpha$
7. build nodal voltage equations of equivalent network:
   $\tilde{\mathbf{Y}}_{\alpha\alpha} \cdot \vec{v}_\alpha = \vec{I}_\alpha + \tilde{\vec{\imath}}_\alpha$ with block diagonal admittance matrix $\tilde{\mathbf{Y}}_{\alpha\alpha}$ of divided subnetworks
8. build relation between voltages and currents of removed network:
   $\tilde{\vec{e}}_\psi = \mathbf{Z}_{\psi\psi} \cdot \vec{\imath}_\psi$
9. build fundamental equations of Diakoptics:
   $\mathbf{Z}_{\psi\psi} \cdot \vec{\imath}_\psi = -\mathbf{C}^t_{\psi\alpha} \cdot \vec{v}_\alpha$
   $\tilde{\mathbf{Y}}_{\alpha\alpha} \cdot \vec{v}_\alpha = \vec{I}_\alpha + \mathbf{C}_{\alpha\psi} \cdot \vec{\imath}_\psi$
10. build solution for unknown nodal voltages $\vec{v}_\alpha$:
    $\vec{v}_\alpha = \tilde{\mathbf{Y}}^{-1}_{\alpha\alpha} \cdot \left(\vec{I}_\alpha - \mathbf{C}_{\alpha\psi} \cdot \tilde{\mathbf{Z}}^{-1}_{\psi\psi} \cdot \mathbf{C}^t_{\psi\alpha} \cdot \tilde{\mathbf{Y}}^{-1}_{\alpha\alpha} \cdot \vec{I}_\alpha\right)$
    with $\tilde{\mathbf{Z}}_{\psi\psi} = \mathbf{Z}_{\psi\psi} + \mathbf{C}^t_{\psi\alpha} \cdot \tilde{\mathbf{Y}}^{-1}_{\alpha\alpha} \cdot \mathbf{C}_{\alpha\psi}$

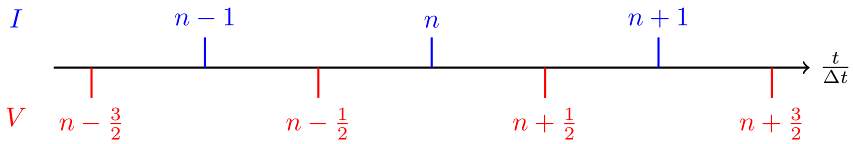
5 Latency Insertion Method (LIM)

- latency generates update equations for branch currents and node voltages
- optimally efficient algorithm, computational effort linear to size of system
- no matrix inversion needed, component by component solved independently
- could be parallelized, which increases simulation speed
- maximum time step $\Delta t_{max} < \min\left(\sqrt{L_{ij} \cdot C_i}\right)$
- useable as connection tool: many blocks solved with other methods, LIM used for connecting them
- order of calculation: $[I^0 \rightarrow V^{\frac{1}{2}} \rightarrow] I^1 \rightarrow V^{\frac{3}{2}} \rightarrow I^2 \rightarrow \ldots$

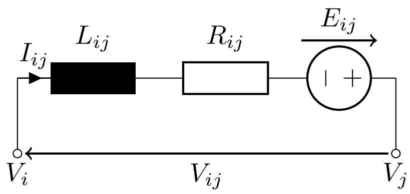
requirements for topology:

- each branch must contain an inductance, otherwise a small inductance is inserted
- each node must provide a capacitive path to ground, otherwise a small shunt capacitance is added

leap-frog algorithm:



branch algorithm:

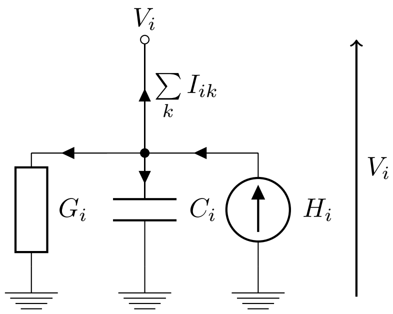


$$V_{ij} = V_i - V_j = L_{ij} \cdot \frac{dI_{ij}}{dt} + R_{ij} \cdot I_{ij} - E_{ij}$$

$$\iff V_i^{n+\frac{1}{2}} - V_j^{n+\frac{1}{2}} = L_{ij} \cdot \left(\frac{I_{ij}^{n+1} - I_{ij}^n}{\Delta t}\right) + R_{ij} \cdot I_{ij}^n - E_{ij}^{n+\frac{1}{2}}$$

$$\iff I_{ij}^{n+1} = I_{ij}^n + \frac{\Delta t}{L_{ij}} \cdot \left(V_i^{n+\frac{1}{2}} - V_j^{n+\frac{1}{2}} - R_{ij} \cdot I_{ij}^n + E_{ij}^{n+\frac{1}{2}}\right)$$

node algorithm:



$$\sum_k I_{ik} + G_i \cdot V_i + C_i \cdot \frac{dV_i}{dt} = H_i$$

$$\iff \sum_k I_{ik}^n + G_i \cdot V_i^{n+\frac{1}{2}} + C_i \cdot \left(\frac{V_i^{n+\frac{1}{2}} - V_i^{n-\frac{1}{2}}}{\Delta t}\right) = H_i^n$$

$$\iff V_i^{n+\frac{1}{2}} = \frac{\frac{C_i \cdot V_i^{n-\frac{1}{2}}}{\Delta t} + H_i^n - \sum_k I_{ik}^n}{\frac{C_i}{\Delta t} + G_i}$$

mutual inductance:

- leads to matrix connecting old and new values of both currents
- matrix has to be inverted only once

non-linear component:

- handle non-linear behaviour $i = f(v)$ of circuit elements by using iterative Newton-Raphson algorithm
- in branch: $V^{n+\frac{1}{2}} = f^{-1}(I_{ij}^{n+1})$
- at node: $I^n = f(V_i^{n+\frac{1}{2}})$
- iterations only on non-linear branches or nodes needed, non-linearity solved locally
- huge computational advantage compared to MNA for non-linear circuit components

other circuit elements:

- represented by resistive companion model
- e.g. branch capacitance or shunt inductance (Euler Backward)

6 Real-time simulation

- difficult to test a power system device under real conditions or in its working environment
- replace some simulation models of a system by one or several physical components
- controller HIL: low power levels with $\pm 10\,\mathrm{V}$
- power HIL: absorbs/sinks real power

Soft Real-time:
average response time of system is met

Hard Real-time:
requires that guaranteed response time is met

Firing Signal Averaging (FSA) method:
- averages external very high frequency signal to simulation time step
- else it can't be tracked by the simulator
- e.g. switching signal of power electronics converter

conservation of energy at system's boundaries:
- for Power Hardware In the Loop (PHIL)
- power electronics interface between HUT (Hardware Under Test) and ROS (Rest Of System)
- ROS as a model in the simulator
- stability problems/erroneous results may occur due to delays in communication between interface and simulator
- power electronics interface must be much fast than HUT to be transparent, otherwise interferences possible

algorithm for conservation of energy at system's boundary:
- power electronics hardware interface based on time-variant first order approximation (TFA) of dynamics of HUT
- this compensates for delays introduced by D/A- and A/D-conversion as well as computation